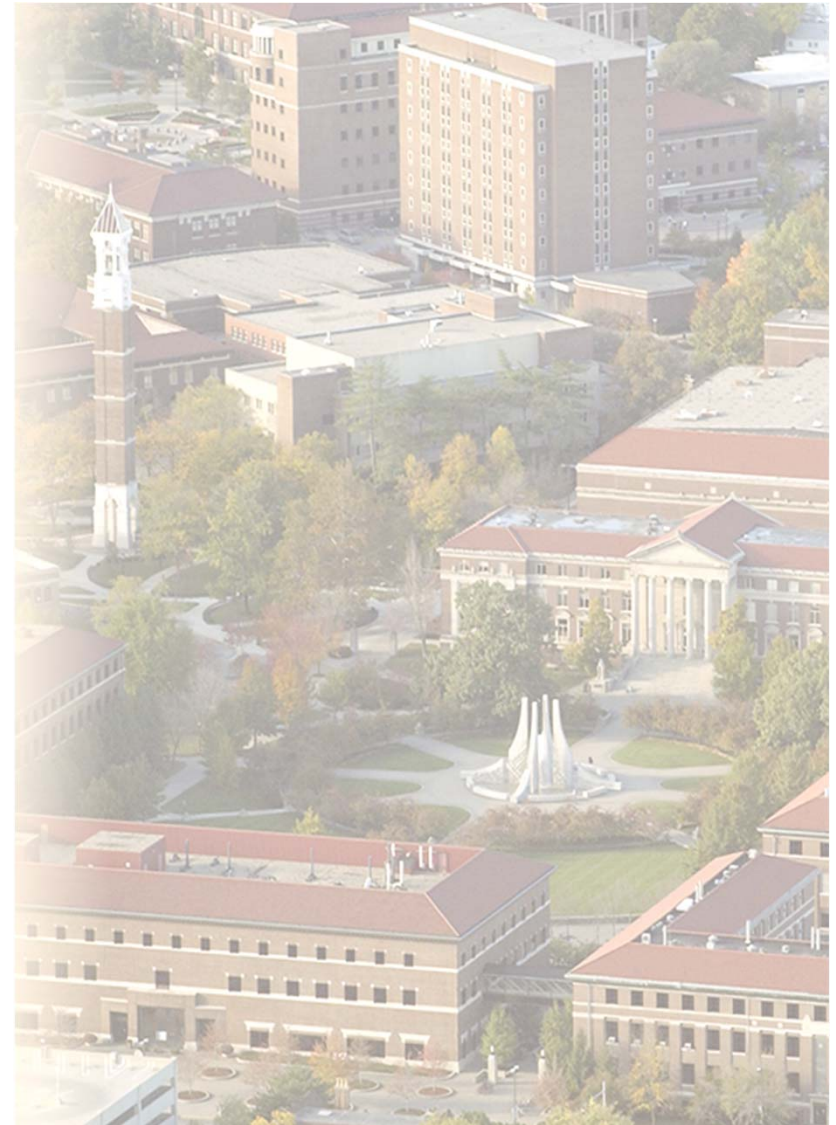


# **Promoting Affordability in Defense Acquisitions: A Multi-Period Portfolio Approach**

**NPS Acquisition Research  
Symposium**  
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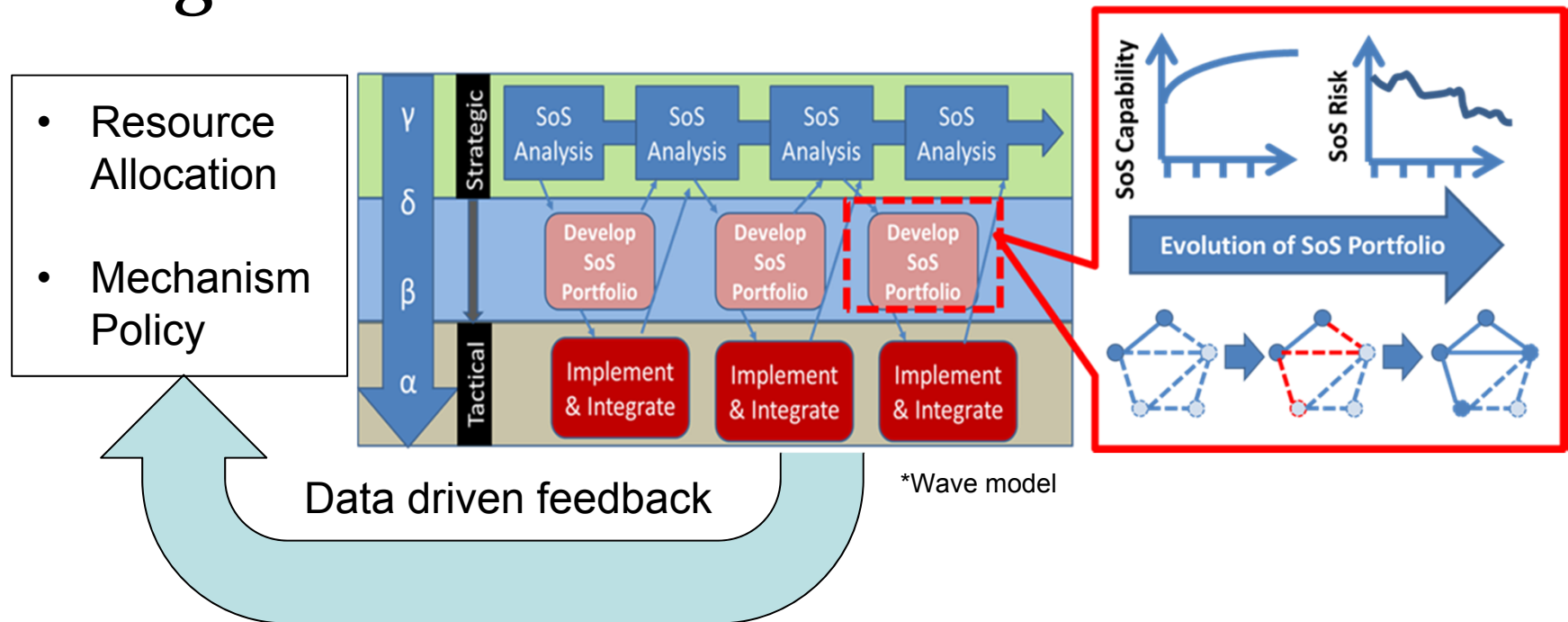
Center for Integrated Systems in Aerospace (CISA)  
Purdue University



