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Availability-Based Importance Framework for Supplier Selection

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Availability-Based Importance Framework for Supplier Selection

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Abstract

The availability of aging systems, particularly weapons systems within the Department of Defense, is of significant concern as budgets tighten and system replacement is infeasible. This work addresses the selection of sole suppliers according to their ability to provide component parts that strengthen availability of the system. We extend a popular multi-criteria decision-making approach, TOPSIS, by (i) considering the availability of individual components as the criteria in the decision problem and (ii) weighting those criteria according to the value of component importance measures while (iii) accounting for uncertainty in underlying reliability and maintainability parameters with interval numbers.

Introduction

The Department of Defense (DoD) uses three primary metrics to measure the quality of one of its systems: reliability, maintainability, and availability (DoD, 2005). Particularly within the U.S. Air Force, high quality aircraft equipment requires high performance values for all three metrics: reliable (ability to last as long as intended) and maintainable (ability to be fixed with minimum effort and time) to make the aircraft equipment available (accessible when needed). Availability, or the probability that a system is performing its required function at a given point in time when operated and maintained in a prescribed manner (Ebeling, 2010), is perhaps the key metric of the three.

The Government Accountability Office (GAO; 2011) recently found that the DoD does not effectively consider tradeoffs among cost, schedule, and performance when analyzing system requirements. The DoD has recently adopted a "Better Buying Power" mantra (*Defense AT&L*, 2011), identifying 23 efficiency-related initiatives, including mandating affordability within system requirements. Ashton B. Carter (2010), Under Secretary of Defense, stated,

> We have a continuing responsibility to procure the critical goods and services our forces need in the years ahead, but we will not have ever-increasing budgets to pay for them. We must therefore strive to achieve what economists call productivity growth: in simple terms, to do more without more.

The availability of DoD systems is threatened by obsolescence. For example, in the U.S. Air Force, the cost to replace over 500 KC-135s, which debuted in the mid-1950s, has been estimated in the tens of billions of dollars with a replacement plan lasting for several decades (GAO, 2004). A budget reduction of about 29% since 1990 has "forced the branches of the military to extend the life of current legacy systems with significant reductions in new acquisitions of replacement systems" (Maithaisel, 2008). One particular area of need within the DoD is in considering maintenance resources during supplier selection (e.g., for the F-35 Joint Strike Fighter [GAO, 2013]).

Routinely in the DoD, reliability and availability have been the focus of maintenance decision making for weapon systems, but not necessarily for the individual parts or components that make up the system. To build an available system, availability must be considered at the component level. In this paper, we focus on making appropriate supplier selection decisions to emphasize component availability. We assume that several suppliers can provide the same component part but with varying levels of reliability (mean time between failures) and maintainability (mean time to repair), the two constituents of availability. As such, we develop a supplier selection framework driven by component availability importance.

We address the need to make acquisition decisions with system availability in mind, proposing an availability-based sole supplier selection framework that accounts for uncertainty in the reliability and maintainability perspectives of availability. In doing so, we provide a supplier selection framework to "do more without more" by accounting for availability, thereby addressing concepts of reliability and maintainability in the procurement process. Often, supplier selection decisions ultimately come down to procurement cost, though we look beyond to availability, a driver of subsequent costs. We extend a traditional weighted multi-criteria discrete comparison technique, TOPSIS, to make this comparison. This work is primarily derived from two published papers by Gravette and Barker (2014) and Hague et al. (2015).

Background

This section provides background on availability and importance measure calculations, interval arithmetic, and some approaches for making comparisons among discrete alternatives.

Availability

Mentioned previously, availability is a very common measure in reliability engineering, particularly for systems whose function is needed at a moment's notice. Availability is calculated from uptime and downtime. *Uptime*, a function of reliability, is defined as the average time during which an asset or system is either fully operational or is ready to perform its intended function. *Downtime* measures how long a system is not in function, likely due to maintenance activities, suggesting that downtime is synonymous with maintainability. The traditional functional relationship for availability is shown in Equation 1, with mean time between failure (MTBF) as a measure of uptime and mean time to repair (MTTR) as a measure of downtime (Lie et al., 1977).

$$\text{Availability} = \frac{\text{uptime}}{\text{uptime} + \text{downtime}} = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}} \tag{1}$$

System Configurations and Importance Measures

A system is comprised of multiple components or subsystems. Common system configurations are shown in Figure 1. A simple series system with *n* components, where each component must be in operating condition for the system to operate, is represented in Figure 1a. Figure 1b portrays a parallel system with *m* components, where each individual component does not have to be in operating condition for the system to operate due to redundancy. Slightly more complex system structures appear in Figure 1c, a series-parallel system (a series of *n* subsystems each with parallel component configurations). Figure 1d is a parallel-series system (*m* series subsystems in parallel).

