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# A Framework to Determine New System Requirements Under Design Parameter and Demand Uncertainties

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# A Framework to Determine New System Requirements Under Design Parameter and Demand Uncertainties

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## Abstract

Identifying optimal design requirements of new systems that operate along with existing systems to provide a set of overarching capabilities is a challenging endeavor for acquisition practitioners. Difficulties arise due to the tightly coupled effects that setting requirements on a system's design (here, military transportation aircraft) can have on how the system is being used (here, how the aircraft is allocated to carry cargo on specified routes). The task of identification becomes even more difficult when considering uncertainties in the way the system is used, and in the demand for its services. Our research extends a previously developed quantitative approach that generates optimum design requirements of new, yet-to-be-designed systems that, when serving alongside other systems, will optimize *fleet-level* objectives. We extend the approach to now include the effect of various forms of operational uncertainties. Our approach relegates quantitative complexities of decision-making to the method and designates trade-space exploration to the practitioner. We demonstrate the approach by exploring tradeoffs between fleet-level cost and fleet-wide performance for the USAF Air Mobility Command fleet.

## Introduction

### Nomenclature

| | | |
|---|---|---|
| $AR_X$ | = | aspect ratio of aircraft type X |
| $B_p$ | = | maximum average daily utilization of each aircraft (16 hours) |
| $BH_{p,k,i,j}$ | = | number of block hours for *kth* trip of aircraft *p* from base *i* to base *j* |
| $C_{p,k,i,j}$ | = | cost coefficient for $k^{th}$ trip of aircraft *p* from base *i* to base *j* |
| $Cap_{p,k,i,j}$ | = | pallet carrying capacity for $k^{th}$ trip of aircraft *p* from base *i* to base *j* |
| $C_{D_O}$ | = | Parasite drag coefficient |
| $Dem_{i,j}$ | = | demand from base *i* to base *j* in number of pallets |
| *DOC* | = | Direct Operating Cost |
| *DOC/BH* | = | Direct Operating Cost per block hour ($/hr) |
| *M* | = | Fleet-level DOC or fuel limit |
| $O_{p,I}$ | = | indicates if airport *i* is the initial location (e.g., home base) of an aircraft *p* |
| *PalletX* | = | number of pallets carried by aircraft type X |

| | | |
|---|---|---|
| $Prod_{p,k,i,j}$ | = | productivity coefficient for $k^{th}$ trip of aircraft *p* from base *i* to base *j* |
| $Range_X$ | = | design range of aircraft type X (nm) |
| SFC | = | Specific Fuel Consumption (1/hr) |
| $Speed_X$ | = | cruise speed of aircraft type X (knots) |
| $S_{TO}$ | = | take off field length |
| $(T/W)_X$ | = | thrust-to-weight ratio of aircraft type X |
| $W_E$ | = | empty weight (lbs) |
| $(W/S)_X$ | = | wing loading of aircraft type X (lb/ft2) |
| $x_{p,k,i,j}$ | = | Boolean variable for $k^{th}$ trip flown by aircraft *p* from base *i* to base *j* |

## Research Issue

The *Energy Efficiency Starts With the Acquisition Process* factsheet states that "neither current requirements or acquisition processes accurately explore tradeoff opportunities using fuel as an independent variable" (Deputy Under Secretary of Defense for Acquisition, Technology, and Logistics [DUSD(AT&L)], 2012). The factsheet also states, "Current processes undervalue technologies with the potential to improve energy efficiency." The complexity of dealing with many interdependent systems, the impact of changing characteristics of such systems, and uncertainties related to allocations of such systems becomes cognitively impossible to manage without a decision-support framework. Current acquisition processes focus on development at the *system-level* (e.g., aircraft performance), with little consideration on how the interactive effects between the newly designed systems will have when set to work with other existing systems.

Typically, engineering systems are designed to meet specific performance objective(s) during their lifespan. A system may not be able to obtain or consistently provide a predicted value of a performance objective, because of uncertainties in the estimation, the physical implementation of the system, and in the operating environment, as illustrated in Figure 1 (Schuëller & Jensen, 2008). Epistemic (reducible) uncertainties in the design parameters (assumptions in model inputs) reflect uncertainties in the use of the system under varying operational conditions (e.g., during design, a nominal inlet temperature for the engine may be assumed but, in reality, varying operational conditions can change this). The varying nature of daily demand for the use of these systems also presents uncertainty. The research presented here addresses uncertainty in fleet operations and in the design parameters of the new system.

