Endogenous Split Awards as a Protest Management Tool:
A Modeling & Computational Approach

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Bid Protests & Split Awards: Agenda

- **Managing bid protests** in DoD procurement
- **Simple model** of bidding & protest process
- **Split awards** as a protest management tool
- Key question: What is the **right split?**
- Bids & prices with **fixed** split awards
- Bids & prices with **endogeneous** split awards
- **Conclusions**
- **Research agenda** moving forward

"Managing" Bid Protests

- **Objective is <u>not</u> to minimize** number of bid protests
- Protests intended to **correct procurement mistakes**
  - **Honest mistake**: Limited information & bounded rationality
  - **Dishonest mistake:** Bias or fraud by procurement officials
- Objective is to "**right size**" number of protests
  - Encourage protests that correct (significant) mistakes
  - Discourage protests that don't make significant corrections
- **Modeling** the process could help identify, compare, & characterize **levers of control** for managing protests

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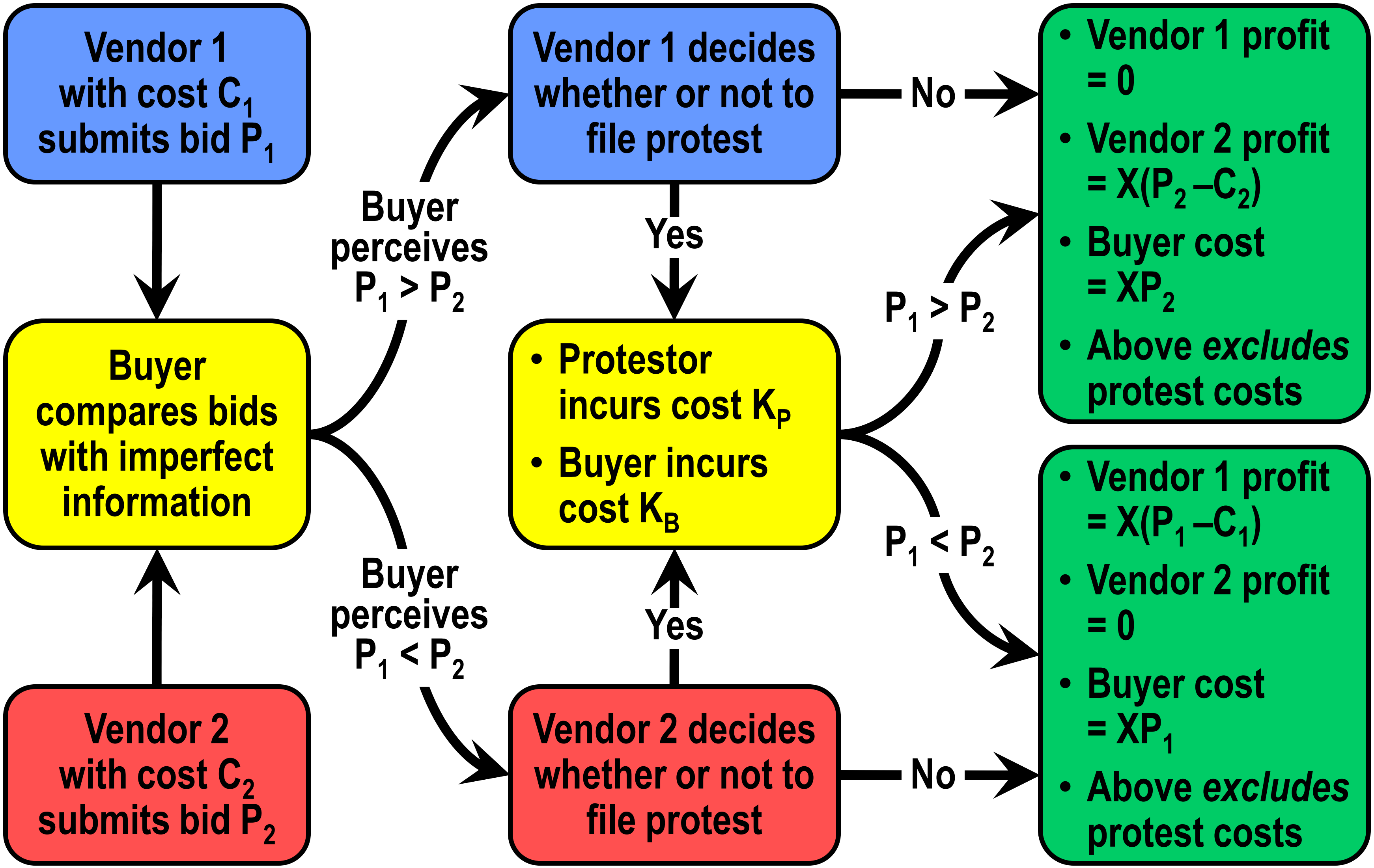
Modeling Bid Protests

- As noted, the intended **role of bid protests** is, in the most general terms, to correct procurement mistakes
- Such mistakes – whether honest or dishonest – result from some form of **imperfect decision-making**
  - **How best to model** such imperfection?
- Consider a model driven by **imperfect information**
  - Imperfect info ➔ **small mistake** more likely than **big mistake**
  - **Bias** ➔ small injustice more likely than big injustice
- Imperfect information consistent with **empirical results**
  - "**Agency mis-evaluation**" is by far the most commonly cited reason for sustaining a DoD bid protest (Gansler, et al.)

Simple Model of Bid & Protest Process



Managing Vendor Protest Incentives

- **Losing vendor 1 protests iff** $\text{Prob}(P_1<P_2)\times X-K_P > 0$
- Recall the **two goals** of protest management:
  1. Encourage/allow "good" or **efficient protests**
  2. Discourage "bad" or **inefficient protests**
- **Levers of control?**
  - **$\text{Prob}(P_1<P_2)$** ➔ Influence initial assessment accuracy
    ➔ Change or shift burden of proof
  - **K_P** ➔ Influence expected costs
    ➔ Different costs for successful vs. failed protests
  - **X** ➔ Influence gain from successful protest
    ➔ **Split awards**

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Bidding with *Fixed* Award Splits

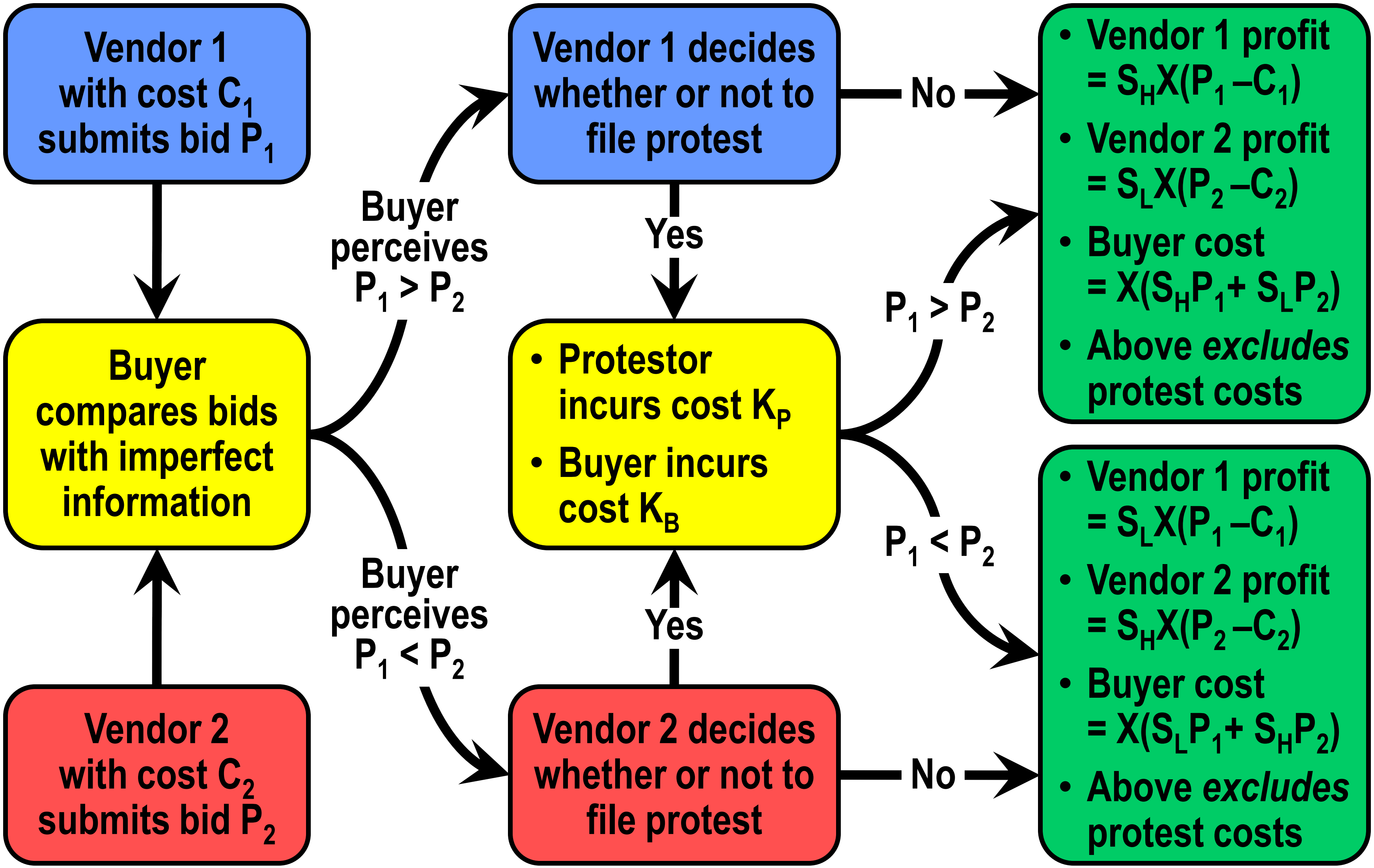
Contract splits:

- S_L = Share or split awarded **low-price** bidder
- S_H = Share or split awarded **high-price** bidder
- $S_L + S_H = 1$
- $0 \leq S_H \leq ½$ & $½ \leq S_L \leq 1$

Award Determination:

- If final decision is that $P_1 < P_2$:
  - Vendor 1 awarded contract to produce $S_L X$ units
  - Vendor 2 awarded contract to produce $S_H X$ units
- If final decision is that $P_1 > P_2$:
  - Vendor 1 awarded contract to produce $S_H X$ units
  - Vendor 2 awarded contract to produce $S_L X$ units

Bid & Protest Process with Split Awards



Revised Vendor Protest Incentives

- **Winner-take-all awards:** Losing vendor 1 protests **iff** $\text{Prob}(P_1 < P_2) \times X - K_P > 0$
- **Split awards:** Losing vendor 1 protests **iff** $\text{Prob}(P_1 < P_2) \times (S_L - S_H)X - K_P > 0$
- Split awards **raise the hurdle** for profitable protest
  - Is the hurdle high enough to limit "bad" protests?
  - Is the hurdle low enough to allow "good" protests?
- **Defacto split awards** already a *response* to protests
  - Alternative contracts, subcontracts, agency settlements, "Fed mail" buy-offs
  - Why not formalize this "under the table" process?

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Key Question: What is the Right Split?

- Higher S_H ➔ **lower protest incentive**
  - $E\Pi_1(\text{protest}) = \text{Prob}(P_1<P_2) \times (1-2S_H)X - K$
  - $\delta E\Pi_1(\text{protest})/\delta S_H = -2X \times \text{Prob}(P_1<P_2)$
- Higher S_H ➔ **higher total contract expense**
  - Winner-take-all cost = XP_L
  - Split-award cost = $X(S_H P_H+(1-S_H)P_L)$
  - Difference = $XS_H(P_H-P_L)$
- Higher S_H ➔ **incentive to submit higher bid**

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Equilibrium Bidding with Fixed Splits

Focus on bid-stage only (for now):

- Ignore "continuation value" of protest stage
  - Effect of protest on bidding strategy ambiguous
- Also ignore buyer's imperfect information
  - Assume buyer perfectly informed regarding P_1 & P_2
  - Symmetric imperfect info ➔ neutral impact

Expected profit function:

- $E\Pi_1(P_1) = X(P_1 - C_1)[Prob(P_1 > P_2)S_H + Prob(P_1 < P_2)S_L]$

  $= X(P_1 - C_1)[S_L - Prob(P_1 > P_2)(S_L - S_H)]$

Equilibrium Bidding with Fixed Splits

Expected profit function:

- Assume C_1, C_2 identically & independently distributed over interval [0,M]
- Symmetric bidding strategy λ(C)
  - λ: [0,M] ~ [0,M]
  - λ(M) = M

Equilibrium bidding strategy:

- $$l(C_1) = \frac{S_H M + (S_L - S_H)\left(1 - F(C_1)\right) E\left(C_2 \middle| C_2 > C_1\right)}{S_L - \left(S_L - S_H\right) F(C_1)}$$
- Complete derivation included in appendix

Equilibrium Bidding with Fixed Splits

- Let C_1, C_2 ~ U[0,100] ➔

$$I(C_1) = \frac{S_H M + (S_L - S_H)\left(1 - F(C_1)\right)E\left(C_2 \middle| C_2 > C_1\right)}{S_L - (S_L - S_H)F(C_1)}$$

$$I(C_1) = \frac{100S_H + (S_L - S_H)\left(1 - \tfrac{1}{100}C_1\right)\tfrac{1}{2}\left(C_1 + 100\right)}{S_L - \tfrac{1}{100}(S_L - S_H)C_1}$$

$$I(C_1) = \frac{20{,}000S_H + (S_L - S_H)\left(100 - C_1\right)\left(C_1 + 100\right)}{200S_L - 2(S_L - S_H)C_1}$$

$$I(C_1) = \frac{20{,}000S_H + \left(1 - 2S_H\right)\left(10{,}000 - C_1^2\right)}{200S_L - 2C_1(S_L - S_H)}$$

$$I(C_1) = \frac{20{,}000S_H + 10{,}000 - C_1^2 - 20{,}000S_H + 2S_H C_1^2}{200S_L - 2C_1(S_L - S_H)}$$

$$I(C_1) = \frac{10{,}000 + (2S_H - 1)C_1^2}{200S_L - 2C_1(S_L - S_H)} = \frac{10{,}000 - (S_L - S_H)C_1^2}{200S_L - 2C_1(S_L - S_H)}$$

Equilibrium Bidding with Fixed Splits

$$S_H = 0 \Rightarrow \quad \lambda(C_1) = \frac{10{,}000 - C_1^2}{200 - 2C_1} = \frac{(100 - C_1)(100 + C_1)}{2(100 - C_1)} = 50 + \tfrac{1}{2}C_1$$

$$S_H = 0.1 \Rightarrow \quad S_L = 0.9 \Rightarrow \quad \lambda(C_1) = \frac{10{,}000 - 0.8C_1^2}{180 - 1.6C_1}$$

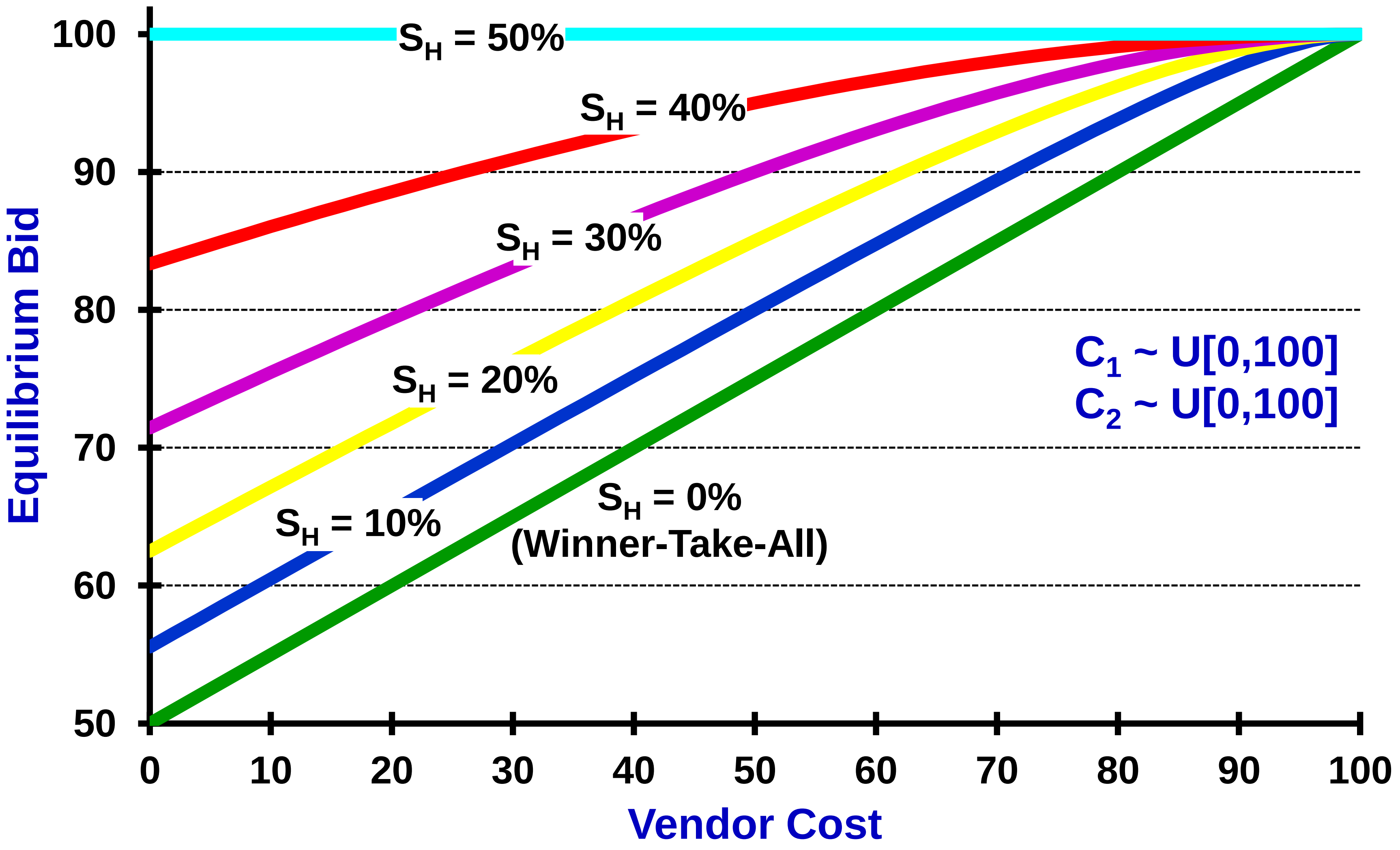
$$S_H = 0.2 \Rightarrow \quad S_L = 0.8 \Rightarrow \quad \lambda(C_1) = \frac{10{,}000 - 0.6C_1^2}{160 - 1.2C_1}$$