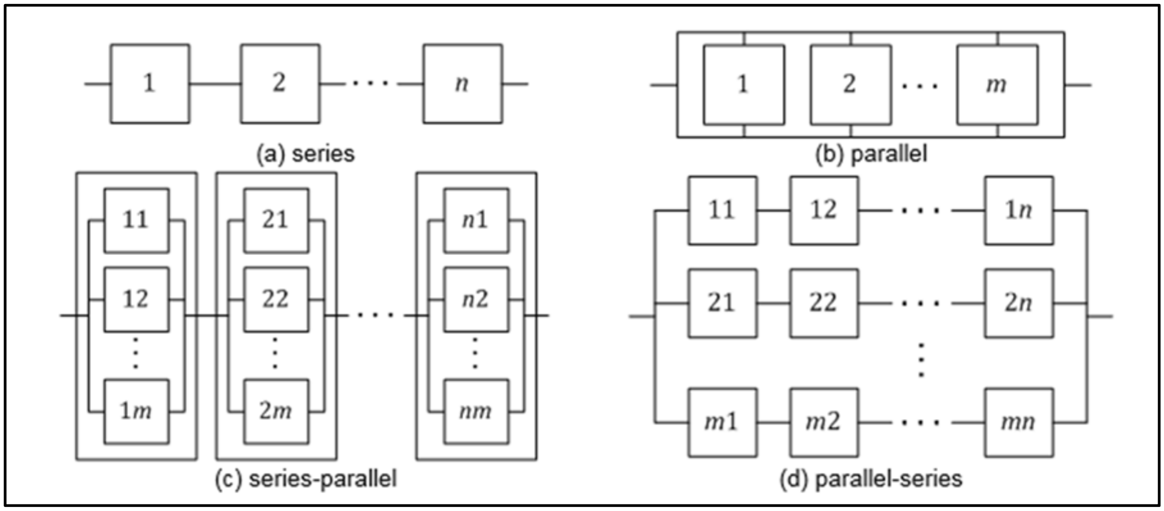

Figure 1. Four Primary Configurations That Describe the Structure of Most Systems

The illustrative example used subsequently in this paper is the aircraft servo-actuation series-parallel system. In a series-parallel system, there are multiple components, each with their own criticality to the performance of the system. We can measure the importance of each component in contributing to overall system performance with the calculation of component importance measures (IMs). IMs allow for the ranking of components from most important to least.

Reliability is the most common measure of system performance for applying IMs (Kuo & Zuo, 2003; Modarres et al., 2010). That is, in the reliability context, IMs highlight the components that are most critical to system reliability. IM examples include risk reduction worth (RRW), an index that quantifies the potential damage to a system caused by a particular component, and the reliability achievement worth (RAW) of a component, or the maximum proportion increase in system reliability generated by that component (Ramirez-Marquez et al., 2006). This work will focus on the Birnbaum IM (Birnbaum, 1969), shown in Equation 2. Where R_s measures system reliability and R_i measures the reliability of component *i*, the Birnbaum IM, I_i^B, measures the change in system reliability due to a change in the reliability of component *i*. The component with the largest I_i^B value is the component that offers the greatest improvement in system reliability when its reliability is improved.

$$\mathrm{I}_i^B = \frac{\partial R_s}{\partial R_i} \qquad (2)$$

As availability is the primary system performance measure of interest in this paper, we adopt a Birnbaum importance measure for availability (Barabady & Kumar, 2007; Cassady et al., 2004; Gravette & Barker, 2014), shown in Equation 3. The availability of the system and the availability of component *i* are represented with A_s and A_i, respectively.

$$I_i^A = \frac{\partial A_s}{\partial A_i} \qquad (3)$$

The availability of a series-parallel system is provided in Equation 4. Applying Equation 3 to this series-parallel system results in the importance measure in Equation 5 (Gravette & Barker, 2014). We will subsequently use Equation 5 to rank the importance of system components to prioritize suppliers of these components.