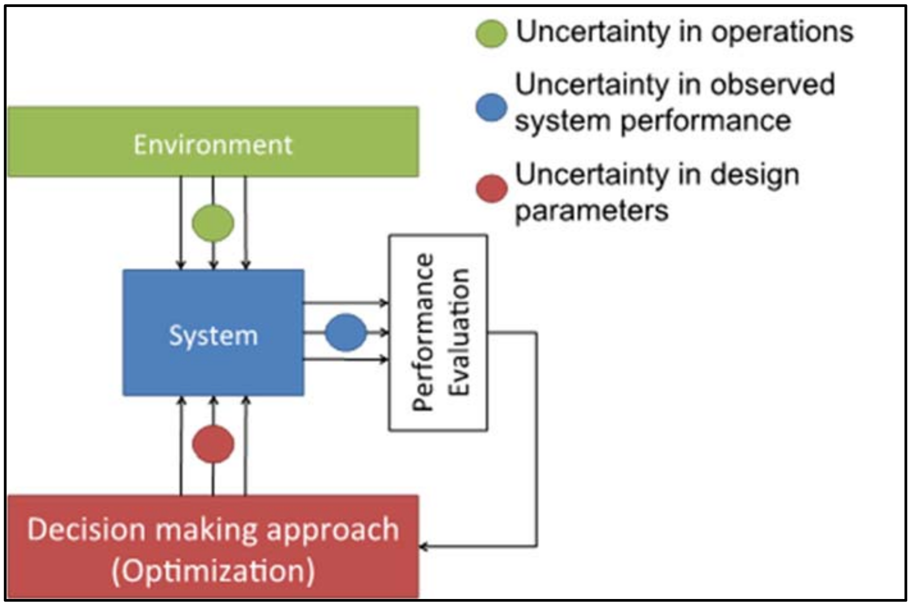


**Figure 1. Classes of Uncertainties**

Our approach treats design requirements of new systems as decision variables in an optimization problem to minimize (or maximize) fleet-level objectives—the solution being the selection, based on mathematical techniques, of decision variable values that yield the best fleet-level objectives. Our problem of designing a new system (here, an aircraft) and determining the optimal allocation of the new aircraft, together with existing aircraft as a fleet to meet air transportation demand, is mathematically difficult when considering the effects of uncertainty, and may become impossible to solve in practical time—even for small-sized problems with few decision variables. Prior Acquisition Research Program–funded work demonstrated that, for a simple air cargo transportation example representing operations of the USAF Air Mobility Command (AMC) fleet, breaking down the complex mathematical problem into a series of smaller, well-posed sub-problems makes the problem computationally tractable; this strategy is known as a subspace decomposition strategy. The research presented here extends prior work (Govindaraju et al., 2014) to encompass investigating tradeoffs between objectives of productivity/mission effectiveness and cost. These two competing objectives (maximizing productivity increases cost and minimizing cost decreases productivity) often play a critical role in determining new system requirements.

## Scope and Method of Approach

The early stages of the design process have substantial uncertainty due to the lack of high fidelity information from a realized design. The conventional approach to designing engineering systems typically involves analyses of deterministic models and parameters. However, most engineering systems experience variations in operating conditions and/or the environment in which these systems operate. Deterministic models account for these variations by considering the extreme or nominal values, and/or using safety factors. These assumptions of the nominal/extreme factors in deterministic models lead to sub-optimal performance of these systems when deployed (Beyer & Sendhoff, 2007; Schuëller & Jensen, 2008). Missing performance targets by incorrectly using nominal, deterministic predictions or greatly over-designing systems by using safety factors applied to deterministic predictions can both lead to cost overruns and increased risk. The observed inefficiencies that stem from the deterministic assumption have sparked research over the last two decades in addressing uncertainty in the design of complex systems (Mavris et al., 1999).

Aircraft design optimization efforts have traditionally focused on improving the aircraft's performance (e.g., minimizing takeoff gross weight, cost), with the new aircraft expected to improve fleet-level performances, albeit indirectly. Identifying optimal design requirements of new systems that operate along with existing systems to provide a set of overarching capabilities is challenging due to the tightly coupled effects that setting requirements on a system's design can have on how the system is being used (Mane et al., 2007; Taylor & Weck, 2007). The task of identification becomes even more difficult when considering uncertainties in the way the system is used and in the demand for its services. Previous work by the author (Choi et al., 2013a, 2013b; Govindaraju et al., 2014) and others (Baker et al., 2002; Listes & Dekker, 2005) address uncertainties at the system-level or at the resource allocation level *separately*, and do not consider the impact and propagation of uncertainty that setting system-level requirements has on the way the same system might be operated along with other systems to provide a set of capabilities.

Quantifying and addressing uncertainty in multiple disciplines is often characterized by high computational expense leading to the design problem being intractable. A computationally efficient approach is necessary to effectively conduct studies that examine several scenarios using different predictions of demand, cost of operating the fleet, and so forth. Furthermore, multi-objective analyses performed using the resulting methodology will address the current need for analytical frameworks, which enable decision-makers and/or acquisition practitioners to assess tradeoffs that design choices may have on fleet-level metrics of interest.

***Monolithic Problem Formulation***

The approach in this study seeks to maximize (or minimize) a fleet-level objective function by searching for the optimal values of a set of decision variables that describe the new system design features and also determine the assignment of the new and existing systems to perform operational missions. Solving the monolithic problem that combines the selection of the new aircraft design features and the assignment of new and existing aircraft to meet demand requirements can be challenging. The monolithic representation of the variable resource allocation problem considered for this study involves the combination of resource assignment and aircraft design problems under uncertainty. The resulting problem is a stochastic mixed integer non-linear programming (MINLP) problem (stochastic—because of the uncertainties in demand and design parameters; mixed integer—because of the presence of continuous decision variables such as aspect ratio, wing loading, etc., and integer decision variables such as pallet capacity; and non-linear— because of the non-linearity of the constraints such as the aircraft takeoff distance constraint) and is represented by the following equations:

Maximize

$$E\left[\begin{array}{l}\sum_{p=1}^{P}\sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{N}x_{p,k,i,j}\cdot\left(Speed_{p,k,i,j}\cdot Pallet_{p,k,i,j}\right)+\\ \left(x_{p,k,i,j}\cdot\left(Speed_{p,k,i,j}\cdot Pallet_{p,k,i,j}\right)\left(\left(AR\right)_X,\left(W/S\right)_X,\left(T/W\right)_X\right)\right)_X\end{array}\right] \text{(Productivity = Speed x Capacity)} \qquad (0)$$

Subject to

$$\sum_{p=1}^{P}\sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{N}x_{p,k,i,j}\cdot C_{p,k,i,j}+\left(x_{p,k,i,j}\cdot C_{p,k,i,j}\left(Pallet_X,Speed_X,\left(AR\right)_X,\left(W/S\right)_X,\left(T/W\right)_X\right)\right)_X\le M$$

(DOC or Fleet fuel limits) (0)

$$\sum_{i=1}^{N} x_{p,k,i,j} \geq \sum_{i=1}^{N} x_{p,k+1,i,j} \quad \forall k = 1,2,3...K, \quad \forall p = 1,2,3...P, \quad \forall j = 1,2,3...N$$ (Node balance constraints) (0)

$$\sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{N} x_{p,k,i,j} \cdot BH_{p,k,i,j} \leq B_P \quad \forall p = 1,2,3...P$$ (Trip constraints) (0)

$$\sum_{p=1}^{P}\sum_{k=1}^{K} Cap_{p,k,i,j} \cdot x_{p,k,i,j} \geq dem_{i,j} \quad \forall i = 1,2,3...N \quad \forall j = 1,2,3...N$$ (Demand constraint) (0)

$$\sum_{i=1}^{N} x_{p,1,i,k} \leq O_{p,i} \quad \forall p = 1,2,3...P, \forall i = 1,2,3...N$$ (Starting location constraints) (0)

$$S_{TO}\left(Pallet_X, Speed_X, (AR)_X, (W/S)_X, (T/W)_X\right) \leq D$$ (Aircraft takeoff distance) (0)

$$14 \leq Pallet_X \leq 38$$ (Design pallet capacity bounds) (0)

$$2400 \leq Range_X \leq 3800$$ (Range at max. payload bounds) (0)

$$350 \leq Speed_X \leq 550$$ (Cruise speed bounds in knots) (0)

$$6.0 \leq (AR)_X \leq 9.5$$ (Wing aspect ratio bounds) (0)

$$65 \leq (W/S)_X \leq 161$$ (Wing loading bounds) (0)

$$0.18 \leq (T/W)_X \leq 0.35$$ (Thrust-to-weight ratio bounds) (0)

$$x_{p,k,i,j} \in \{0,1\}$$ (Binary variable) (0)

$$(AR)_X, (W/S)_X, (T/W)_X$$ (Continuous aircraft design variables) (0)

$$Pallet_X, Speed_X, Range_X$$ (Discrete aircraft design variables) (0)