$$S_H = 0.3 \Rightarrow \quad S_L = 0.7 \Rightarrow \quad \lambda(C_1) = \frac{10{,}000 - 0.4C_1^2}{140 - 0.8C_1}$$

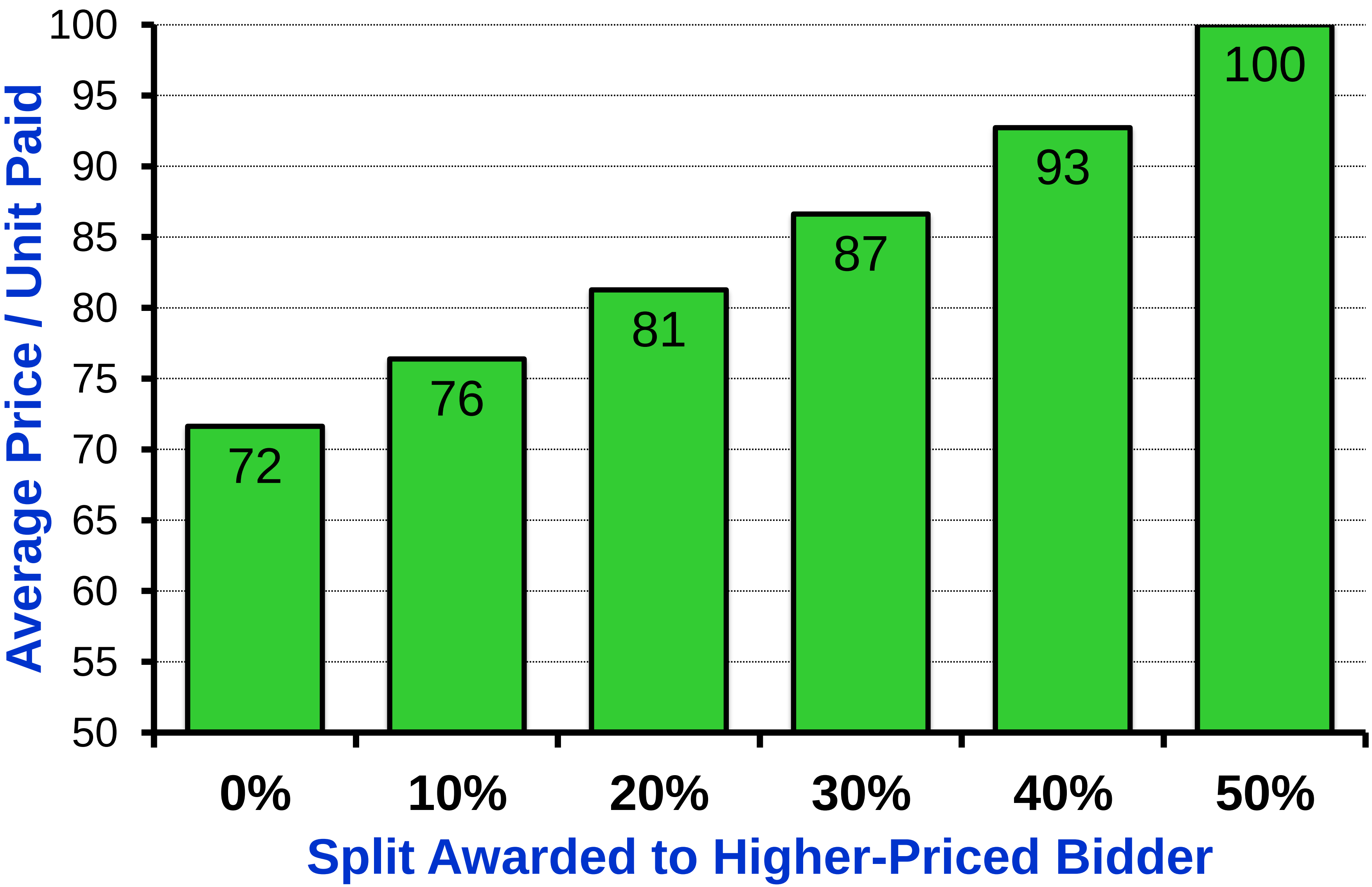
$$S_H = 0.4 \Rightarrow \quad S_L = 0.6 \Rightarrow \quad \lambda(C_1) = \frac{10{,}000 - 0.2C_1^2}{120 - 0.4C_1}$$

$$S_H = 0.5 \Rightarrow \quad S_L = 0.5 \Rightarrow \quad \lambda(C_1) = \frac{10{,}000 - 0}{100 - 0} = 100$$

Equilibrium Bidding with Fixed Splits



Average Price / Unit with Fixed Splits



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Extension: Endogenous Split Awards

- Split awards **reduce frequency** of bid protest ☺
- BUT **2 cost inflation effects** from split awards ☹
  - Direct additional cost = $XS_H(P_H - P_L)$
  - Indirect additional cost = bid inflation
- ***Note:*** Both inflation effects mitigated if size of S_H is **inversely related to $(P_H - P_L)$**
- Potential solution: **Endogenous split awards**
  - Let $R_L = P_L / P_H$ (such that $0 \leq R_L \leq 1$)
  - Let $S_H = F(R_L)$
  - $0 \leq F(R_L) \leq ½$
  - $F(R_L)$ increasing in R_L

Example Split Award Function

- Let $S_H = \alpha R_L^{\beta}$
  - α = **maximum share** to high-price bidder ($0 \leq \alpha \leq ½$)
  - $\beta \geq 0$
  - S_H is **increasing in α & R_L**
  - S_H is **decreasing in β**
- Buyer decision: What are the **best α & β?**