# Overview

- The Big Picture
- A Portfolio Approach: Background
- Current Efforts
  - Robust Multi-Period Optimization
  - Policy construction w/ Mechanism Design
  - An Approximate Dynamic Programming Approach
- Future work

# The Big Picture



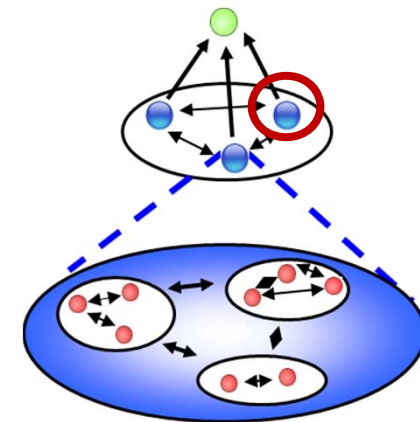
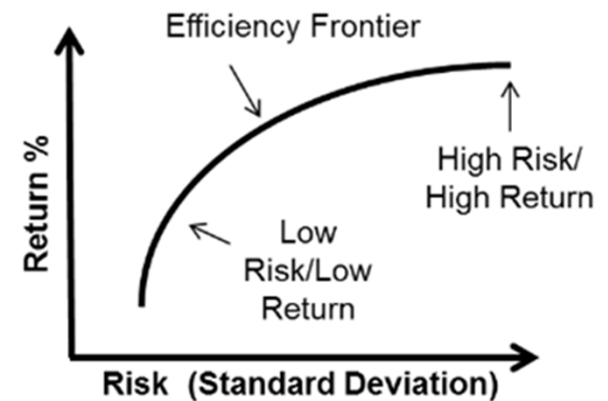
- Prior step acquisition decisions affect latter capability, risk, cost, schedule.
- Many potential interdependencies and choices in connections → larger set for AoAs (open architecture, modularity, competition)
- Very complex tradespace - how do we support acquisitions in this setting?

# Our Research Efforts

- A Robust Multi-period Optimization approach
  - All future states are assumed known w/ uncertainty bounds
  - Strategic level thinking for initial acquisition phases.
  - Determine sequence of choices based on prescribed uncertainty
- Acquisition Policy Construction
  - Apply innovations in Robust Optimization to policy design for acquisition programs
  - Utilize data from McNew survey
- Multi-Period approach using Dynamic Programming
  - Approximate dynamic programming balance decision now w/ future states

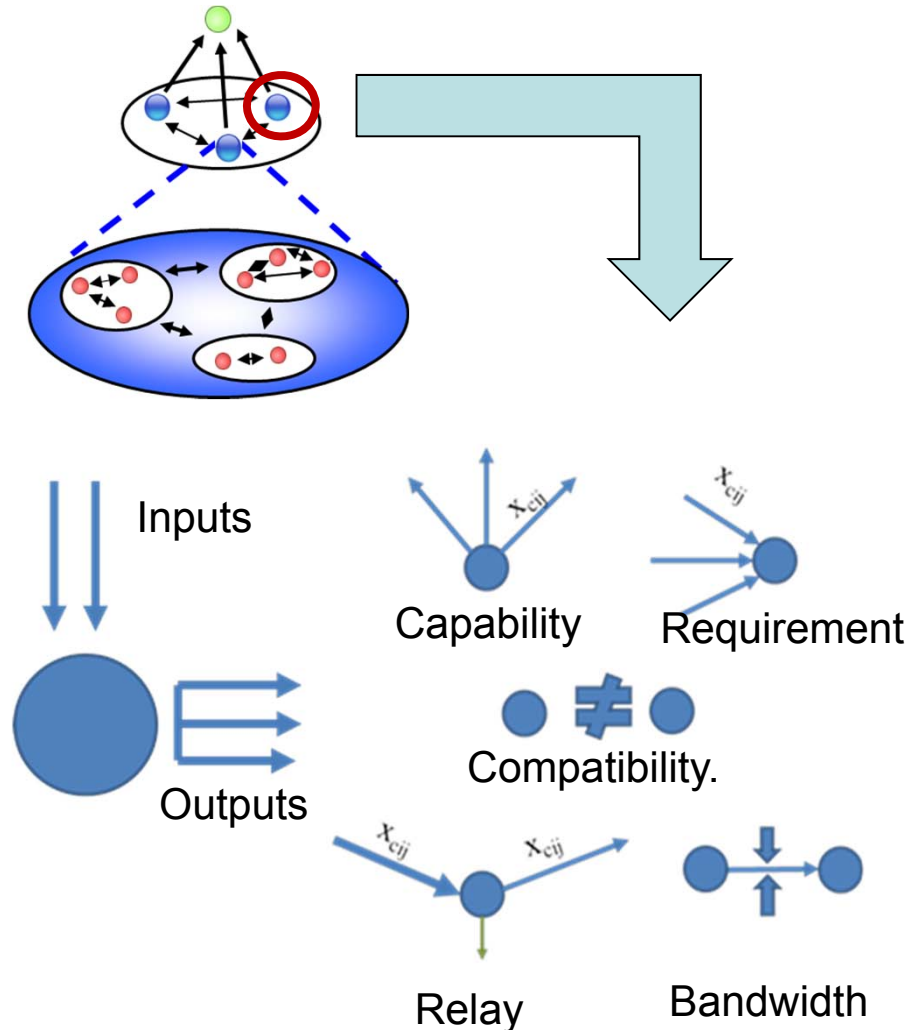
# A Portfolio Approach: Background

- Balance expected profit (performance) against risk (variance) in investments (Markowitz 1952)
- Efficiency frontier of optimal portfolios given investor risk averseness
- Extends to multi-period case with various effects (e.g. transaction costs, uncertainty)
- Tools in optimization from Operations Research, Statistics, etc.
- Model systems or policies in acquisitions as 'nodes'