Equation 1 is the objective function that seeks to maximize the expected fleet-level productivity where $Speed_{p,k,i,j}$ x $Pallet_{p,k,i,j}$ indicates the productivity coefficient of the trip for $k^{th}$ trip for aircraft *p* from base *i* to base *j*. The objective function is the expected (average) value because of the uncertainty in the aircraft design parameters and in the pallet demand in the service network. Equation 2 limits the fleet-level fuel consumption or cost to a pre-defined limit (*M*); the limit is varied and the problem is re-solved for each varied value of limit to generate a set of *Pareto optimal* solutions; the equation has two parts where $x_{p,k,i,j} \cdot C_{p,k,i,j}$ refers to the cost of use of utilizing aircraft type *p* for $k^{th}$ trip from base *i* to base *j.* The rest of the terms in Equation 2 refer to the cost of allocating the new yet-to-be-designed aircraft, where the new aircraft is a function of design variables (aspect ratio, thrust-to-weight ratio, and wing loading) and design requirements (pallet capacity, design range, and cruise speed). Equation 3 is the balance and sequencing constraint that ensures that the $(k+1)^{th}$ trip of an aircraft out of a base occurs only after $k^{th}$ trip into that base—this constraint ensures that an aircraft needs to already be at a base prior to completing a subsequent segment trip out of the same base. Equation 4 limits flights to only occur within daily utilization limit $B_p$ (assumption of 16 hours) of the aircraft, where $BH_{p,k,i,j}$ indicates the block hour for the $k^{th}$ trip for aircraft *p* from base *i* to base *j*. Equation 5 ensures that carrying capacity of combined trips flows by all aircraft on a specific route meets the demand on that route, where $Cap_{p,k,i,j}$ indicates the pallet carrying capacity of the $k^{th}$ trip for aircraft *p* from base *i* to base *j*. Equation 6 ensures that the first trip of each aircraft originates at the initial location, $O_{p,i}$ (home base), which is randomly generated. The non-availability of the aircraft starting location information necessitates the random distribution of the starting location of each aircraft. Equation 7 limits the new aircraft design based on maximum takeoff distance *(D)* to ensure that the new aircraft can operate at all bases in the network. Equations 8–10 describe bounds on the aircraft design variables of payload, design range (in nautical miles) at maximum payload, and cruise speed (in knots) capabilities of the new aircraft. The chosen limits are within ranges exhibited by current military cargo aircraft. The continuous aircraft design variables of aspect ratio $(AR)_X$, thrust-to-weight ratio $(T/W)_x$ , and wing loading $(W/S)_x$ (in lb/ft$^2$), describing the new aircraft, are bounded within the range of values associated with current cargo aircraft; the bounds appear in Equations 11–13.

### *Subspace Decomposition Strategy*

The subspace decomposition strategy, as shown in Figure 2, decomposes the MINLP problem into smaller optimization problems—each sub-problem follows the natural boundaries of disciplines involved in formulating the original problem. The top-level problem helps explore the requirements space for the new yet-to-be-introduced aircraft based on fleet-level metrics. In this research, the top-level optimization problem is tackled using quasi-enumeration and seeks to maximize the expected fleet-level productivity using pallet capacity, range, and cruise speed of the new, yet-to-be-introduced aircraft type X.

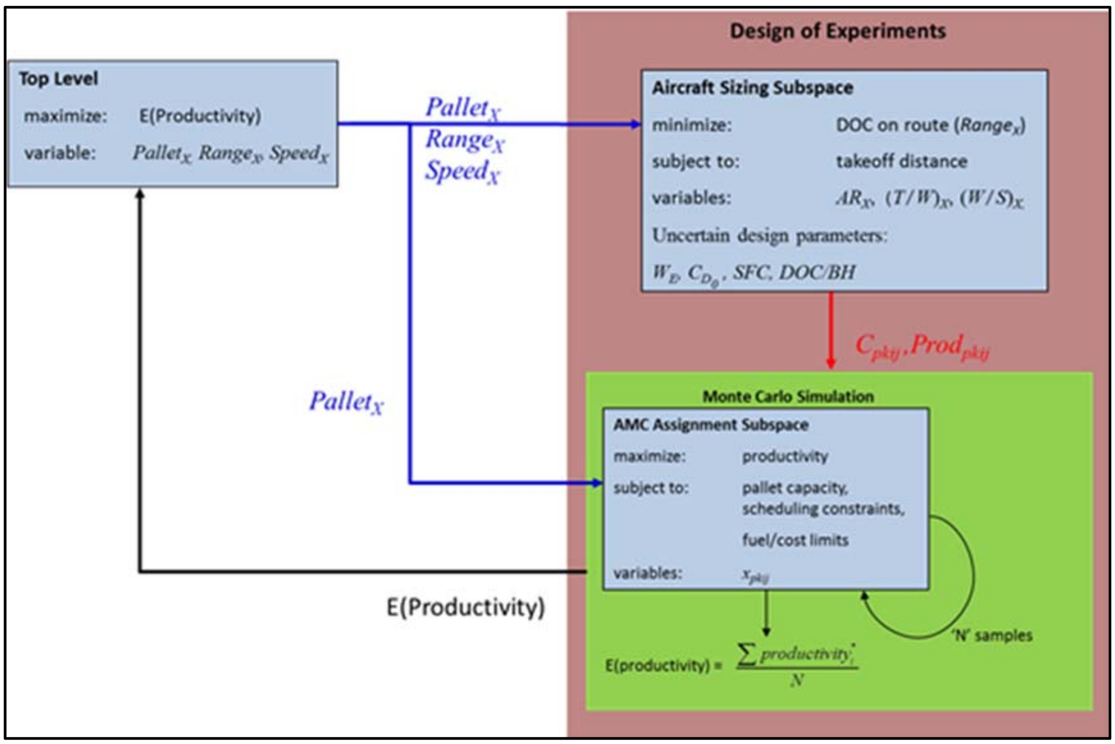


**Figure 2. Subspace Decomposition of Monolithic Optimization Problem With Monte Carlo Sampling and Design of Experiments**