$$A^{\mathrm{SP}} = \prod_{i=1}^{n}\left[\coprod_{j=1}^{m} A_{\mathrm{a}_{ij}}\right] = \prod_{i=1}^{n}\left[1 - \prod_{j=1}^{m}\left(1 - \frac{\mathrm{MTBF}_{ij}}{\mathrm{MTBF}_{ij} + \mathrm{MTTR}_{ij}}\right)\right] \tag{4}$$

$$\mathrm{I}_{ij}^{\mathrm{SP}} = \frac{\partial \mathrm{A}^{SP}}{\partial \mathrm{A}_{ij}} = \prod_{k \neq i}^{n}\left[1 - \prod_{l=1}^{m}(1 - \mathrm{A}_{kl})\right] \times \prod_{l \neq j}^{m}(1 - \mathrm{A}_{il}) \tag{5}$$

$$= \prod_{k \neq i}^{n}\left[1 - \prod_{l=1}^{m}\left(1 - \frac{\mathrm{MTBF}_{kl}}{\mathrm{MTBF}_{kl} + \mathrm{MTTR}_{kl}}\right)\right] \times \prod_{l \neq j}^{m}\left(1 - \frac{\mathrm{MTBF}_{il}}{\mathrm{MTBF}_{il} + \mathrm{MTTR}_{il}}\right)$$

Comparing Discrete Alternatives

The Technique for Order Preferences by Similarity to an Ideal Solution (TOPSIS; Hwang & Yoon, 1981) is a tool for multi-criteria decision making that operationalizes the concept of the compromise solution, or the option that is nearest to the best solution (or positive ideal solution) and farthest from the worst solution (or negative ideal solution). The idea behind TOPSIS is rooted in reference-dependent theory, wherein consumers evaluate alternatives in terms of gains and losses to a subjective reference point (Kahneman & Tversky, 1979). Multiple criteria are considered when determining the positive and negative ideal solution, and those criteria are weighted separately depending on decision-maker preferences. Several recent applications of comparing discrete alternatives with TOPSIS include project selection (Khalili-Damghani et al., 2013; Taylan et al., 2014), manufacturing decision making (Azadeh et al., 2011; Goyal et al., 2012), and enterprise systems (Rouhani et al., 2012; Ye, 2010).

For *B* different discrete alternatives, $b = 1, \ldots, B$, and C different objectives or performance criteria, $c = 1, \ldots, C$, each alternative exhibits performance ratings contained in set $X = \{x_{bc} | b = 1, \ldots, B; c = 1, \ldots, C\}$. As some criteria may be more important than others to the decision maker, the criteria are weighted with w_c, $c = 1, \ldots, C$. Performance ratings x_{bc} can be normalized if the various performance criteria exhibit different ranges (e.g., reliability on [0,1] along with costs in millions of dollars) with a variety of normalization approaches. For the application discussed subsequently, normalization will not be necessary.

The weighted performance rating of alternative *b* for criterion *c* is found in Equation 6. A number of approaches to assess attribute weights from decision makers could be used, including the Analytical Hierarchy Process (Saaty, 1990) or rank reciprocal approach (Barron & Barrett, 1996). We describe the use of the availability IM for determining this weight in the section titled Integrated Framework for Supplier Selection.

$$v_{bc} = w_c x_{bc} \tag{6}$$

The positive ideal solution has all the best attainable criteria values, while the negative ideal solution has all worst possible criteria values. The positive ideal solution, B^+, is found with Equation 7. Set C^+ represents the set of benefit criteria, where larger values of the criteria are preferred (e.g., profit, time between failure). Set C^- is the set of cost criteria, where smaller values of the criteria are preferred (e.g., expenditures, losses, travel time).

Equation 7 suggests that the positive ideal solution consists of those weighted performance ratings that maximize benefit criteria and minimize cost criteria. Likewise, the negative ideal solution, or the weighted performance ratings that represent the smallest from set C^+ and largest from set C^-, is provided in Equation 8.

$$B^+ = \{v_1^+, \dots, v_c^+, \dots, v_C^+\} = \left\{\left(\max_b v_{bc} \,|c \in C^+\right), \left(\min_b v_{bc} \,|c \in C^-\right)\right\} \quad (7)$$

$$B^- = \{v_1^-, \dots, v_c^-, \dots, v_C^-\} = \left\{\left(\min_b v_{bc} |c \in C^+\right), \left(\max_b v_{bc} |c \in C^-\right)\right\} \quad (8)$$

The Euclidean distance between the performance ratings of alternative b and B^+ is found in Equation 9. The distance from the positive ideal solution for alternative b is referred to as D_b^+. Likewise, the Euclidean distance between alternative b and B^- is found in Equation 10 and is referred to as D_b^-.

$$D_b^+ = \sqrt{\sum_{c=1}^{C} (v_{bc} - v_c^+)^2} \quad (9)$$

$$D_b^- = \sqrt{\sum_{c=1}^{C} (v_{bc} - v_c^-)^2} \quad (10)$$

The preference order of alternatives can then be generated by ordering the measure in Equation 11 in descending order. $D_b^\star$ is a measure of the similarity to the positive ideal solution.

$$D_b^\star = \frac{D_b^-}{D_b^+ + D_b^-} \quad (11)$$

Interval Arithmetic

Point estimates (e.g., MTBF, MTTR) often do not effectively portray the uncertainty associated with their underlying random variables (e.g., time between failures, repair time). As such, we opt to not use point estimates for failure time and repair time in the calculation of availability in Equation 1. An approach where these uncertain parameters are described by probability distributions is always preferred when distributions are known, as one could address the problem with, for example, Monte Carlo simulation. However, when such probability distributions are not known, "forcing" a distribution may do more harm to the decision-making process than good (Huber, 2010). This is particularly true when developing distributions for failure time or repair time during the requirements development process in system design.