# Portfolio Approach: Modelling

- Model individual system/policy as 'nodes'
  - Functional & Physical representation
- Rules for node connectivity reflect connection behaviors between systems
  - Compatibility between nodes
  - Bandwidth of linkages
  - Supply (Capability)
  - Demand (Requirements)
  - Relay capability
- Can involve acquisition of system or selection of a policy for control



# 1) Robust Multi-Period Portfolio

Maximize Performance Index

$$\max \left( \sum_q \left( \frac{S_{qc} - R_c}{R_c} \cdot w \cdot X_{q=T}^B \right) \right)$$

subject to:

Transactions (e.g. Purchase, Salvage)

$$\begin{cases} X_{q,t}^B = X_{q,t-1}^B + U_{q,t}^B + V_{q,t}^B \\ C_t^{trans} = C_q^B U_{q,t}^B + C_q^S V_{q,t}^S \end{cases}$$

Budget Requirements

$$\sum_{t=0}^T C_t^{trans} \leq \text{Budget}$$

Capability meets Requirements

$$\sum_q S_{qtC} X_{q,t}^B \geq \sum_q S_{qtR} X_{q,t}^B$$

Selection Rules

$$(X_{i,t}^B + \dots + X_{n,t}^B)_{j,t} = M_{j,t} \quad j=1\dots k$$

$$X_{q,t}^B, X_{q,t-1}^B, U_{q,t}^B, V_{q,t}^B \in [0,1] \quad t=0\dots T \text{ (timesteps)}$$

Constraints

# Robust Optimization (Bertsimas-Sim)

- Represent linear coefficient uncertainties as uncertainty sets
- Adjust conservatism based on *a priori* knowledge
- Cost of solving is equivalent to same LP; extends to discrete case

$$[A] \{X_q\} = \{b\}$$

Adjust conservatism  $\Gamma_i$  term to control probability of **constraint violation**

Conservatism Added  
(This can be converted to an LP == easy to solve even for large problems)

$$\max \left( \sum_q \left( \frac{S_{qc} - R_c}{R_c} \cdot w \cdot X_{q=T}^B \right) \right)$$

subject to:

$$\begin{aligned} X_{q,t}^B &= X_{q,t-1}^B + U_{q,t}^B + V_{q,t}^B \\ C_t^{trans} &= C_q^B U_{q,t}^B + C_q^S V_{q,t}^S \\ \sum_{t=0}^T C_t^{trans} &\leq \text{Budget} \\ \sum_q S_{qC} X_{q,t}^B &\geq \sum_q S_{qR} X_{q,t}^B \\ (X_{i,t}^B + \dots + X_{n,t}^B)_{j,t} &= M_{j,t} \quad j=1\dots k \end{aligned}$$

$$X_{q,t}^B, X_{q,t-1}^B, U_{q,t}^B, V_{q,t}^B \in [0,1] \quad t=0\dots T \text{ (timesteps)}$$

$$\sum_q S_{qc} X_q^B + \max \{ \hat{S}_{qc} y_j + (\Gamma_i - [\Gamma_i] \hat{S}_{it_i} y_t) \} \leq b_i$$

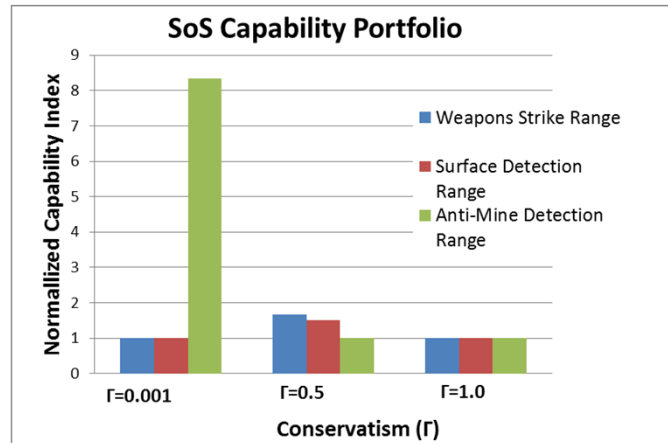
$$\begin{aligned} -y_j &\leq X_q^B \leq y_j \\ y &\geq 0 \end{aligned}$$



# A Simple Acquisition Scenario

- Uncertainties in acquisition cost, retirement cost
- Buy now, buy-now, sell and then replace, or hold and buy later? (reqs. at each time step)
- Maximize end portfolio capabilities, meet total budget within cost uncertainty brackets