The resulting pallet capacity *(Pallet$_x$)*, design range *(Range$_x$)*, and cruise speed (*Speed$_x$*) from the top-level problem then become inputs to the aircraft sizing problem. Here, the aircraft sizing problem seeks to minimize the direct operating cost of the new yet-to-be-introduced aircraft for the value of range, pallet capacity, and cruise speed from the top-level optimization problem. The aircraft design problem is subject to constraints on take-off distance. Actual aircraft design of this kind would need additional constraints; here, using only the takeoff distance simplifies the problem, while demonstrating that the approach can incorporate performance constraints. The design variables in the aircraft sizing subspace are the main drivers of aircraft design, namely the wing aspect ratio $(AR)_x$, thrust-to-weight ratio $(T/W)_x$, and wing loading $(W/S)_x$. The aircraft sizing code computes performance metrics such as cost estimates, fuel consumed, empty weight, and so forth of the new aircraft design, which are then used for solving the aircraft sizing problem After the aircraft sizing problem is solved, the outputs of the aircraft sizing problem and the top-level optimization problem, namely the productivity coefficients and cost coefficients, and pallet capacity, then become inputs to the aircraft assignment problem. Here, the assignment problem's objective is to maximize the fleet-level productivity using characteristics of the new, yet-to-be-introduced aircraft (cost, pallet capacity, speed), subject to capacity, fleet-level cost, and aircraft trip limits. The aircraft sizing problem and the assignment problem are solved multiple times using samples of demand instantiations and for different factor settings (design of experiments) to account for uncertainty in the current approach. The average or expected (mean) value of the fleet productivity is calculated from the results of the sampled demand runs.

### *Aircraft Sizing Subspace*

The problem formulation requires estimates of the cost, block time, and fuel consumed by each aircraft type in the fleet to determine the appropriate assignment of aircraft to the various routes in the network. A Purdue in-house aircraft sizing code, written in MATLAB, provides these estimates in the aircraft sizing subspace shown in Figure 2. Jane's Aircraft database (Jackson et al., 2004) provided the input parameters for the three existing aircraft types (C-5, C-17, 747-F) used in this study. Table 1 summarizes the technical characteristics of the various aircraft types in the existing AMC fleet considered for this study. The MATLAB sizing code's predictions of the existing aircraft size, weight, and performance have been validated with published data.

**Table 1. Technical Characteristics of Different Aircraft Types in Existing Fleet**

| Parameter | C-5 | C-17 | B747-F |
|---|---|---|---|
| Range (nmi) | 2982 | 2420 | 4445 |
| Pallet capacity | 36 | 18 | 29 |
| Cruise speed (knots) | 490 | 450 | 490 |
| $W/S$ (lb/ft$^2$) | 135.48 | 161.84 | 137.34 |
| $T/W$ | 0.205 | 0.263 | 0.286 |
| $AR$ | 7.75 | 7.20 | 7.70 |

Direct operating cost (DOC) estimates include fuel costs, crew costs, maintenance, depreciation, and insurance. DOC estimates are also dependent on the payload, route distance, empty weight, landing weight, and takeoff gross weight. To estimate the fuel weight necessary for flying the route distance, the fuel required for each mission segment is computed and aggregated. The fuel weight fractions for the different mission segments such as warm-up and take-off, climb, landing and taxi, and reserves are based on empirical data (Raymer, 2006). The aircraft design variables are aspect ratio $(AR)_x$, thrust-to-weight ratio $(T/W)_x$ , and wing loading $(W/S)_x$. There are many other design variables, but these three have significant impact on the size, weight, and performance of the aircraft. The aircraft sizing problem is a nonlinear programming (NLP) problem and is described using Equations 17–21.

Minimize $f = (DOC_{pallet,range,speed})_X$ (17)

Subject to $S_{TO}(Pallet_X, Speed_X, (AR)_X, (W/S)_X, (T/W)_X) \leq D$ (Aircraft takeoff distance) (18)

$6.0 \leq (AR)_X \leq 9.5$ (Wing aspect ratio bounds) (19)

$65 \leq (W/S)_X \leq 161$ (Wing loading bounds) (20)

$$0.18 \leq (T/W)_X \leq 0.35 \qquad \text{(Thrust-to-weight ratio bounds)(21)}$$

Equation 17 is the objective function that seeks to minimize DOC or fuel cost of the new aircraft *X*. The aircraft *X* design input variables are pallet carrying capacity of the aircraft, design maximum range at maximum loading condition, and cruise speed as shown in Equations 8–10. Equation 18 limits the aircraft design based on maximum takeoff distance to ensure that the new aircraft can operate at bases in the network within the bounds of modern day cargo aircraft descriptions shown in Equations 19–21.

## Uncertainty in Design Parameters

The conceptual phase of the aircraft design process is based on empirical equations and simplified physics models. The limited knowledge available at this phase of the design process combined with the modeling fidelity results in high uncertainty. Therefore, the design parameters of the system are not certain and this type of uncertainty enters the system in the form of perturbations of the design variables. It is to be noted that the perturbations may or may not depend on the design variables. For example, an aircraft is typically sized for its design mission based on a set of nominal values on operating conditions (e.g., engine inlet air temperature). However, when evaluating the "operating missions" to determine block time and fuel consumed on the flight, there might be a variation in winds aloft, which would alter the block time and fuel consumed. To account for the uncertainties in the design parameters, it is necessary to perform simulations in the absence of closed form mathematical expressions for the objective function and constraints. Implementing robust counterparts for the original objective function and reliability-based optimization techniques have been widely employed to solve problems involving design parameter uncertainty (Schuëller & Jensen, 2008). As an initial step to capture the inherent uncertainty present in the conceptual phase of the design process, we have devised a three-factor full factorial design of experiment (DOE) study (Montgomery, 2008). The four uncertain design parameters chosen for this study are empty weight ($W_E$), specific fuel consumption (SFC), parasite drag coefficient ($C_{D_0}$), and direct operating cost per block hour (DOC/BH). Table 2 summarizes the uncertain model parameters and their range of values. The three factor values for each uncertain model parameter are the baseline value, 90% of baseline value, and 110% of baseline value.