Addressing such uncertainty in the TOPSIS technique has been done with an extension using fuzzy numbers to deal with uncertainty in the set of performance ratings, *X* (e.g., Chen et al., 2006; Samvedi et al., 2013; Vahdani & Zandieh, 2010). We instead represent uncertainty in these failure time and repair time parameters with interval values, assuming we can bound the parameters with minimum and maximum values. If we can only assume the upper and lower bounds, we should "consider what decisions we could reach for all possible values of those data that are consistent with those interval constraints" (Huber, 2010).

An interval number is an ordered pair of real numbers $\left[\underline{y}, \overline{y}\right]$ such that $\underline{y} \leq \overline{y}$, where the underbar represents the lower bound of the interval and the overbar represents the upper bound. For interval numbers $Y = [\underline{y}, \overline{y}]$ and $Z = [\underline{z}, \overline{z}]$, the following algebraic relationships hold (Moore, 1966).

$$Y + Z = [\underline{y}, \overline{y}] + [\underline{z}, \overline{z}] = [\underline{y} + \underline{z}, \overline{y} + \overline{z}] \quad (12)$$

$$Y - Z = [\underline{y}, \overline{y}] - [\underline{z}, \overline{z}] = [\underline{y} - \overline{z}, \overline{y} - \underline{z}] \quad (13)$$

$$Y \times Z = [\underline{y}, \overline{y}] \times [\underline{z}, \overline{z}] = \left[\min\left(\underline{y} \times \underline{z}, \underline{y} \times \overline{z}, \overline{y} \times \underline{z}, \overline{y} \times \overline{z}\right), \max\left(\underline{y} \times \underline{z}, \underline{y} \times \overline{z}, \overline{y} \times \underline{z}, \overline{y} \times \overline{z}\right)\right] \quad (14)$$

$$Y/Z = [\underline{y}, \overline{y}] / [\underline{z}, \overline{z}] = \left[\min\left(\underline{y}/\underline{z}, \underline{y}/\overline{z}, \overline{y}/\underline{z}, \overline{y}/\overline{z}\right), \max\left(\underline{y}/\underline{z}, \underline{y}/\overline{z}, \overline{y}/\underline{z}, \overline{y}/\overline{z}\right)\right], \text{ where } 0 \notin [\underline{z}, \overline{z}] \quad (15)$$

$$Y^2 = \left[\min\left(\underline{y}^2, \left|\underline{y} \times \overline{y}\right|, \overline{y}^2\right), \max\left(\underline{y}^2, \left|\underline{y} \times \overline{y}\right|, \overline{y}^2\right)\right] \quad (16)$$

Other properties include the following (Neumaier, 1990):

$$\frac{1}{Y} = \left[\frac{1}{\overline{y}}, \frac{1}{\underline{y}}\right], \text{where } 0 < y_1 < y_2 \quad (17)$$

$$\alpha \times Y = \alpha \times [\underline{y}, \overline{y}] = [\underline{y}, \overline{y}] \times \alpha = [\alpha \times \underline{y}, \alpha \times \overline{y}] \text{ , for real constant } \alpha \geq 0 \quad (18)$$

Ultimately, there are instances where two intervals will be compared to each other (e.g., determining which suppliers' interval availability is preferred to another). For intervals $Y = [\underline{y}, \overline{y}]$ and $Z = [\underline{z}, \overline{z}]$, assume that *Y* is preferred to *Z* when a maximum value of the interval is sought. Barker and Rocco (2011) provide several decision rules for comparing intervals shown in Equation 19 that reflect different levels of risk aversion.

$$Y \succ Z \Leftrightarrow \begin{cases} \underline{y} > \underline{z} & \text{Best case} \\ \overline{y} > \overline{z} & \text{Worst case} \\ \left(\underline{y} + \overline{y}\right) > \left(\underline{z} + \overline{z}\right) & \text{Laplace} \\ \theta\left(\underline{y} - \underline{z}\right) > (1-\theta)(\overline{y} - \overline{z}), \theta \in [0,1] & \text{Hurwicz} \\ \left(\overline{y} - \underline{z}\right) > \left(\overline{z} - \underline{y}\right) & \text{Min regret} \end{cases} \quad (19)$$