Split Award Scenarios with $S_H = \alpha R_L^{\beta}$

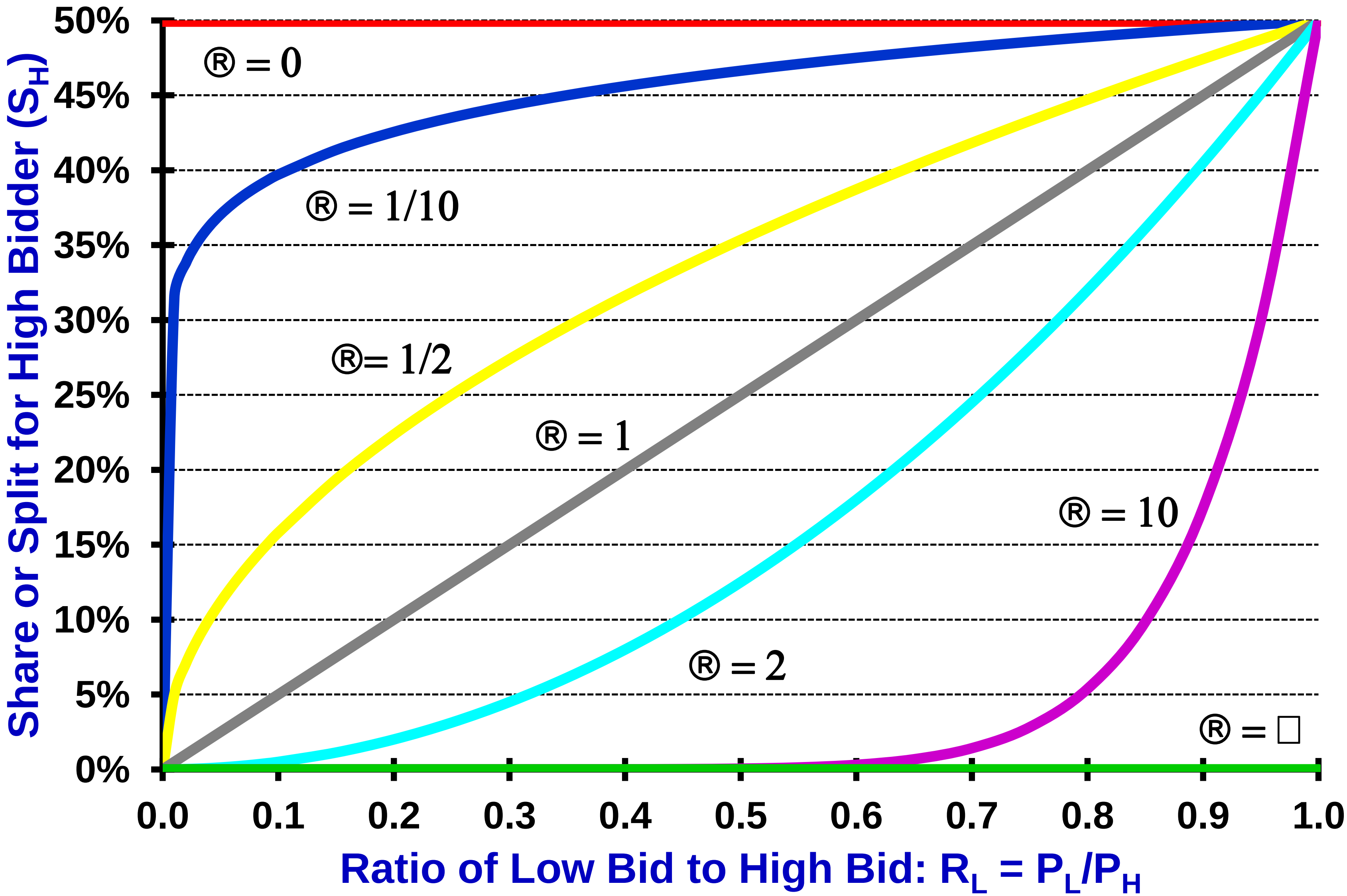
	$\beta = 0$	$0 < \beta < 1$	$\beta = 1$	$1 < \beta < \infty$	$\beta = \infty$
$\alpha = 0$	$S_H = 0$ **Winner-Take-All**	$S_H = 0$ **Winner-Take-All**	$S_H = 0$ **Winner-Take-All**	$S_H = 0$ **Winner-Take-All**	$S_H = 0$ **Winner-Take-All**
$0 < \alpha < ½$	$S_H = \alpha$ **Fixed Split**	$0 \le S_H \le \alpha$ $S_H > \alpha R_L$	$0 \le S_H \le \alpha$ $S_H = \alpha R_L$	$0 \le S_H \le \alpha$ $S_H < \alpha R_L$	$S_H = 0$ **Winner-Take-All**
$\alpha = ½$	$S_H = ½$ **Even Split**	$0 \le S_H \le ½$ $S_H > ½R_L$	$0 \le S_H \le ½$ $S_H = ½R_L$	$0 \le S_H \le ½$ $S_H < ½R_L$	$S_H = 0$ **Winner-Take-All**

←→

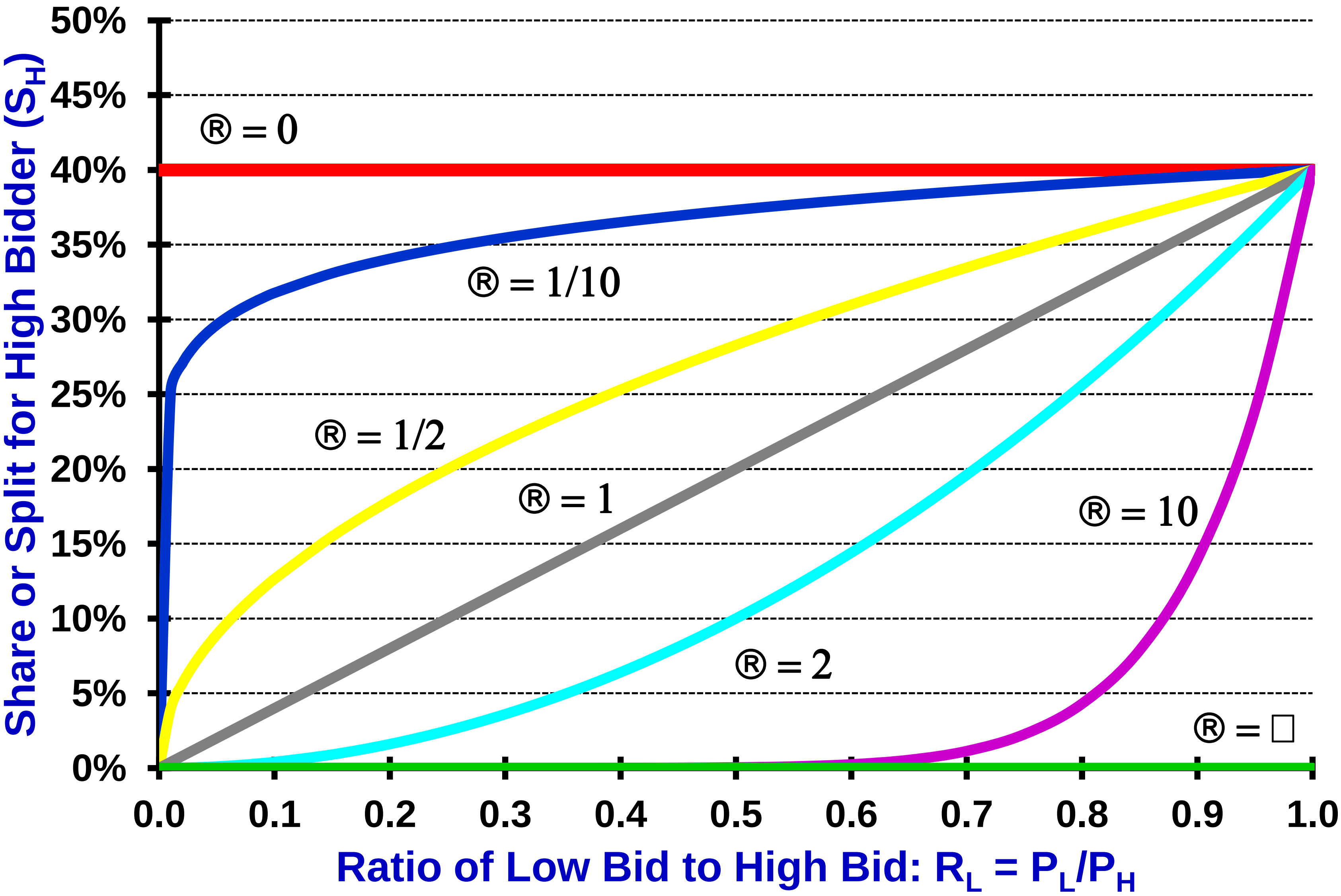
Better for High Bidder
Worse for Low Bidder