**Table 2. Uncertain Design Parameters for the Aircraft Sizing Problem**

| Uncertain design parameter | Range of values |
|---|---|
| $W_E$ (lbs) | ±10% |
| $C_{D_0}$ | ±10% |
| DOC/BH ($/hr) | ±10% |
| SFC (1/hr) | ±10% (Baseline value: 0.5) |

We use a full factorial design of experiments approach to determine an optimal description of the new aircraft. A full factorial DOE generates $n^k$ experiment designs, where $n$ is the number of variables and $k$ is the number of factors. The results from the DOE are used to identify the significance of the different factors of the design parameters and their impact on fleet-level performance metrics. Since the aircraft sizing and the assignment problems are embedded within the DOE method, for each experiment involving a set of factor values of the design parameters, the new aircraft (obtained from solving the aircraft sizing problem) and the existing aircraft are then assigned to routes in the service network; the fleet-level cost and productivity values are then computed from the optimal assignment.

These fleet-level performance metrics for each of the DOE runs are then passed on to the top-level problem.

The DOE approach combined with the subspace decomposition strategy can also accommodate economic uncertainty analyses that might be of interest to acquisition practitioners. For instance, uncertainty in future budget estimates for new aircraft acquisitions will affect the DOC predictions of the new aircraft, and this uncertainty propagates to the assignment problem. The acquisition decision of the new aircraft is dependent on the budget, and the number of new aircraft in service and uncertainty in fleet operations will affect fleet-level performance metrics. The DOE approach can quantify the resulting uncertainty that propagated from the *system design* subspace to the *fleet operations* subspace.

***AMC Assignment Subspace***

The fleet assignment subspace identifies the optimal assignment of the fleet's aircraft to meet demand obligations; this includes allocation of the new aircraft—as described by the preceding aircraft sizing subspace—along with existing aircraft in the fleet. The resulting mathematical program of the allocation problem is given by the following set of equations:

Maximize

$$\sum_{p=1}^{P}\sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{N} x_{p,k,i,j}\cdot\left(Speed_{p,k,i,j}\cdot Pallet_{p,k,i,j}\right)$$ (Productivity = Speed x Capacity) (1)

Subject to

$$\sum_{p=1}^{P}\sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{N} x_{p,k,i,j}\cdot C_{p,k,i,j}\le M$$ (Fleet-level DOC or fuel limits) (2)

$$\sum_{i=1}^{N} x_{p,k,i,j}\ge \sum_{i=1}^{N} x_{p,k+1,i,j}\quad \forall k=1,2,3\ldots K,\ \forall p=1,2,3\ldots P,\quad \forall j=1,2,3\ldots N$$ (Node balance constraints) (3)

$$\sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{N} x_{p,k,i,j}\cdot BH_{p,k,i,j}\le B_P\quad \forall p=1,2,3\ldots P$$ (Trip constraints) (4)

$$\sum_{p=1}^{P}\sum_{k=1}^{K} Cap_{p,k,i,j}\cdot x_{p,k,i,j}\ge dem_{i,j}\quad \forall i=1,2,3\ldots N\ \ \forall j=1,2,3\ldots N$$ (Demand constraint) (5)

$$\sum_{i=1}^{N} x_{p,1,i,k}\le O_{p,i}\quad \forall p=1,2,3\ldots P,\ \forall i=1,2,3\ldots N$$ (Starting location constraints) (6)

$$x_{p,k,i,j}\in\{0,1\}$$ (Binary variable) (7)

Equation 26 describes of the objective of maximizing the fleet-level productivity, subject to a fleet DOC limit that enables a subsequent multi-objective optimization, node balance, trip, demand, and aircraft starting location constraints described in Equations 22–28; these follow the same description as Equations 2–6 of the monolithic problem formulation. Equation 28 describes the binary decision variable of the assignment subspace that takes a value of 1, if the $k^{th}$ trip of aircraft *p* is flown from base *i* to base *j*, and 0, otherwise. The "scheduling-like" formulation used in the problem is a surrogate for the cost and operation model, and it does not consider aircraft or pilot scheduling. However, the aircraft in the fleet are not required to return to their home base at the end of the day (a round-trip assumption is removed), and may continue servicing different routes. The Generic Algebraic Modeling System (GAMS) software package, accessed through a MATLAB interface, is used to solve the assignment problem, using the CPLEX solver option (Ferris, 1998). The “scheduling-like” formulation using node balance constraints allows individual aircraft to make multiple flight segments in one day (within a time limit), allows for pallets to be carried from their origin to destination on possibly multiple aircraft, and tracks each individual aircraft by “tail number.” These features more directly model AMC operations than some of the previous models from the author and his colleagues considering passenger airline transportation (Govindaraju et al., 2013; Mane et al., 2007).

### *Uncertainty in Fleet Operations*

The cost of operating a fleet is subject to the trip demand characteristics—a quantity that is typically uncertain. While passenger demand between origin-destination pairs is fairly constant for commercial or passenger airline route networks, the same cannot be said for AMC operations, which typically experience high levels of uncertainty in demanded trips and cargo size (Choi et al., 2013). To develop the example problem with relevance to AMC, the Global Air Transportation Execution System (GATES) dataset provides historical route and cargo demand data. These data enable the reconstruction of the route network, pallet demand characteristics, and existing fleet size for the fleet assignment problem. The GATES dataset reveals the variation in pallet demand (number of pallets transported on a route) over a year, reflecting the uncertainty associated with pallet demand in AMC operations. Thus, it becomes imperative for any systems designer/planner to consider the uncertainty in the network as part of the decision-making framework. Figure 3a shows the severity of fluctuation indicating the wide variation in the number of the pallets transported daily between two popular bases in the GATES dataset. Figure 3b, showing the histogram of the number of pallets transported per aircraft per day, reveals that for many of the days, aircraft are very lightly loaded. The demand uncertainty in the service network is addressed through a Monte Carlo Sampling (MCS) approach from literature (Mane & Crossley, 2012). The MCS technique uses prior demand data distribution information to generate random samples of demand realizations, based on the historical demand distributions for each route. The expected (arithmetic mean) fleet-level performance metrics are then averaged from the total number of Monte Carlo runs. We adopt a naïve MCS strategy for this work due to its ease of implementation. However, the MCS technique can prove to be computationally expensive with increasing sample sizes, due to the computational complexity of solving an integer program for each realized sample of demand and starting location for each aircraft in the fleet, and may warrant further work in using more efficient sampling techniques. The process of solving one aircraft sizing problem and several assignment problems is repeated for different factorial experiments. The expected fleet-level productivity and cost values from each of the DOE experiments are then passed on to the top level.