Integrated Framework for Supplier Selection

Dickson (1966) introduced 23 supplier selection criteria still found in literature today, including quality, delivery, performance history, and price. Many have recently applied TOPSIS to a subset of these criteria for supplier selection (Awasthi et al., 2010; Kasirian & Yusuff, 2013; Liao & Kao, 2011; Wang et al., 2009). In this work, we focus on the *availability* aspect of supplier quality. That is, we want to select component (or service) suppliers based on their ability to maintain a level of availability in the system of interest. And an innovation of this work comes from how we weight the importance of component availability with the availability CIM provided in Equation 3. Ultimately, we choose a sole supplier who can supply the most important components of the system such that system availability is maintained.

We assume that the desired component and system availability can be derived from system requirements or from documentation from the original equipment manufacturer (OEM). We will derive component importance from these original requirements and later compare how different suppliers meet these availability requirements. However, when we assume that component design specifications come from system requirements, we could naturally conclude some uncertainty associated with the two main elements of the availability calculation, MTBF and MTTR. Such uncertainty could exist particularly when a new system is under development or is being redesigned, and failure or repair histories do not exist. Assume, however, that intervals can effectively quantify these design parameters. Notation for the intervals of the two availability parameters are $\mathrm{MTBF} = [\underline{\mathrm{MTBF}}, \overline{\mathrm{MTBF}}]$ and $\mathrm{MTTR} = [\underline{\mathrm{MTTR}}, \overline{\mathrm{MTTR}}]$.

The following subsections develop the four steps of the interval-valued availability framework for supplier selection. The series-parallel configuration in Figure 2 illustrates the framework within each step. The system, which mentioned previously could represent an aircraft servo-actuation system, consists of three subsystems in series, where each subsystem is a collection of components (servo controllers, servo actuators, and power sources) arranged in parallel. While this example is notional, a servo-actuation system is an important subsystem in an aircraft flight control system. Other configurations beyond the series-parallel system could be explored, including network configurations, assuming that computational requirements for calculating system availability and availability component importance are not too great.

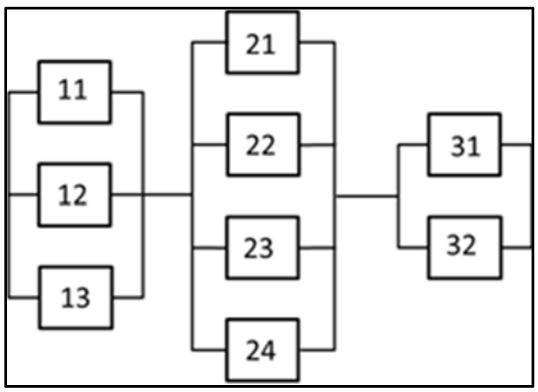

Figure 2. The Series-Parallel System Serving as the Illustrative Example for the Supplier Selection Framework

Step 1: Calculate the Interval-Valued Availability Importance for Each Component

Before considering any of the suppliers, we want to understand the importance of each component with respect to its contribution to system availability. For the general series-parallel representation in Figure 1c, Equation 20 integrates the availability IM with the interval representations of mean failure and repair times.

$$\mathrm{I}_{ij}^{SP} = \frac{\partial \mathrm{A}^{SP}}{\partial \mathrm{A}_{ij}} = \prod_{k \neq i}^{n} \left[1 - \prod_{l=1}^{m} \left(1 - \frac{[\underline{MTBF}, \overline{MTBF}]_{kl}}{[\underline{\mathrm{MTBF}}, \overline{MTBF}]_{kl} + [\underline{\mathrm{MTTR}}, \overline{MTTR}]_{kl}} \right) \right] \times \prod_{l \neq j}^{m} \left(1 - \frac{[\underline{MTBF}, \overline{MTBF}]_{il}}{[\underline{MTBF}, \overline{MTBF}]_{il} + [\underline{\mathrm{MTTR}}, \overline{MTTR}]_{il}} \right) \quad (20)$$

Applying the interval arithmetic rules in Equations 12, 15, and 18, the ratio in Equation 20 becomes the following.

