Determining the Value of a Prototype

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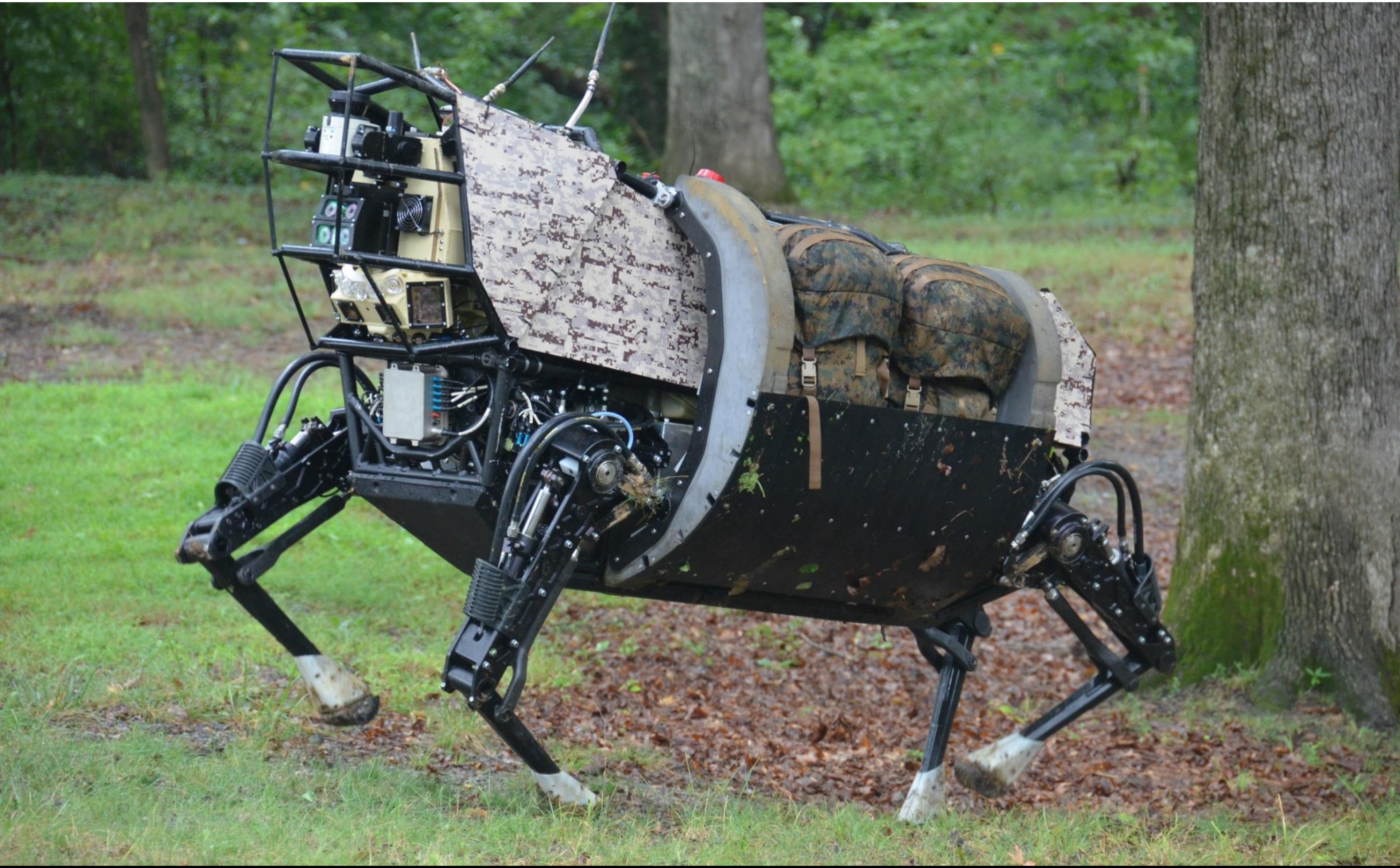
What are prototypes worth?