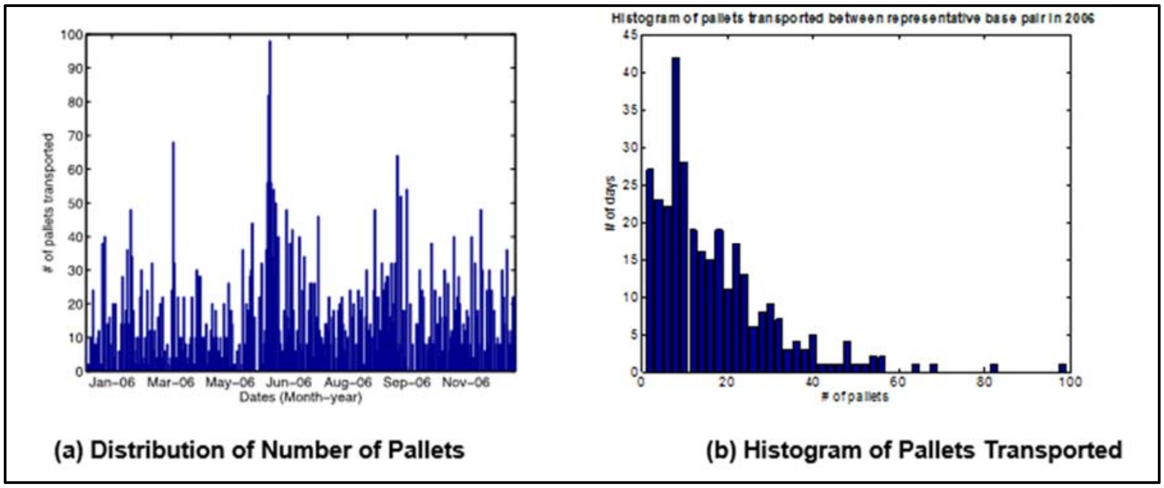


**Figure 3. Pallets Transported on a Sample Route From GATES Dataset**

## Three-Base Network Problem

### *Network Description*

A simple, illustrative sample problem for AMC operations, consisting of six directional routes between three bases is devised as an initial study. The airbase locations and the route data are extracted from the GATES dataset. Figure 4 depicts the geographical locations of the bases and the average daily pallet demand between the bases of the three-base network problem. The intent is to assign aircraft on various routes to satisfy the network cargo demand.


**Figure 4. Schematic of Three-Base Network Problem**

### *Three-Base Network Results*

The three bases in the sample network (ETAR, OKAS, and OTBH,) are the most flown routes in the GATES dataset for May 2006. The actual size of the strategic airlift fleet dedicated to cargo transport is obtained from the GATES dataset by accumulating unique tail numbers, resulting in a fleet composition of 92 C-5s, 145 C-17s, and 69 B747-Fs. The reduced existing fleet size for the three-base network problem consists of three of each aircraft type (C-5, C-17, and B747-F, which is assumed to be operated as a chartered aircraft). One new aircraft (aircraft type X) is introduced to the fleet. The new aircraft, in addition to the existing fleet, serve to satisfy the pallet cargo demand on various routes subject to the scheduling and capacity constraints.

Using the aforementioned subspace decomposition and a partial enumeration for the top-level search, the author conducted a full factorial DOE with numerous Monte Carlo simulations to obtain a description of the new aircraft requirements (top-level decision variables), the new aircraft description (the aircraft sizing design variables), and the corresponding allocation of the new aircraft along with the existing aircraft fleet (the assignment decision variables). For each discrete combination of the top-level values (pallet capacity, design range, and cruise speed), we conducted a DOE in the uncertain aircraft design. For each “experiment” in the DOE, one aircraft sizing subspace problem solution describes the new aircraft. Then, for each “experiment” in the DOE, considering the assignment problem with uncertain demand, the Monte Carlo approach solves several assignment problems, each with a different cargo demand from samples generated using the AMC GATES data. Each of these assignment solutions must meet a fixed constraint on fleet DOC, in addition to the other operational constraints. The subspace decomposition approach is solved sequentially until the top-level converges. Then, to identify the tradeoff between fleet operating cost, which is strongly related to fuel consumption, and the expected productivity, the entire process repeats using a different limit value on fleet DOC.