$$\frac{[\underline{\mathrm{MTBF}}, \overline{\mathrm{MTBF}}]}{[\underline{\mathrm{MTBF}}, \overline{\mathrm{MTBF}}] + [\underline{\mathrm{MTTR}}, \overline{\mathrm{MTTR}}]} = \Big[\min\Big\{(\underline{\mathrm{MTBF}}^2 + (\underline{\mathrm{MTBF}} \times \underline{\mathrm{MTTR}})), ((\underline{\mathrm{MTBF}} \times \overline{\mathrm{MTBF}}) + (\underline{\mathrm{MTBF}} \times \overline{\mathrm{MTTR}})), ((\overline{\mathrm{MTBF}} * \underline{\mathrm{MTBF}}) + (\overline{\mathrm{MTBF}} \times \underline{\mathrm{MTTR}})), (\overline{\mathrm{MTBF}}^2 + (\overline{\mathrm{MTBF}} \times \overline{\mathrm{MTTR}})\Big\}, \max\Big\{(\underline{\mathrm{MTBF}}^2 + (\underline{\mathrm{MTBF}} \times \underline{\mathrm{MTTR}})), ((\underline{\mathrm{MTBF}} \times \overline{\mathrm{MTBF}}) + (\underline{\mathrm{MTBF}} \times \overline{\mathrm{MTTR}})), ((\overline{\mathrm{MTBF}} \times \underline{\mathrm{MTBF}}) + (\overline{\mathrm{MTBF}} \times \underline{\mathrm{MTTR}})), (\overline{\mathrm{MTBF}}^2 + (\overline{\mathrm{MTBF}} \times \overline{\mathrm{MTTR}})\Big\}\Big] \tag{21}$$

Using constant ψ, the interval-valued availability importance measure is simplified in Equation 23.

$$\psi = \Big[(\underline{\mathrm{MTBF}}^2 + (\underline{\mathrm{MTBF}} \times \underline{\mathrm{MTTR}})), ((\underline{\mathrm{MTBF}} \times \overline{\mathrm{MTBF}}) + (\underline{\mathrm{MTBF}} \times \overline{\mathrm{MTTR}})), ((\overline{\mathrm{MTBF}} \times \underline{\mathrm{MTBF}}) + (\overline{\mathrm{MTBF}} \times \underline{\mathrm{MTTR}})), (\overline{\mathrm{MTBF}}^2 + (\overline{\mathrm{MTBF}} \times \overline{\mathrm{MTTR}})\Big] \tag{22}$$

$$\mathrm{I}_{ij}^{SP} = \frac{\partial \mathrm{A}^{SP}}{\partial \mathrm{A}_{ij}} = \prod_{k \neq i}^{n}\left[1 - \prod_{l=1}^{m}(1 - [\min(\psi_{kl}), \max(\psi_{kl})])\right] \times \prod_{l \neq j}^{m}(1 - [\min(\psi_{il}), \max(\psi_{il})]) \tag{23}$$

The importance of the components to system availability is a function of the design of the system, not the suppliers. Therefore, system requirements for MTBF and MTTR are used to parameterize Equation 23. For the illustrative example in Figure 2, the interval bounds for the availability parameters from the system design requirements for each component are found in Table 1.

Table 1. Component MTBF and MTTR Intervals, in Days

Component	$\underline{MTBF}$	$\overline{MTBF}$	$\underline{\mathrm{MTTR}}$	$\overline{MTTR}$
C_{11}	25	35	1	5
C_{12}	365	395	2	7
C_{13}	150	165	1	8
C_{21}	150	200	2	5
C_{22}	75	110	1	6
C_{23}	185	200	3	5
C_{24}	120	125	1	3
C_{31}	365	465	1	1.5
C_{32}	365	485	1	2

When applying the interval division rule in Equation 15, there will always be an instance where the denominator is less than the numerator when dividing one interval's maximum by another's minimum. This is problematic when calculating availability values, as the definition of availability requires that it be on (0,1). As such, we eliminate these possibilities with Equation 24, a reformulation of the interval division arithmetic. Resulting component availability and importance measure calculations are found in Table 2. IM results are given in several decimal places as some are very small in magnitude.

$$\frac{Y}{Z} = \frac{[\underline{y}, \overline{y}]}{[\underline{z}, \overline{z}]} = \left[\min\left(\frac{\underline{y}}{\underline{z}}, \frac{\underline{y}}{\overline{z}}, \frac{\overline{y}}{\underline{z}}, \frac{\overline{y}}{\overline{z}}\right), \max\left(\frac{\underline{y}}{\underline{z}}, \frac{\underline{y}}{\overline{z}}, \frac{\overline{y}}{\underline{z}}, \frac{\overline{y}}{\overline{z}}\right)\right], where\ 0 \leq \frac{Y}{Z} \leq 1 \quad (24)$$