Worse for High Bidder
Better for Low Bidder

Split Award Scenarios with $S_H = {}^1/_2 R_L^{\beta}$



Split Award Scenarios with $S_H = {}^{2}/_{5} R_L^{\beta}$



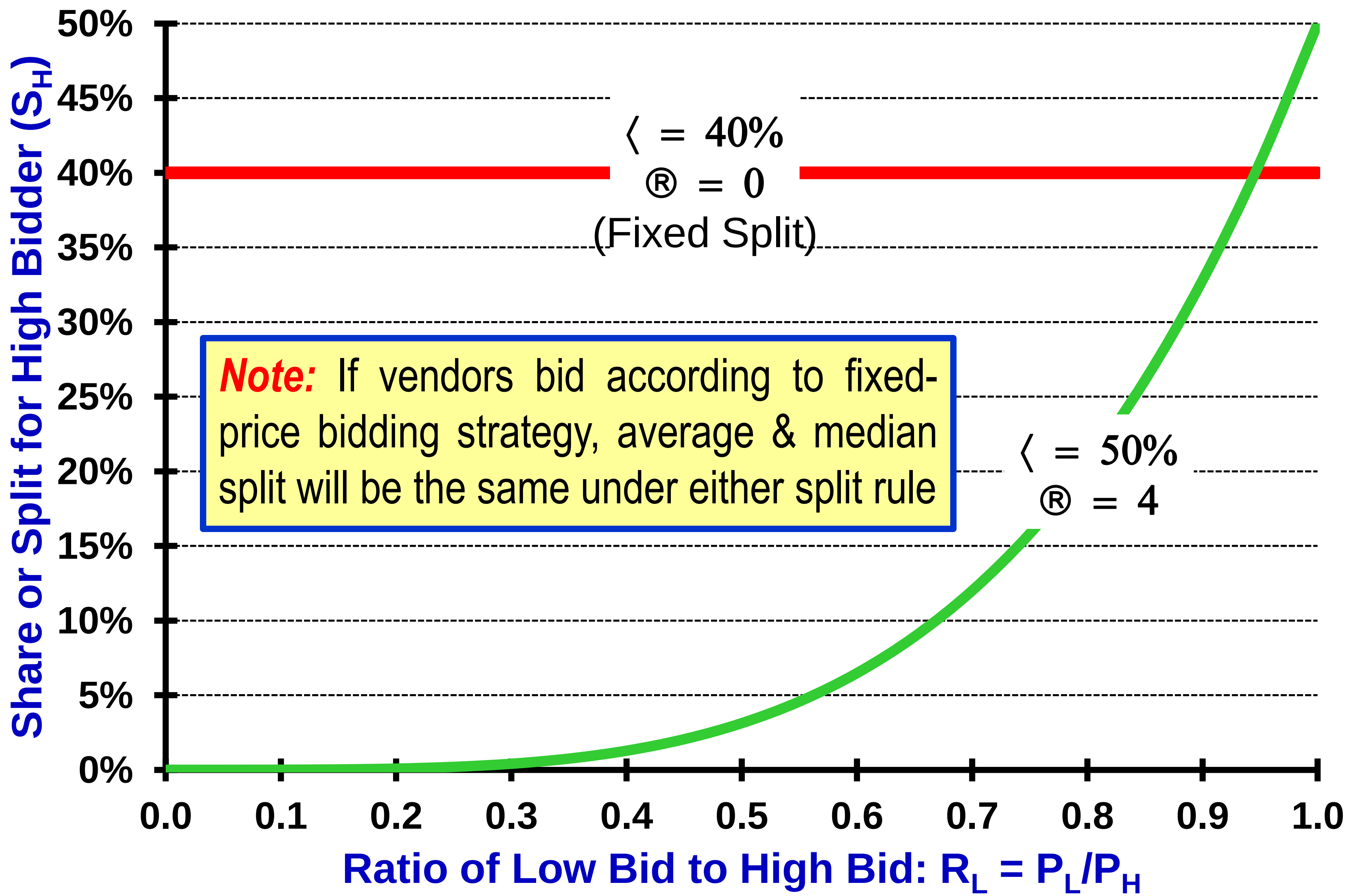
From Fixed Splits to Endogenous Splits

- Recall that the **equilibrium bidding strategy** under **fixed splits** of $\mathbf{S_H = 0.4}$ & $\mathbf{S_L = 0.6}$ with C_1, C_2 ~ U[0,100] was given by:

$$P_j = I(C_j) = \frac{10{,}000 - 0.2C_j^2}{120 - 0.4C_j}$$

  - In equilibrium, this yielded an expected **price per unit** of **93**
- Now, consider the following **endogenous split award function**:
  - $S_H = \alpha R_L^{\beta}$ with $\alpha = ½$ & $\beta = 4$
  - $\mathbf{S_H = ½R_L^4}$
- If both vendors continue to bid according to the above fixed-split equilibrium bidding strategy, we have:
  - Average split (average value of $S_H = ½R_L^4$) = 0.4
  - Median split (median value of $S_H = ½R_L^4$) = 0.4
  - Thus, **"apples-to-apples" comparison** to compare bidding under these two award rules (one fixed, one endogenous)

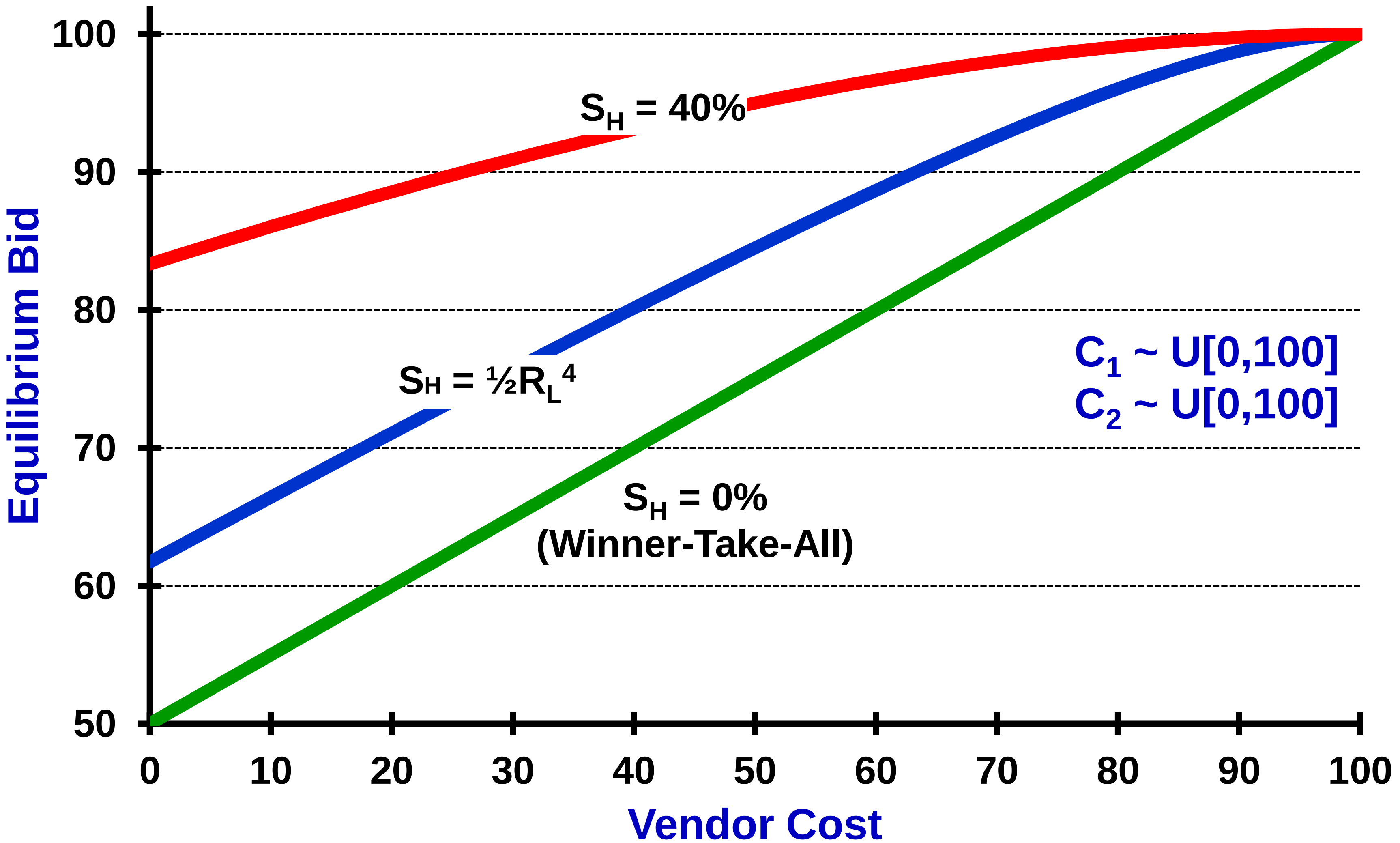
Split Award Scenarios with $S_H = \alpha R_L^{\beta}$