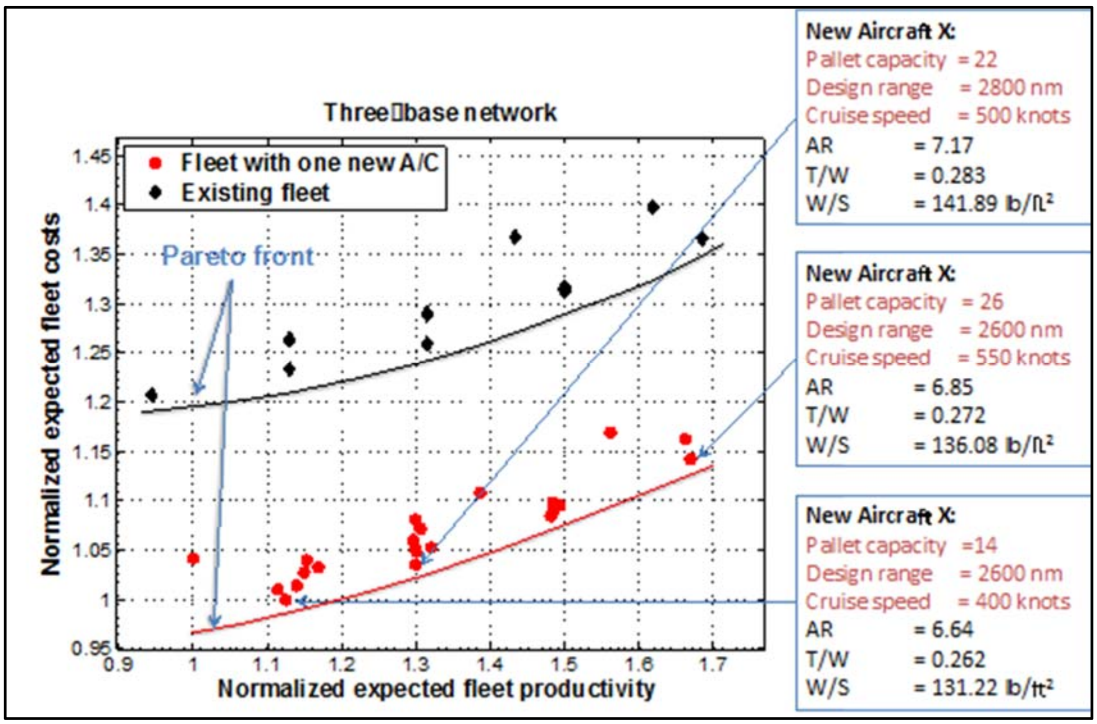


**Figure 5. Pareto Front of Normalized Expected Fleet Productivity and Normalized Expected Fleet Costs for Existing Fleet, and for Fleet Comprising of Existing Aircraft and One New Aircraft in the Three-Base Network Problem**

Figure 5 shows the Pareto front obtained from solving the multi-objective problem of the example three-base network. The plot shows only the expected values and does not depict the uncertainty associated with those computations. The expected fleet productivity and cost values shown in Figure 5 have been normalized with respect to the minimum productivity and minimum cost values from the outputs of the “Fleet with one new A/C” scenario. In other words, the lowest value of expected productivity obtained from the decomposition process and the top-level partial enumeration equals 1.0 for the “Fleet with one new A/C” scenario; similarly, the lowest value of fleet DOC obtained equals 1.0.

The Pareto front is the set of *Pareto optimal* solutions, that is, solutions from multi-objective problems that cannot be improved without worsening at least one of the other objectives. These solutions are called *non-inferior* or *non-dominated* solutions. The Pareto front provides a quantitative measure of how the optimal design of the new system varies, as a particular objective is traded off with another objective. As we move along the Pareto front depicted by the red curve, the design requirements of the optimum design of the new aircraft changes to maximize the fleet-level productivity at different fleet-level DOC limits. The Pareto front depicted by the black curve shows the variation in normalized fleet-level productivity and normalized fleet-level costs for a fleet comprising only of existing aircraft. The existing fleet serves as a “baseline” to measure the impact of the introduction of the new aircraft into the existing fleet. As Figure 5 indicates, the introduction of the new aircraft X decreases the fleet-level costs for specific fleet-level productivity values. For instance, the new aircraft X with a normalized expected fleet productivity value of 1.12 and a normalized expected fleet-level costs value of 1.0, is a “small” aircraft capable of transporting 14 pallets for a range of 2,600 nautical miles at a speed of 400 knots; this aircraft reduces the expected fleet-level costs by 21.2% in comparison to the “Existing fleet” scenario for the same normalized expected fleet productivity value. However, estimates of the expected fleet-level performance metrics vary because of the uncertainties present in the design of the new aircraft and in the fleet operations.

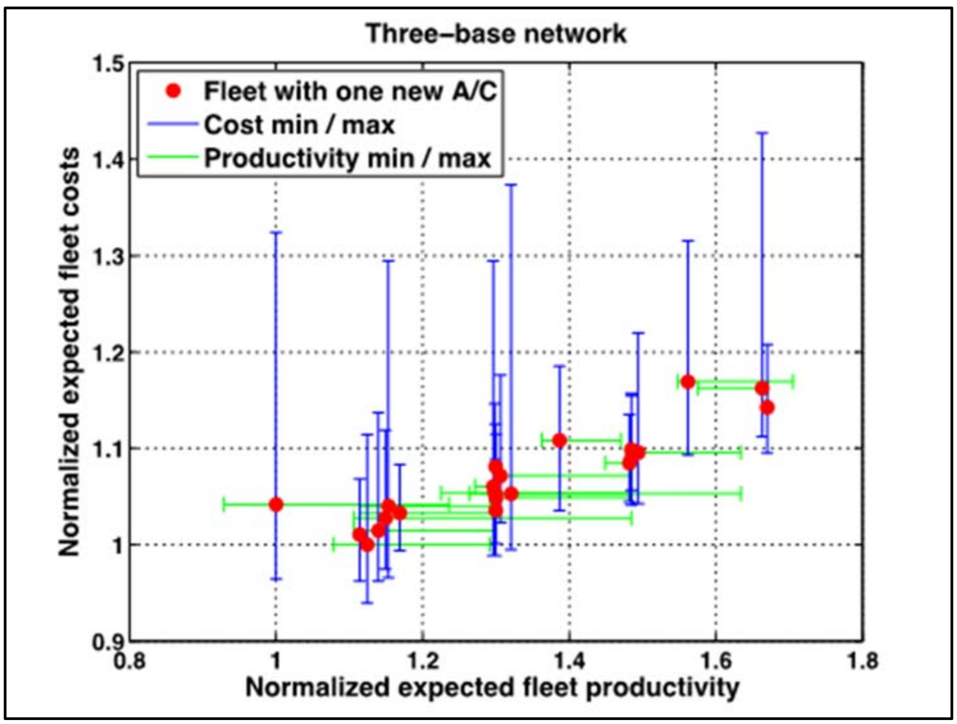


**Figure 6. Variation of Normalized Expected Fleet Productivity as a Function of Normalized Expected Fleet Costs for Fleet Comprising of Existing Aircraft and One New Aircraft in the Three-Base Network Problem**

Figure 6 shows the variation in fleet-level performance metrics and their corresponding maximum/minimum values. The solutions for the “Fleet with one new A/C” scenario shown in Figure 6 are identical to those shown in Figure 5. Each horizontal green error bar shows the maximum/minimum of the normalized expected fleet-level productivity, and each vertical blue error bar shows the maximum/minimum of the normalized expected fleet-level costs. As Figure 6 indicates, the “worst case” (maximum value) for the new aircraft X with a normalized expected fleet-level costs value of 1.12 still reduces the fleet-level costs by 9.8% in comparison to the "Existing fleet" scenario for the same normalized expected fleet productivity value. These results suggest that for several points along the Pareto front, the introduction of the new aircraft improves the fleet-level performance metrics significantly, even after accounting for uncertainties.