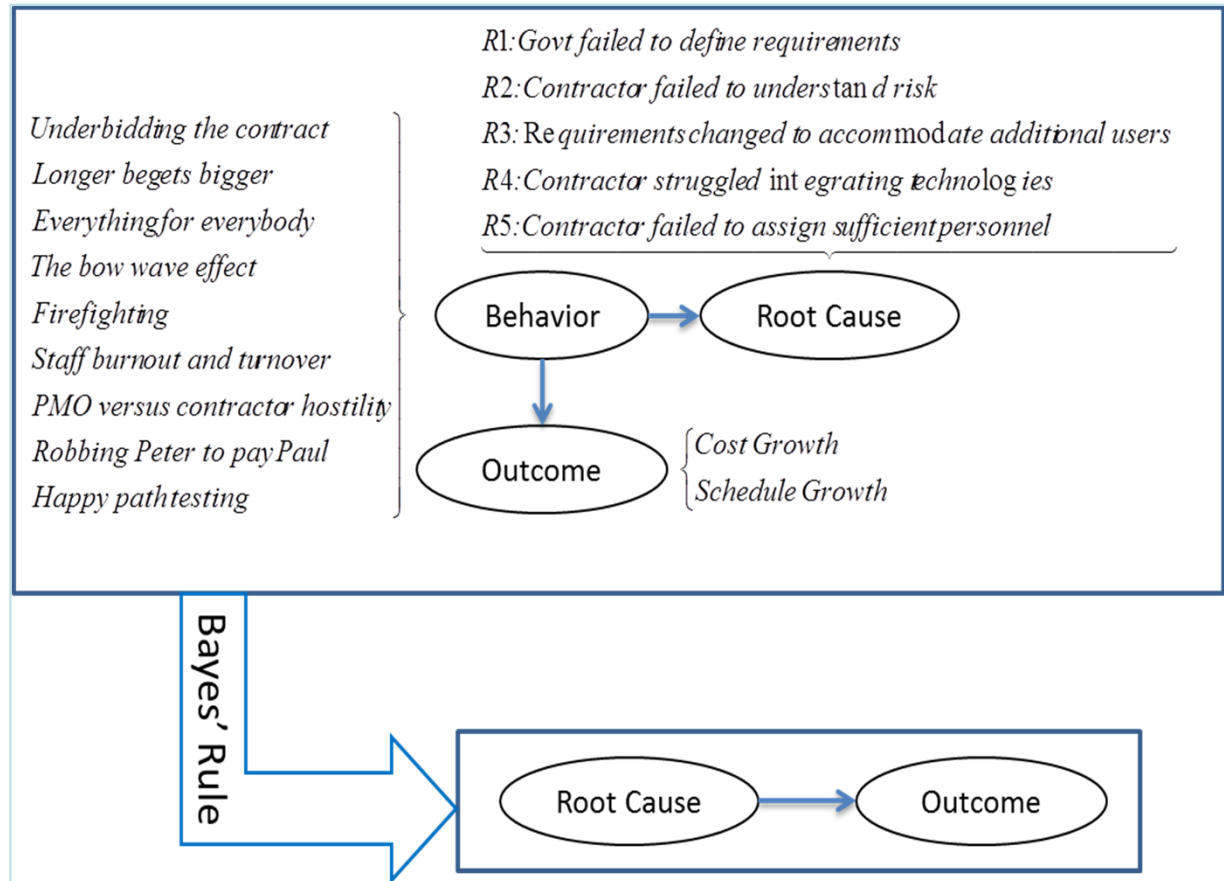
Category	System	Weapon	Surface	Anti Mine	Comm	Power	Power	Comm	Acquisition	Retiring	Uncertainty	Uncertainty
		Strike	Detection	Detection							Comm	Power
		Range	Range	Range	Bandwidth	Bandwidth	Required	Required	Cost	Cost	Cost	Cost
ASW	Variable Depth	0	50	0	0	0	95	100	1.00E+05	1.00E+05	9.84E+01	3.04E+01
	Multi Fcn Tow	0	40	0	0	0	90	120	2.00E+05	2.00E+05	1.74E+02	1.83E+02
	Lightweight tow	0	30	0	0	0	75	100	3.00E+05	3.00E+05	1.15E+02	2.37E+02
MCM	RAMCS II	0	0	10	0	0	70	120	1.00E+05	1.00E+05	7.80E+01	9.05E+00
	ALMDS (MH-60)	0	0	20	0	0	90	150	2.00E+05	2.00E+05	1.91E+01	1.33E+02
SUW	New Prototype 1	0	0	30	0	0	100	170	3.00E+05	3.00E+05	2.58E+02	1.91E+02
	N-LOS Missiles	25	0	0	0	0	0	250	1.00E+05	1.00E+05	3.49E+01	9.19E+01
	Griffin Missiles	3	0	0	0	0	0	100	2.00E+05	2.00E+05	1.69E+02	8.05E+01
Seaframe	New Prototype 1	30	0	0	0	0	0	300	3.00E+05	3.00E+05	1.72E+02	2.91E+01
	Package System 1	0	0	0	0	300	0	0	1.00E+05	1.00E+05	7.02E+01	4.72E+01
	Package System 2	0	0	0	0	450	0	0	2.00E+05	2.00E+05	1.54E+02	1.42E+02
	Package System 3	0	0	0	0	500	0	0	3.00E+05	3.00E+05	2.41E+02	2.60E+01
Comm.	Comm System 1	0	40	0	180	0	100	0	1.00E+05	1.00E+05	1.26E+01	3.59E+01
	Comm System 2	0	0	0	200	0	120	0	2.00E+05	2.00E+05	1.24E+02	9.83E+01
	Comm System 3	0	0	0	240	0	140	0	3.00E+05	3.00E+05	2.17E+02	7.00E+01
	Comm System 4	0	0	0	300	0	160	0	4.00E+05	4.00E+05	2.20E+02	3.98E+02
	Comm System 5	0	0	0	360	0	180	0	5.00E+05	5.00E+05	7.03E+01	4.15E+02
	Comm System 6	0	0	0	380	0	200	0	6.00E+05	6.00E+05	4.09E+02	4.62E+02



System Description	System Package	$\Gamma$ (Conservatism)								
		0.001			0.5			1		
		t=0	t=1	t=2	t=0	t=1	t=2	t=0	t=1	t=2
ASW	Variable Depth	0	0	0	0	0	1	0	0	0
	Multi Fcn Tow	0	0	0	0	0	0	0	0	0
	Lightweight tow	1	1	1	1	1	0	1	1	1
MCM	RAMCS II	0	0	0	1	0	0	0	0	0
	ALMDS (MH-60)	1	1	1	0	0	0	1	1	1
	New Prototype 1	0	0	0	0	1	1	0	0	0
SUW	N-LOS Missiles	0	1	1	0	0	0	0	0	0
	Griffin Missiles	1	0	0	1	1	1	1	1	1
	New Prototype 1	0	0	0	0	0	0	0	0	0
Seaframe	Package System 1	0	0	0	0	0	0	0	0	0
	Package System 2	1	1	1	1	1	1	1	1	1
	Package System 3	0	0	0	0	0	0	0	0	0
Communications	Comm System 1	1	1	1	1	1	1	1	1	1
	Comm System 2	1	0	0	1	1	1	1	1	1
	Comm System 3	0	0	0	0	0	0	0	0	0
	Comm System 4	0	0	0	0	0	0	0	0	0
	Comm System 5	0	1	1	0	0	0	0	0	0
	Comm System 6	0	0	0	0	0	0	0	0	0