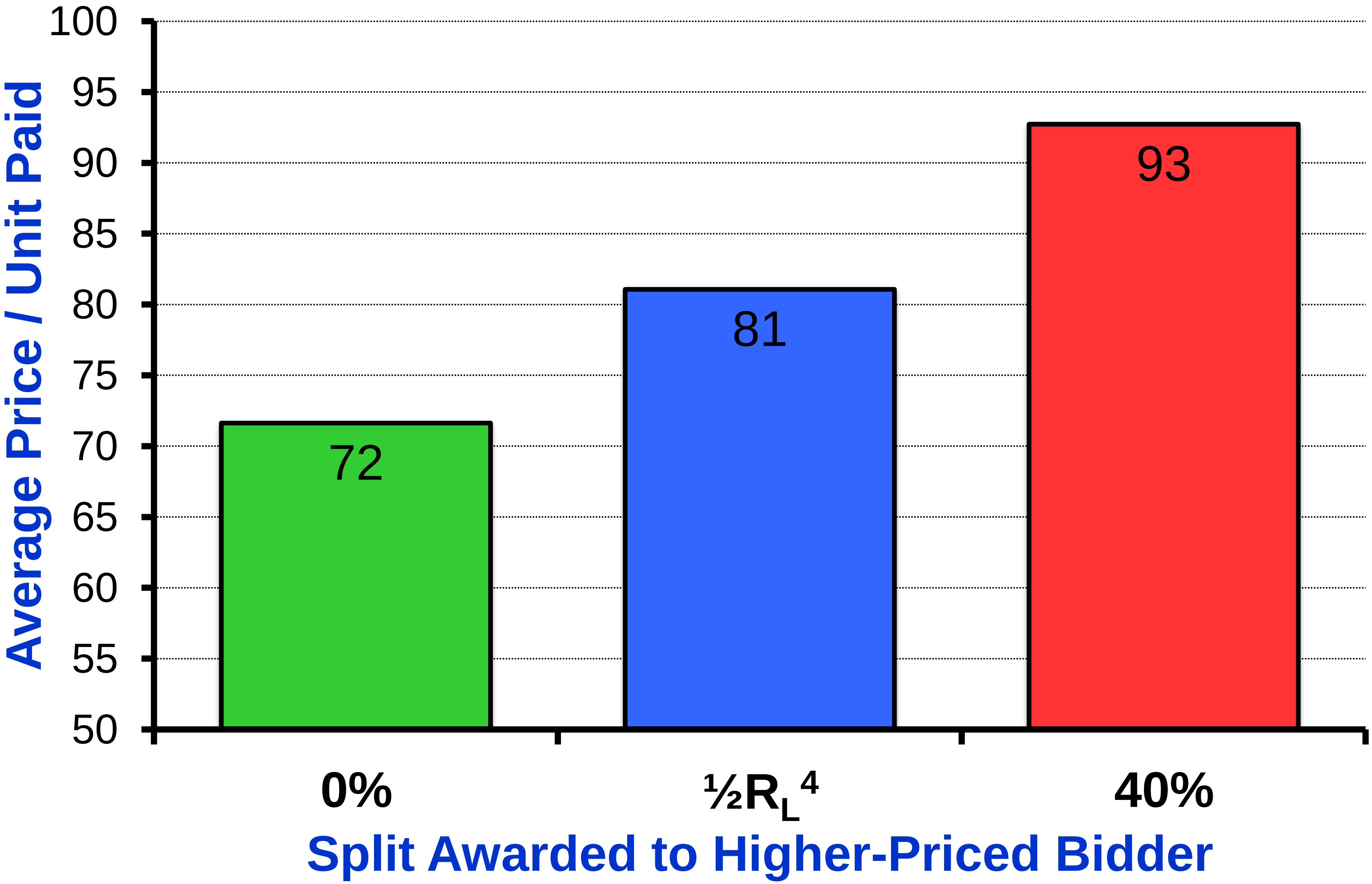
From Fixed Splits to Endogenous Splits

- If vendors follow fixed-split bidding strategy for $S_H = 0.4$, expected & median values of the endogenous split should **still** be $\mathbf{S_H = 0.4}$
  - But is this strategy still optimal when splits are endogenous?
- So, when contract **splits are endogenous** & given by $S_H = ½R_L^4$:
  - What is the **equilibrium bidding strategy**?
  - What is the **average price per u**nit paid by the buyer?
- We answered these questions **computationally**
  - **Closed-form solution** to equilibrium calculation is problematic
  - Thus, solve via **"iterative best-response"**
    1. Start: Assume vendor 1 follows given fixed-price bid strategy
    2. Compute: What is vendor 2's best-response bidding strategy?
    3. Iterate: What is vendor 1's best-response to 2's best-response?
    4. Repeat: Until you reach a "fixed-point" solution

Equilibrium Bidding with Endogenous Splits



Average Price/Unit with Different Splits



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Bid Protests & Split Awards: Conclusions

- Objective is to **manage**, not minimize protests
  - Encourage protests that correct (significant) mistakes
  - Discourage protests that do not
- **Split awards** are lever for protest management
  - Raise the hurdle for profitable protest
  - Filters out unmerited protests more than merited
- Challenge is determining the **right split**
  - Higher split to 2nd-vendor reduces protest incentive
  - BUT higher 2nd-vendor split also increases costs
  - Higher fixed 2nd-vendor split induces bid inflation
- **Endogenous** split awards offer potential solution
  - Retains protest "filtering" benefits
  - Reduces inflation of bids & average price paid

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Research Agenda Moving Forward

- **Research questions:**
  - What is the **optimal split award function?**
    - » Minimize expected and/or long-term buyer cost
    - » Including cost of protests & corrective benefit of protests
    - » Include impact of other benefits of split awards
  - What is the impact of **changes in key variables?**
    - » Vendor & buyer information, costs of protest, etc.
  - What is the impact of **repeated procurements?**
    - » Inter-temporal effects: Experience & innovation
- **Research methodology:**
  - Closed-form **game-theoretic solutions** & dynamics
  - Numerical **computation** & **simulation**

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A Modeling & Computational Approach

Appendix: Equilibrium Bid Strategy with Fixed-Splits

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Temporary Simplifying Assumptions

- For now, ignore “continuation value” of protest stage
  - Effect of protest on bidding strategy ambiguous
- For now, also ignore buyer’s imperfect information
  - Assume buyer perfectly informed regarding P_1 & P_2
  - Symmetric imperfect info ➔ neutral impact

Expected Profit Function (Bid Stage)

- $E\Pi_1(P_1) = X(P_1 - C_1)[\text{Prob}(P_1 > P_2)S_H + \text{Prob}(P_1 < P_2)S_L]$

  $= X(P_1 - C_1)[\text{Prob}(P_1 > P_2)S_H + [1 - \text{Pr}(P_1 > P_2)]S_L]$

  $= X(P_1 - C_1)[S_L + \text{Prob}(P_1 > P_2)(S_H - S_L)]$

  $= X(P_1 - C_1)[S_L - \text{Prob}(P_1 > P_2)(S_L - S_H)]$

Cost Distribution & Bidding Strategy

- Assume C_1, C_2 identically & independently distributed over interval [0,M]
  - Distribution function F
  - Density function f = F'
- Symmetric bidding strategy $\lambda(C)$
  - λ: [0,M] ~ [0,M]
  - $\lambda(M) = M$

Equilibrium Bidding with Fixed Splits

- Calculate optimal bid P_1 for vendor 1 assuming:
  - Vendor 1 has cost C_1
  - Vendor 2 is bidding according to strategy $\lambda(C_2)$
- $\text{Prob}(P_2<P_1) = \text{Prob}[\lambda(C_2)<P_1] = \text{Prob}[C_2< \lambda^{-1}(P_1)]$
  $= F(\lambda^{-1}(P_1))$
- $E\Pi_1(P_1) = X(P_1-C_1)[S_L-\text{Prob}(P_1>P_2)(S_L-S_H)]$
  $= X(P_1-C_1)[S_L-F(\lambda^{-1}(P_1))(S_L-S_H)]$
- Chain rule + inverse derivative theorem ➔
  $\delta E\Pi_1/\delta P_1 = X[S_L-F(\lambda^{-1}(P_1))(S_L-S_H)]$
  $- X(P_1-C_1)(S_L-S_H)f(\lambda^{-1}(P_1))/\lambda'(\lambda^{-1}(P_1))$

Equilibrium Bidding with Fixed Splits

- First-order condition ➔

$$S_L - F(\lambda^{-1}(P_1))(S_L - S_H) = (P_1 - C_1)(S_L - S_H) f(\lambda^{-1}(P_1)) / \lambda'(\lambda^{-1}(P_1))$$

$$\lambda'(\lambda^{-1}(P_1))[S_L - F(\lambda^{-1}(P_1))(S_L - S_H)] = (P_1 - C_1)(S_L - S_H) f(\lambda^{-1}(P_1))$$

- At symmetric equilibrium, $P_1 = \lambda(C_1)$ ➔ $\lambda^{-1}(P_1) = C_1$ ➔

$$\lambda'(C_1)[S_L - F(C_1)(S_L - S_H)] = (\lambda(C_1) - C_1)(S_L - S_H) f(C_1)$$

$$S_L \lambda'(C_1) = (S_L - S_H)[F(C_1)\lambda'(C_1) + \lambda(C_1) f(C_1) - C_1 f(C_1)]$$

$$(S_L - S_H)[F(C_1)\lambda'(C_1) + f(C_1)\lambda(C_1)] = S_L \lambda'(C_1) + C_1 (S_L - S_H) f(C_1)$$

$$(S_L - S_H) \frac{\partial}{\partial C_1} \left[F(C_1) \lambda(C_1) \right] = S_L \lambda'(C_1) + C_1 (S_L - S_H) f(C_1)$$