Table 2. Component Availability Intervals

Component	$\underline{I_{ij}^{SP}}$	$\overline{I_{ij}^{SP}}$
C_{11}	0.000036089	0.012236505
C_{12}	0.000254704	0.049855491
C_{13}	0.000209593	0.034514925
C_{21}	0.000001735	0.002155172
C_{22}	0.000001735	0.001635931
C_{23}	0.000001430	0.005926724
C_{24}	0.000002762	0.009251472
C_{31}	0.002732233	0.250513347
C_{32}	0.002732233	0.217577706

Step 2: Rank the Components According to Availability Importance

As Equation 23 is a function of interval values, the resulting availability IM takes the form of an interval, as shown in Table 2. The larger the value of I_{ij}^{SP}, the more important is component ij. Discussed previously, a ranking of I_{ij}^{SP} provides a prioritization of components from most important to system availability to least. However, given that the availability IM is interval-valued, ordering the components in Table 2 is not a straightforward task. For example, the intervals for components 11 and 12 overlap with each other, making them indistinguishable without a decision rule.

We use the Laplace criterion from Equation 19 to show the order relationship when a maximum value is sought. The ranking of components appears in Table 3. Based on the system requirements, Table 3 suggests that components 31 and 32, the components in subsystem 3, are the most important in their contribution to system availability. Components within subsystem 2 would appear to be the least important. Different decision rules take different optimistic and pessimistic perspectives on the rankings of the intervals, though the Laplace rule is fairly risk neutral. The min regret rule from Equation 19 also produces the same ranking.

Table 3. Ranking of Components According to Their Interval-Valued Importance Measure Results

Component	Laplace criterion $\left(\underline{\mathrm{I}_{ij}^{SP}} + \overline{\mathrm{I}_{ij}^{SP}}\right)$	Rank
C_{11}	0.0123	5
C_{12}	0.0501	3
C_{13}	0.0347	4
C_{21}	0.0022	8
C_{22}	0.0016	9
C_{23}	0.0059	6
C_{24}	0.0093	7
C_{31}	0.2532	1
C_{32}	0.2203	2

Step 3: Calculate Weights for Components

Mentioned previously, we adopt the TOPSIS approach for selecting among alternatives when different evaluation criteria are considered. In this application, we select a sole supplier for all components based on its ability to supply component parts with good availability. Therefore, we consider every component to be an "evaluation criterion" in the selection of a supplier.

The TOPSIS approach requires that a weight be applied to each evaluation criterion. The availability importance measure from Step 2 gives us a means to weight each component. To scale the Laplace criterion result from Table 3 such that all weights sum to 1, Equation 25 is applied.

$$\mathrm{w}_{ij} = \frac{\left(\underline{\mathrm{I}_{ij}^{SP}} + \overline{\mathrm{I}_{ij}^{SP}}\right)}{\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\underline{\mathrm{I}_{ij}^{SP}} + \overline{\mathrm{I}_{ij}^{SP}}\right)} \tag{25}$$

The results of Table 3 and Equation 25 provide an objective approach to weighting the components according to their availability importance based on system design requirements. The result is provided in Table 4.

Table 4. Component Weighting Using Scaled Interval-Valued Importance Results

Component	$\left(\underline{I_{ij}^{SP}} + \overline{I_{ij}^{SP}}\right)$	Weight
C_{11}	0.0123	0.0208
C_{12}	0.0501	0.0850
C_{13}	0.0347	0.0589
C_{21}	0.0022	0.0037
C_{22}	0.0016	0.0028
C_{23}	0.0059	0.0101
C_{24}	0.0093	0.0157
C_{31}	0.2532	0.4295
C_{32}	0.2203	0.3736

Step 4: Apply TOPSIS to Select Supplier

The final step in the framework is to select a sole supplier. As we are selecting a sole supplier, any supplier alternatives that are unable to meet the system requirements are not be considered: We only choose among those suppliers whose availability (via MTBF and MTTR) outperforms the requirements. For each supplier S_b we evaluate their availability for each component, criterion c. Availability is an interval number, $\left[\underline{A_{S_b,c}}, \overline{A_{S_b,c}}\right]$. The results from the TOPSIS analysis will provide the supplier that is closest to the best availability for each component and farthest from the worst availability for each component, the differences for which are weighted according to each component's importance and summed across all components.

Table 5 depicts four suppliers and their interval availability for each component. There is considerable overlap among the component availabilities for each supplier, therefore requiring an analytical approach to determine which supplier is ideal.