## Concluding Statements and Future Work

The results from this paper show promise in the applicability of the subspace decomposition framework combined with the DOE and Monte Carlo sampling approaches, to determine the optimum design requirements of engineering system(s) that work along with existing systems to provide a capability or set of capabilities considering the uncertainty in design parameters and in fleet operations. This methodology could help acquisition practitioners with a decision-support framework to determine the design requirements and the optimal design for different tradeoff opportunities of performance and cost under varying levels of uncertainties. Also, explicitly addressing uncertainty in design process can lead to more reliable/robust design recommendations for acquisition practitioners to achieve fleet-level objectives. Solutions from these "design under uncertainty" problems provide insight about new systems, and these insights can inform acquisition decisions.

Also, solutions to multi-objective analyses will advance the knowledge base regarding how to perform "fuel/cost as an independent variable" tradeoffs and will enhance understanding about what features this kind of process should entail under various operational scenarios. This methodology can also provide a quantitative measure of the impact of the optimum design under uncertainty to the fleet-level objectives in comparison to the optimum design determined due to deterministic methods. While demonstrated for the design of aircraft systems, the subspace decomposition approach appears generalizable to other systems that require effective decision-making in setting requirements for new systems and platforms.

The studies presented here describe our initial steps in understanding the impact of considering design parameter uncertainties and demand uncertainties simultaneously from a *fleet-level* perspective. Future work will incorporate methods from robust optimization, statistical sampling, and reliability-based optimization methods to address computational tractability in dealing with these two forms of uncertainty.

## References


Allardice, R. R. (2012, March 5). AMC vice commander: Saving fuel secures the future—one gallon at a time. *Inside AMC*. Retrieved from http://www.amc.af.mil/news/story.asp?id=123292555

Baker, S. F., Morton, D. P., Rosenthal, R. E., & Williams, L. M. (2002). Optimizing military airlift. *Operations Research, 50*(4), 582–602.

Beyer, H. G., & Sendhoff, B. (2007). Robust optimization—A comprehensive survey. Computer Methods in *Applied Mechanics and Engineering, 196*(33), 3190–3218.

Bolkcom, C. (2007, June). *Military airlift: C-17 Aircraft Program* (Congressional Research Service Report RL30685). Retrieved from http://www.dtic.mil/cgi-bin/GetTRDoc?AD=ADA470260

Choi, J., Govindaraju, P., Davendralingam, N., & Crossley, W. (2013a). Platform design for fleet-level efficiency: Application for Air Mobility Command (AMC). In *Proceedings of the 10th Annual Acquisition Research Symposium.* Monterey, CA: Naval Postgraduate School.

Choi, J., Govindaraju, P., Davendralingam, N., & Crossley, W. (2013b). Platform design for fleet-level efficiency under uncertain demand: Application for Air Mobility Command (AMC). In *Proceedings of the 2013 Aviation Technology, Integration, and Operations Conference (ATIO).*

Deputy Under Secretary of Defense for Acquisition, Technology, and Logistics (DUSD[AT&L]). (2012). *Energy efficiency starts with the acquisition process* [Fact sheet]. Retrieved from http://www.acq.osd.mil/asda/docs/fact_sheets/energy_efficiency_starts_with_the_acquisition_process.pdf

Ferris, M. C. (1998). MATLAB and GAMS: Interfacing optimization and visualization software. *Mathematical Programming Technical Report* (98-19), 21.

Govindaraju, P., & Crossley, W. A. (2013, August). Profit motivated airline fleet allocation and concurrent aircraft design for multiple airlines. In *Proceedings of the Aviation Technology, Integration, and Operations Conference*.

Govindaraju, P., Davendralingam, N., & Crossley, W. (2014). Multi-objective optimization of fleet-level metrics to determine new system design requirements: An application to military air cargo fuel efficiency. In *Proceedings of the 11th Annual Acquisition Research Symposium*. Monterey, CA: Naval Postgraduate School.

Jackson, J. (2009). *Mobility capabilities and requirements study 2016 accreditation report. Volume 1: Summary* (No. IDA-P-4475). Alexandria, VA: Institute for Defense Analyses.

Jackson, P., Peacock, L. T., & Munson, K. (2004). *Jane's All the World's Aircraft, 2003–2004.* ISBN 0-7106-2537, 5.

Listes, O., & Dekker, R. (2005). A scenario aggregation–based approach for determining a robust airline fleet composition for dynamic capacity allocation. *Transportation Science, 39*(3), 367–382.

Mane, M., & Crossley, W. A. (2006, September). Preliminary cost feasibility study of air taxi operations. In *Proceedings of the 6th Aviation Technology, Integration, and Operations Conference (ATIO).*

Mane, M., & Crossley, W. A. (2012). Assignment and design of aircraft for on-demand air transportation with uncertain operations. *Journal of Aircraft, 49*(1), 141–150.

Mane, M., Crossley, W. A., & Nusawardhana, A. (2007). System-of-systems inspired aircraft sizing and airline resource assignment via decomposition. *Journal of Aircraft, 44*(4), 1222–1235.

Mavris, D. N., Bandte, O., & DeLaurentis, D. A. (1999). Robust design simulation: A probabilistic approach to multidisciplinary design. *Journal of Aircraft, 36*(1), 298–307.

Montgomery, D. C. (2008). *Design and analysis of experiments*. John Wiley & Sons

Raymer, D. P. (2006). *Aircraft design: A conceptual approach*. Reston, VA: American Institute of Aeronautics and Astronautics.

Schuëller, G. I., & Jensen, H. A. (2008). Computational methods in optimization considering uncertainties—An overview. *Computer Methods in Applied Mechanics and Engineering, 198*(1), 2–13.

Taylor, C., & Weck, O. D. (2007). Coupled vehicle design and network flow optimization for air transportation systems. *Journal of Aircraft, 44*(5), 1479–1486.

U.S. Air Force. (2013). *Air Mobility Command* [Fact sheet]. Retrieved from http://www.amc.af.mil/library/factsheets/factsheet.asp?id=229

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