Equilibrium Bidding with Fixed Splits

$$(S_L - S_H)\frac{\partial}{\partial C_1}\left[F(C_1)\lambda(C_1)\right] = S_L\lambda'(C_1) + C_1(S_L - S_H)f(C_1)$$

$$(S_L - S_H)\int_{C_1}^{M}\frac{\partial}{\partial C_1}\left[F(C_1)\lambda(C_1)\right]dC_1$$

$$= S_L\int_{C_1}^{M}\lambda'(C_1)\,dC_1 + (S_L - S_H)\int_{C_1}^{M}C_1 f(C_1)\,dC_1$$

$$(S_L - S_H)\left[F(M)\lambda(M) - F(C_1)\lambda(C_1)\right]$$

$$= S_L\left[\lambda(M) - \lambda(C_1)\right] + (S_L - S_H)\int_{C_1}^{M}C_1 f(C_1)\,dC_1$$

$$(S_L - S_H)\left[M - F(C_1)\lambda(C_1)\right] = S_L\left[M - \lambda(C_1)\right] + (S_L - S_H)\int_{C_1}^{M}C_1 f(C_1)\,dC_1$$

$$-S_H M - (S_L - S_H)\int_{C_1}^{M}C_1 f(C_1)\,dC_1 = \left(S_L - S_H\right)F(C_1)\lambda(C_1) - S_L\lambda(C_1)$$

$$S_H M + (S_L - S_H)\int_{C_1}^{M}C_1 f(C_1)\,dC_1 = \lambda(C_1)S_L\left[1 - F(C_1)\right] + S_H F(C_1)\lambda(C_1)$$

Equilibrium Bidding with Fixed Splits

$$S_H M + (S_L - S_H)\int_{C_1}^{M} C_1 f(C_1)\, dC_1 = \lambda(C_1) S_L \left(1 - F(C_1)\right) + \lambda(C_1) S_H F(C_1)$$

$$S_H M + (S_L - S_H)\left(1 - F(C_1)\right)\frac{\int_{C_1}^{M} C_1 f(C_1)\, dC_1}{1 - F(C_1)}$$

$$= \lambda(C_1)\left[S_L\left(1 - F(C_1)\right) + S_H F(C_1)\right]$$

$$S_H M + (S_L - S_H)\left(1 - F(C_1)\right) E\left(C_2 \middle| C_2 > C_1\right)$$

$$= \lambda(C_1)\left[S_L - S_L F(C_1) + S_H F(C_1)\right] = \lambda(C_1)\left[S_L - \left(S_L - S_H\right) F(C_1)\right]$$

$$\lambda(C_1) = \frac{S_H M + (S_L - S_H)\left(1 - F(C_1)\right) E\left(C_2 \middle| C_2 > C_1\right)}{S_L - \left(S_L - S_H\right) F(C_1)}$$

Equilibrium Bidding with Fixed Splits

- Let $C_1, C_2 \sim U[0,100]$ ➔

$$I(C_1) = \frac{S_H M + (S_L - S_H)\left(1 - F(C_1)\right)E\left(C_2 \middle| C_2 > C_1\right)}{S_L - (S_L - S_H)F(C_1)}$$

$$I(C_1) = \frac{100S_H + (S_L - S_H)\left(1 - \tfrac{1}{100}C_1\right)\tfrac{1}{2}\left(C_1 + 100\right)}{S_L - \tfrac{1}{100}(S_L - S_H)C_1}$$

$$I(C_1) = \frac{20{,}000S_H + (S_L - S_H)\left(100 - C_1\right)\left(C_1 + 100\right)}{200S_L - 2(S_L - S_H)C_1}$$

$$I(C_1) = \frac{20{,}000S_H + \left(1 - 2S_H\right)\left(10{,}000 - C_1^2\right)}{200S_L - 2C_1(S_L - S_H)}$$

$$I(C_1) = \frac{20{,}000S_H + 10{,}000 - C_1^2 - 20{,}000S_H + 2S_H C_1^2}{200S_L - 2C_1(S_L - S_H)}$$

$$I(C_1) = \frac{10{,}000 + (2S_H - 1)C_1^2}{200S_L - 2C_1(S_L - S_H)} = \frac{10{,}000 - (S_L - S_H)C_1^2}{200S_L - 2C_1(S_L - S_H)}$$

Equilibrium Bidding with Fixed Splits

$$S_H = 0 \Rightarrow \quad \lambda(C_1) = \frac{10{,}000 - C_1^2}{200 - 2C_1} = \frac{(100 - C_1)(100 + C_1)}{2(100 - C_1)} = 50 + \tfrac{1}{2}C_1$$

$$S_H = 0.1 \Rightarrow \quad S_L = 0.9 \Rightarrow \quad \lambda(C_1) = \frac{10{,}000 - 0.8C_1^2}{180 - 1.6C_1}$$

$$S_H = 0.2 \Rightarrow \quad S_L = 0.8 \Rightarrow \quad \lambda(C_1) = \frac{10{,}000 - 0.6C_1^2}{160 - 1.2C_1}$$

$$S_H = 0.3 \Rightarrow \quad S_L = 0.7 \Rightarrow \quad \lambda(C_1) = \frac{10{,}000 - 0.4C_1^2}{140 - 0.8C_1}$$

$$S_H = 0.4 \Rightarrow \quad S_L = 0.6 \Rightarrow \quad \lambda(C_1) = \frac{10{,}000 - 0.2C_1^2}{120 - 0.4C_1}$$

$$S_H = 0.5 \Rightarrow \quad S_L = 0.5 \Rightarrow \quad \lambda(C_1) = \frac{10{,}000 - 0}{100 - 0} = 100$$

Endogenous Split Awards as a Protest Management Tool:
A Modeling & Computational Approach

Unused Back-Up Slides

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Bid Protests of Growing Concern

A NEW CYBER-SECURITY PLAN TAKES SHAPE 4

FEDERAL TIMES

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PAY AND BENEFITS

DoD performance pay

The Government Accountability Office warns against pulling the plug on Defense's performance pay system. **Page 4**

On furlough

Seven states are putting Social Security Administration employees on furlough, worsening the claims backlog. **Page 6**

FACILITIES MANAGEMENT

Move to Guam

The Defense Department plans to build a $10 billion Marine Corps base on Guam, the first step in a broadening military presence there. **Page 8**

PROCUREMENT

The rising problem of bid protests

Why large contracts are being sidelined

By ELISE CASTELLI
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When the General Services Administration announced 29 winners of the $50 billion Alliant contract in 2007, agency leaders heralded it as the government's premier contract for information technology purchases.

"With its expansive scope, access to the best in class in the private sector and ability to provide customized solutions tailored to agencies' unique IT needs, we can again prove that GSA is at the forefront of serving the acquisition needs of the federal government," GSA's Federal Acquisition Service Commissioner James Williams declared at the time.

But 18 months later, a lot has changed. The contract was held up because of bid protests from several firms that didn't make the cut. A federal court ordered GSA to re-evaluate all bidders. And it wasn't until two weeks ago that GSA got the giant Alliant contract back on track by awarding it to 59 companies.

Alliant is one of a few high-profile, high-value procurements — another is the Air Force's tanker contract — that have been waylaid by protests in recent years. While these large procurements get all the attention, most bid protests concern smaller contracts.

Overall, the protests rose 44 percent since 2001 — in part because companies were recently allowed to protest not only ad-

See PROTESTS, Page 19

ON THE RISE

The number of **bid protests** lodged each year has increased considerably since 2001:

SOURCE: Government Accountability Office

GSA STIMULUS PLAN

Hundreds of buildings to be upgraded

By TIM KAUFFMAN
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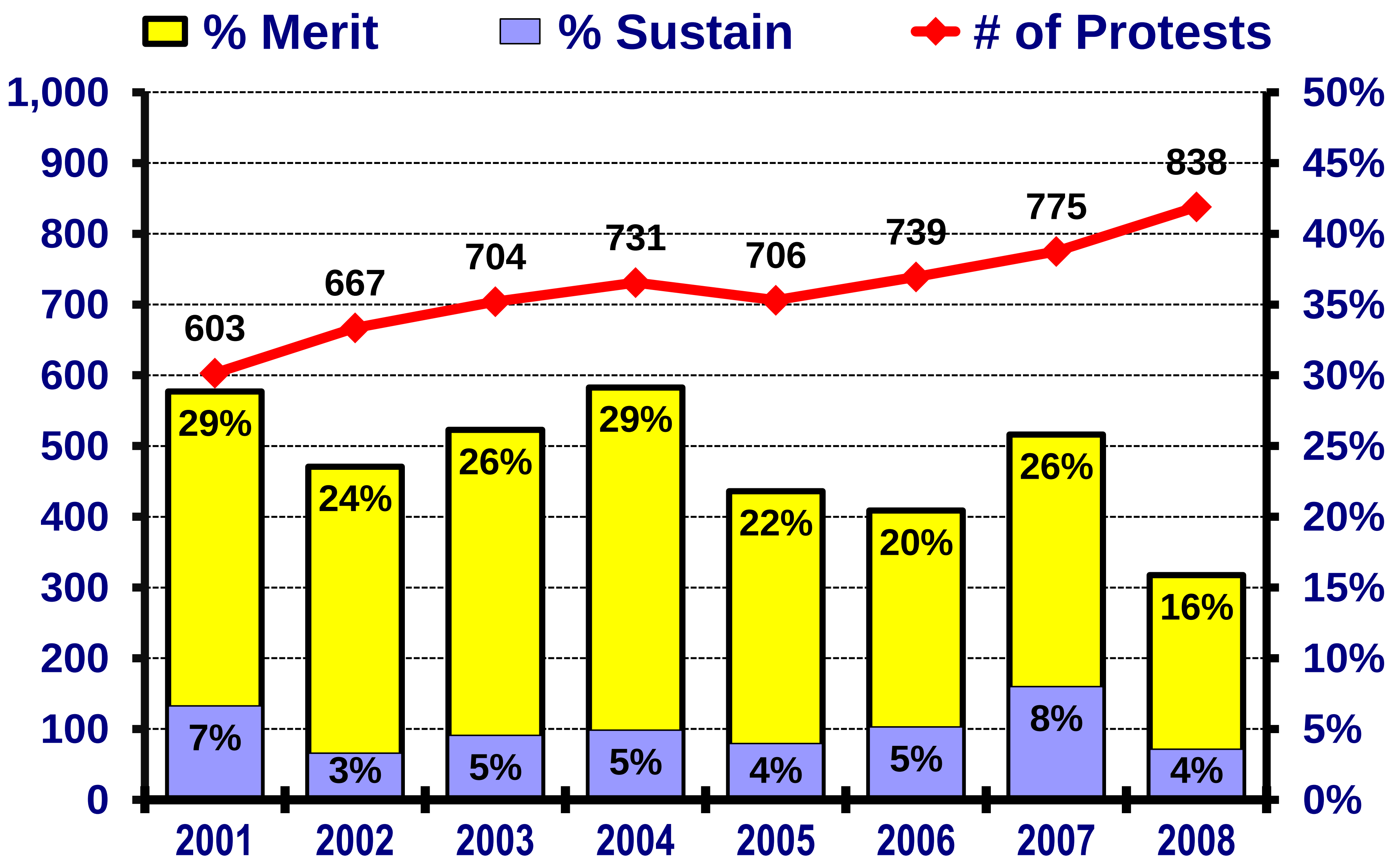
Hundreds of federal buildings across the country will be going green in the next year or two under the General Services Administration's plans for spending more than $5.5 billion in stimulus funds.

Many will benefit from features such as advanced meters to improve monitoring of electricity and water use, lighting controls and sensors that turn off lights when not needed, new or improved heating and air-condi-

DoD Bid Protest Trends



Vendor Protest Incentives

- Expected **Profit** from Protest
  = Expected Benefits – Expected Costs
- Expected **Costs** = K
  = Research + Legal + Reputation + Opportunity Costs
- Expected **Benefits**
  = Probability of Success × Gain if Successful
- **Gain** if Successful = Contract Revenue = X
- **Probability** of Success
  = Prob($P_1 < P_2$) *given* that buyer perceived $P_1 > P_2$
- Expected Profit from Protest = **Prob($P_1 < P_2$) × X – K**

Modeling Buyer Imperfect Information

- Let $\mathbf{R_1} = P_1 / (P_1+P_2)$ & $\mathbf{R_2} = P_2 / (P_1+P_2)$
  - $0 \le R_1 \le 1$ & $0 \le R_2 \le 1$
  - $R_1 + R_2 = 1$
- Let $\mathbf{r_1}$ = buyer's **estimate of R_1**
  - $r_1 = r / N$ where $r \sim Bin(N,R_1)$
  - **Binomial** with N draws & success probability = R_1
  - Higher N ➔ more accurate estimate of R_1
- Let $\mathbf{r_2}$ = buyer's **estimate of R_2**
  - $r_2 = 1 – r_1$

Perceived Probability of Protest Success

- Assume **buyer discloses estimate r_1**
  - $r_1 < ½$ ➔ vendor 1 wins
  - $r_1 > ½$ ➔ vendor 2 wins
- If vendor 1 loses, his estimate of the **probability of a successful protest** is:
  - Prob(P_1<P_2) *given* that buyer perceives P_1>P_2
  - Prob(R_1<½) *given* that buyer estimates R_1 at r_1
  - Prob(R_1<½) *given* Nr_1 successes from Bin(N,R_1)
  - Prob(R_1<½ | Nr_1 out of N)

Perceived Probability of Protest Success

$$\Pr ob\left(R_1 < \tfrac{1}{2} \middle| Nr_1 \text{ out of } N\right)$$

$$= \frac{\Pr ob\left(R_1 < \tfrac{1}{2}\right)\Pr ob\left(Nr_1 \text{ out of } N \middle| R_1 < \tfrac{1}{2}\right)}{\Pr ob\left(Nr_1 \text{ out of } N\right)}$$

$$= \frac{\int_0^{1/2} \Pr ob(z)\Pr ob\left(Nr_1 \text{ out of } N \middle| R_1 = z\right)dz}{\int_0^{1} \Pr ob(z)\Pr ob\left(Nr_1 \text{ out of } N \middle| R_1 = z\right)dz}$$

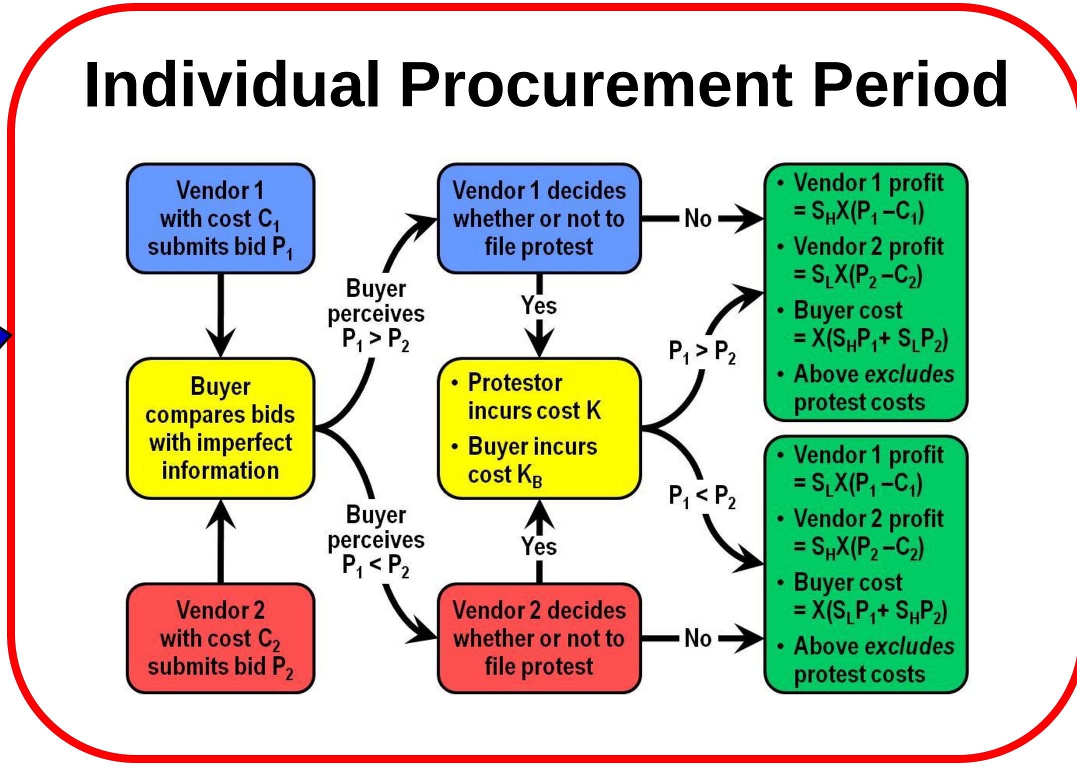
$$= \frac{\int_0^{1/2} \Pr ob(z)\binom{N}{Nr_1} z^{Nr_1}(1-z)^{N(1-r_1)}\,dz}{\int_0^{1} \Pr ob(z)\binom{N}{Nr_1} z^{Nr_1}(1-z)^{N(1-r_1)}\,dz}$$

where $\text{Prob}(z)$ reflects vendor 1's *prior* probability distribution of R_1

Extension: Repeated Procurements

- What are the **other benefits** of split awards?
  - Why are split awards **used currently?**
- Split awards **preserve competition** for repeated or follow-on procurements
- ***Direct* modeling implications:**
  - Appropriate to model as **repeated** bidding game
  - Implies presence of **learning/experience** effects
- ***Indirect* modeling implications:**
  - Incorporate **innovation** to avoid trivial outcomes
  - Innovation driven by “**shocks**” or **investment**

Extension: Repeated Procurements



- **Learning/Experience Effects**
- **Investment & Innovation**
- **Discounting Future Periods**