Table 5. Interval-Valued Component Availabilities for Each Supplier

	Supplier							
	S_1		S_2		S_3		S_4	
Component	$\underline{A_{S_1,c}}$	$\overline{A_{S_1,c}}$	$\underline{A_{S_2,c}}$	$\overline{A_{S_2,c}}$	$\underline{A_{S_3,c}}$	$\overline{A_{S_3,c}}$	$\underline{A_{S_4,c}}$	$\overline{A_{S_4,c}}$
C_{11}	0.85	0.99	0.82	0.98	0.81	0.99	0.86	0.97
C_{12}	0.90	0.99	0.85	0.99	0.89	0.97	0.91	0.99
C_{13}	0.85	0.94	0.91	0.99	0.86	0.92	0.88	0.97
C_{21}	0.84	0.94	0.87	0.96	0.88	0.99	0.91	0.99
C_{22}	0.84	0.94	0.87	0.96	0.88	0.99	0.91	0.99
C_{23}	0.91	0.98	0.90	0.97	0.92	0.97	0.87	0.99
C_{24}	0.91	0.98	0.90	0.97	0.92	0.98	0.87	0.99
C_{31}	0.81	0.95	0.86	0.97	0.92	0.95	0.89	0.93
C_{32}	0.88	0.95	0.93	0.98	0.88	0.96	0.90	0.97

After the component weights from Table 4 are applied to Table 5 using Equation 6, the Laplace criterion is used to determine the positive and negative ideal solutions found in Equations 26 and 27. These solutions are provided in order by criterion, or C_{11} through C_{32}. For example, for component C_{23}, supplier S1 offers the part that provides the best and supplier S4 offers the least desired (weighted) interval availability.

$$B^{+} = \{0.038, 0.161, 0.112, 0.007, 0.005, 0.019, 0.030, 0.803, 0.714\}$$
$$= \{S_1, S_4, S_2, S_4, S_4, S_1, S_3, S_3, S_2\} \quad (26)$$

$$B^{-} = \{0.037, 0.156, 0.105, 0.007, 0.005, 0.019, 0.029, 0.756, 0.684\}$$
$$= \{S_2, S_2, S_3, S_1, S_1, S_4, S_4, S_1, S_1\} \quad (27)$$

To determine which supplier is ideal for *all* components, the separation between each alternative (supplier) and the B^{+} and B^{-} suppliers is calculated using the Euclidean distance equations Equations 9 and 10.

Table 6. Separation Between Each Supplier and the Ideal Solutions

Supplier	$\underline{D_b^+}$	$\overline{D_b^+}$	$\underline{D_b^-}$	$\overline{D_b^-}$
S_1	0.016	0.072	0.017	0.020
S_2	0.023	0.041	0.072	0.084
S_3	0.013	0.039	0.054	0.058
S_4	0.016	0.040	0.003	0.004

Finally, by applying Equation 11, the supplier that is closest to the best availability for each component within the system is calculated. Supplier S_3 provides the best overall offerings of the nine components in the system according to the weighted importance of each component and would therefore be the best sole supplier of the components in the aircraft servo-actuation series-parallel system if availability is the primary metric of interest.

Table 7. Final Supplier Ranking

Supplier	Laplace criterion $\left(\underline{D_b^*} + \overline{D_b^*}\right)$	Rank
S_1	0.631	4
S_2	1.596	3
S_3	1.857	1
S_4	1.607	2

Concluding Remarks

There has been much research in the area of supplier selection, much of which continues to follow Dickson's 23 criteria. The DoD tends to consider procurement cost very strongly when considering suppliers of component parts for weapons systems; however, another major source of subsequent costs is due to the unavailability of such systems. To be mission-ready, DoD systems must be available for use. As such, the objective of this paper is to provide an availability-based framework for choosing a supplier who can provide components that lead to a high system availability, focusing particularly on those components that are most important to system availability. Due to uncertainty in MTBF and MTTR component data, interval arithmetic provides a vehicle for making computations within a known range of data. Note that this is a fairly nuanced extension of a TOPSIS, though most any other multi-criteria decision analysis technique could take its place. Similarly, we focus on availability as a long-term cost driver for supplier selection, though other criteria could be considered in addition to or in place of availability. A novel idea provided in this framework is the treatment of component performance as the criteria in the multi-criteria comparison, with weights being derived by component importance measures from the field of reliability engineering.

With the modern economy and the current budgetary constraints placed upon the DoD, obtaining components with a high availability and reliability is vital to efficiency. In the world of maintaining equipment, having fewer corrective repairs translates into more time for technicians to focus on other tasks such as preventive maintenance, which also is a proponent of equipment reliability. The effects of this element of Better Buying Power can be felt throughout the DoD in the form of reliability, availability, cost avoidance, and better resource allocation.

This work provides an important first step in integrating component importance into supplier selection. A primary limitation of this framework is the assumption that sole suppliers are chosen, though this could certainly be a realistic assumption at the subsystem level (as the type of system represented in Figure 2 would be one of many subsystems in a larger system). Future work will explore the development of a supplier mix to meet reliability and maintainability needs.

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