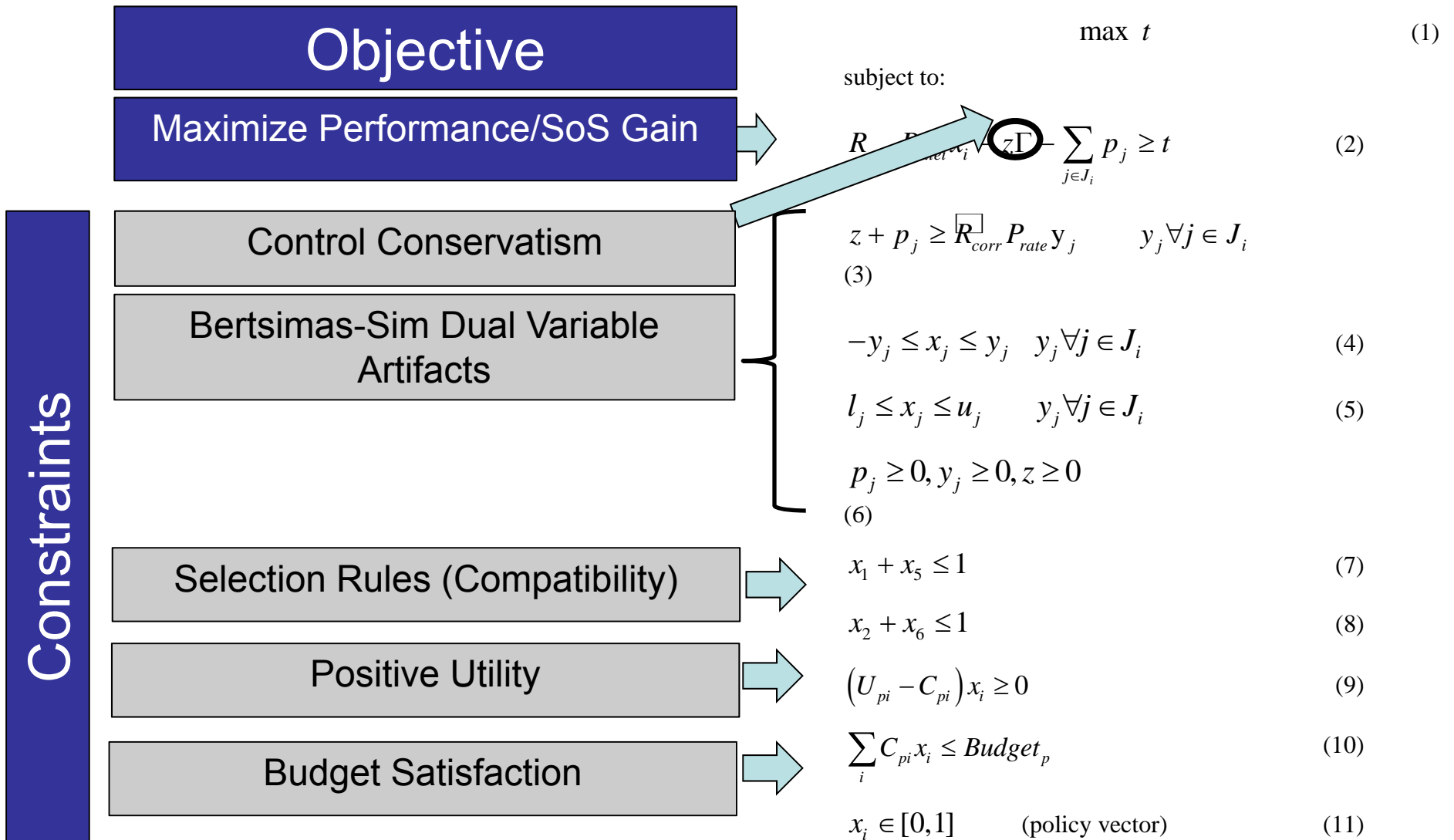
## 2) Myopic Policy Design for Acquisitions

- McNew uses behavior archetypes to structure survey
- 65 program managers surveyed to confirm these 'behaviors' on program
- If present, confirm cost, schedule growth, root cause
- Use Bayes to determine →



$P(\text{outcomes} \mid \text{root cause})$  &  $P(\text{root cause})$

# Robust Optimization Framework (Myopic)



# Simple Myopic Policy Application to McNew Data

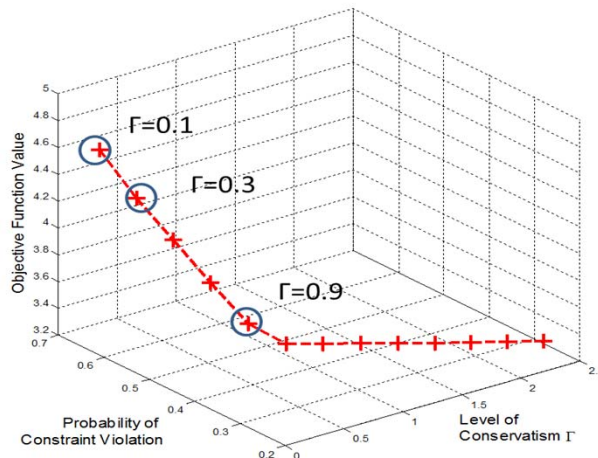
Given:

- Bayesian Analysis of McNew data
- Cost implications
- Model potential gain by using policy ( $x_i$ )
- Uncertainty in correlated gains for policies ( $x_i$ )

Question:

What policies should I effect at various levels of policy robustness, satisfying some mechanism conditions?

	Correlation							P
	R1	R2	R3	R4	R5	SG	CG	
R1	1.0	0.3	0.4	0.2	0.1	0.5	0.5	0.4
R2		1.0	0.4	0.4	0.2	0.4	0.5	0.3
R3			1.0	0.1	0.1	0.4	0.5	0.3
R4				1.0	0.3	0.4	0.3	0.3
R5					1.0	0.3	0.3	0.3
SG						1.0	0.8	0.6
CG							1.0	0.6



Policy 1	1	-	-
Policy 2	1	1	-
Policy 3	1	1	1
Policy 4	1	-	-
Policy 5	-	1	1
Policy 6	-	-	1
Policy 7	1	1	1
Policy 8	1	1	1
Conservatism ( $\Gamma$ )	0.1	0.3	0.9
P(Constraint Viol)	0.64	0.61	0.52

# 3) Multi-Period Portfolios: A Dynamic Programming Approach

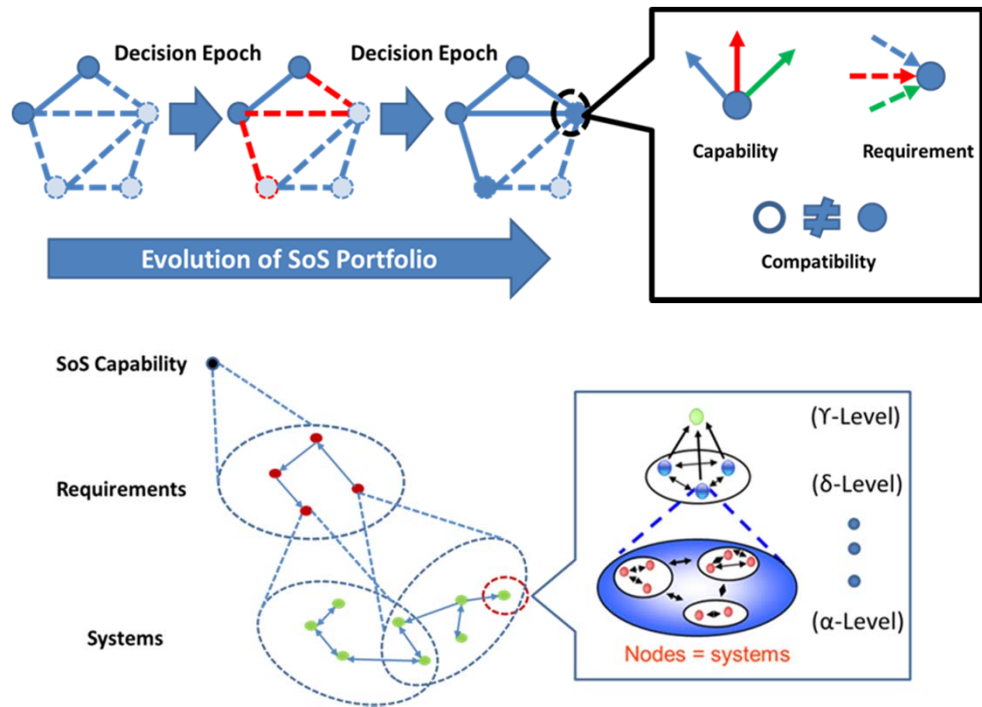
$$V_t(S_t) = \max_{x_t} C_t(S_t, x_t) + V_{t+1}(S_{t+1})$$

$$V_t(S_t) = \max_{\pi} \{ \gamma^t C_t^\pi(S_t, x_t^\pi(S_t)) \}$$

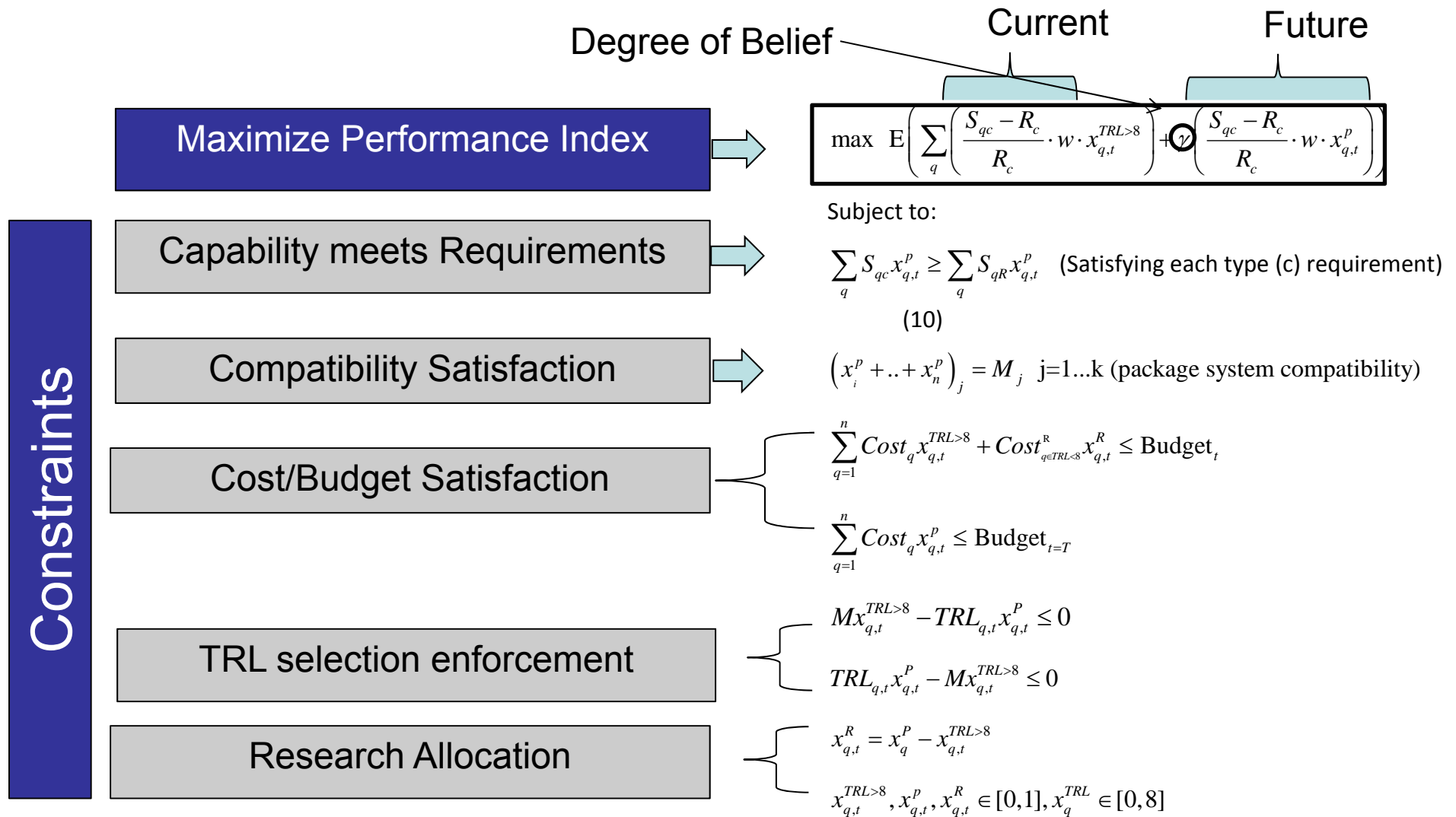
Maximize Performance Index as recursive multi-stage problem

where  $C_t()$  is the reward function of current time step  
 $S_t$  is the current state  
 $x_t$  is the action taken at time (T)  
 $V_{t+1}$  is the value function of being in state  $S_{t+1}$   
 $\gamma$  is a weighting constant,  $\pi$  is a set of all policies

Construction of Value Function Approximations → Value of being in a particular 'state'



# Portfolio Approach: A Simple ADP implement



# A Simple Case Study : Naval Scenario

System Module	Weapon Package	Weapon Strike Range (miles)	Surface Detection Range (miles)	Anti Mine Detection Range (miles)	Unconv Warfare Payload (kg)	Comm. Capacity (Mbps)	Power Capacity (kW)	Power Req. (kW)	Comm. Bandwidth Req. (Mbps)	Cost of Acquisition (USD)	Cost of Research (USD)	TRL
ASW	Variable Depth	0	30	0	0	0	0	50	75	80000	20000	8
	Multi Fcn Tow	0	40	0	0	0	0	100	125	90000	22500	6
	Lightweight tow	0	50	0	0	0	0	150	150	100000	25000	6
	ASW Prototype 1	0	60	0	0	0	0	175	150	120000	30000	7
	ASW Prototype 2	0	70	0	0	0	0	180	100	130000	32500	7
MCM	RAMCS II	0	0	30	0	0	0	100	75	80000	20000	8
	ALMDS (MH-60)	0	0	40	0	0	0	150	125	90000	22500	7
	MCM Prototype 1	0	0	50	0	0	0	200	150	100000	25000	7
	MCM Prototype 2	0	0	60	0	0	0	250	175	120000	30000	7
	MCM Prototype 3	0	0	70	0	0	0	270	185	140000	35000	7
SUW	N-LOS Missiles	3	0	0	0	0	0	150	100	80000	20000	8
	Griffin Missiles	25	0	0	0	0	0	200	200	90000	22500	7
	SUW Prototype 1	50	0	0	0	0	0	250	300	100000	25000	7
	SUW Prototype 2	60	0	0	0	0	0	200	120	120000	30000	6
	SUW Prototype 3	70	0	0	0	0	0	200	300	130000	32500	6
Unconventional Warfare	Package System 1	0	0	0	100	0	0	25	50	70000	17500	8
	Package System 2	0	0	0	150	0	0	50	150	80000	20000	8
	Package System 3	0	0	0	200	0	0	75	200	90000	22500	8
Comm. Package	Package System 1	0	0	0	0	300	0	50	0	80000	20000	8
	Package System 2	0	0	0	0	400	0	75	0	90000	22500	8
	Package System 3	0	0	0	0	450	0	100	0	100000	25000	6
	Package System 4	0	0	0	0	500	0	150	0	100000	25000	6
	Package System 5	0	0	0	0	550	0	200	0	110000	27500	6
Power Package	Package System 1	0	0	0	0	0	350	0	0	80000	20000	8
	Package System 2	0	0	0	0	0	450	0	0	90000	22500	8
	Package System 3	0	0	0	0	0	550	0	0	100000	25000	7
	Package System 4	0	0	0	0	0	650	0	0	110000	27500	7
	Package System 5	0	0	0	0	0	750	0	0	120000	30000	6

# Multi-period NWS Epochs

Gamma Value		Decision Epochs (Acquisitions)											
		1	0.1	1	0.1	1	0.1	1	0.1	1	0.1	1	0.1
System													
ASW	Variable Depth	0	1	0	1	0	1	0	1	0	1	0	1
	Multi Fcn Tow	0	0	0	0	0	0	0	0	0	0	0	0
	Lightweight tow	0	0	0	0	0	0	0	0	0	0	0	0
	ASW Prototype 1	0	0	0	0	0	0	0	0	0	0	0	0
	ASW Prototype 2	0	0	1	0	1	0	1	0	1	0	1	0
MCM	RAMCS II	0	1	0	1	0	1	0	1	0	1	0	1
	ALMDS (MH-60)	0	0	0	0	0	0	0	0	0	0	0	0
	MCM Prototype 1	0	0	0	0	0	0	0	0	0	0	0	0
	MCM Prototype 2	0	0	0	0	0	0	0	0	0	0	0	0
	MCM Prototype 3	0	0	1	0	1	0	1	0	1	0	1	0
SUW	N-LOS Missiles	0	0	0	0	0	0	0	0	0	0	0	0
	Griffin Missiles	0	0	0	0	0	0	0	0	0	0	0	0
	SUW Prototype 1	0	0	0	0	0	0	0	0	0	0	0	0
	SUW Prototype 2	0	0	0	0	0	0	0	0	0	0	0	0
	SUW Prototype 3	0	0	0	0	0	0	0	1	1	1	1	1
nonconventior	Package System 1	0	0	0	0	0	0	0	0	0	0	0	0
Warfare	Package System 2	0	0	0	0	0	0	0	0	0	0	0	0
	Package System 3	1	1	1	1	1	1	1	1	1	1	1	1
	Package System 4	0	0	0	0	1	0	1	0	1	0	1	0
Comm. Package	Package System 1	0	0	0	0	0	0	0	0	1	0	1	
	Package System 2	0	0	0	0	0	0	0	0	0	0	0	0
	Package System 3	0	0	0	0	0	0	0	0	0	0	0	0
	Package System 4	0	0	0	0	1	0	1	0	1	0	1	0
	Package System 5	0	0	0	0	1	0	1	0	1	0	1	0
Power Package	Package System 1	0	0	0	0	0	0	0	0	0	0	1	
	Package System 2	0	0	0	0	0	0	0	0	1	0	1	
	Package System 3	0	0	0	0	0	0	0	0	0	0	0	0
	Package System 4	0	0	0	0	1	0	1	0	1	0	0	0
	Package System 5	0	0	0	0	0	1	0	0	0	0	1	0

Acquisition Allocation Comparison

Gamma Value		Decision Epochs (Research TRL)											
		1	0.1	1	0.1	1	0.1	1	0.1	1	0.1	1	0.1
System													
ASW	Variable Depth	0	0	0	0	0	0	0	0	0	0	0	0
	Multi Fcn Tow	0	0	0	0	0	0	0	0	0	0	0	0
	Lightweight tow	0	0	0	0	0	0	0	0	0	0	0	0
	ASW Prototype 1	0	0	0	0	0	0	0	0	0	0	0	0
	ASW Prototype 2	1	0	0	0	0	0	0	0	0	0	0	0
MCM	RAMCS II	0	0	0	0	0	0	0	0	0	0	0	0
	ALMDS (MH-60)	0	0	0	0	0	0	0	0	0	0	0	0
	MCM Prototype 1	0	0	0	0	0	0	0	0	0	0	0	0
	MCM Prototype 2	0	0	0	0	0	0	0	0	0	0	0	0
	MCM Prototype 3	1	0	0	0	0	0	0	0	0	0	0	0
SUW	N-LOS Missiles	0	0	0	0	0	0	0	0	0	0	0	0
	Griffin Missiles	0	0	0	0	0	0	0	0	0	0	0	0
	SUW Prototype 1	0	0	0	0	0	0	0	0	0	0	0	0
	SUW Prototype 2	0	0	0	0	0	0	0	0	0	0	0	0
	SUW Prototype 3	1	1	1	1	1	1	1	1	1	0	0	0
Warfare	Package System 1	0	0	0	0	0	0	0	0	0	0	0	0
	Package System 2	0	0	0	0	0	0	0	0	0	0	0	0
	Package System 3	0	0	0	0	0	0	0	0	0	0	0	0
	Package System 4	1	1	1	1	0	1	0	0	0	0	0	0
	Package System 5	1	0	1	1	0	1	0	0	0	0	0	0
Comm. Package	Package System 1	0	0	0	0	0	0	0	1	0	0	0	0
	Package System 2	0	0	0	0	0	0	0	1	0	1	0	1
	Package System 3	0	1	0	0	0	0	0	0	0	0	0	0
	Package System 4	1	1	1	1	0	1	0	0	0	0	0	0
	Package System 5	1	0	1	1	0	1	0	0	0	0	0	0
Power Package	Package System 1	0	0	0	0	0	0	0	1	0	1	0	0
	Package System 2	0	0	0	0	0	0	0	1	0	0	1	0
	Package System 3	1	0	1	1	0	1	0	0	0	0	0	0
	Package System 4	1	1	1	0	0	0	0	0	0	0	0	0
	Package System 5	0	1	0	1	1	0	1	0	1	0	0	0

Research Allocation Comparisons



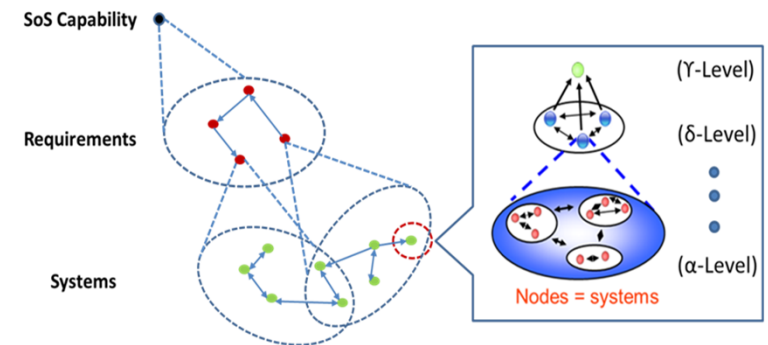
## Summary thoughts:

- A Robust Multi-period Optimization approach
  - Potentially useful for long-term strategic level mapping
  - LP computationally efficient; excellent solvers for integer/discrete case
  - Deals with uncertainty as ‘sets’
  - Discrete consideration makes it amenable to policy
- Multi-Period approach using Dynamic Programming
  - Approximate dynamic programming balance decision now w/ future states
  - Evolutionary decision-making
  - Intuitive interpretation but requires careful selection of value function approximations and policy construction.

# Current Directions

- Adapting ‘exploration’ vs. ‘exploitation’ framework for balancing ‘future’ and ‘current’ gains.
- Formulations that are well bounded and require minimal intervention by an acquisition practitioner but intelligent in approximating future values states.
- Appropriate range of metrics across acquisitions that can well capture salient future state values (e.g. KVA – Housel, Mun).
- Incorporate collaborative information across decision-makers within value function?

$$V_t(S_t) = \max_{x_t} C_t(S_t, x_t) + V_{t+1}(S_{t+1})$$



**THANK YOU**