Setting the Stage – A program is proposed

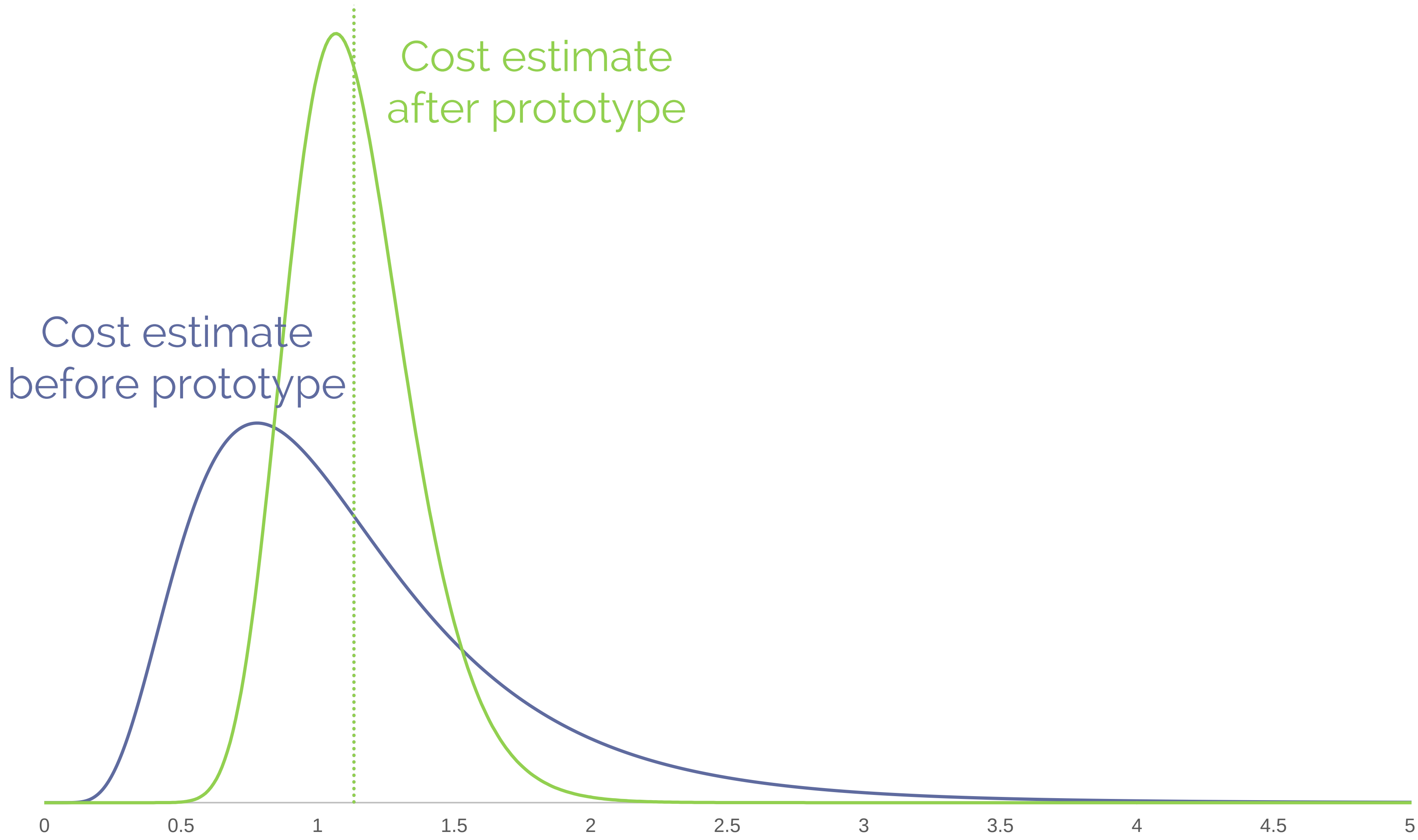
A decisionmaker has three options:

1. Start the program
2. Do not start the program
3. Make a prototype, and then start or do not start the program

An alternative program option exists with a known cost

Cost is the discriminator

Prototypes give us information



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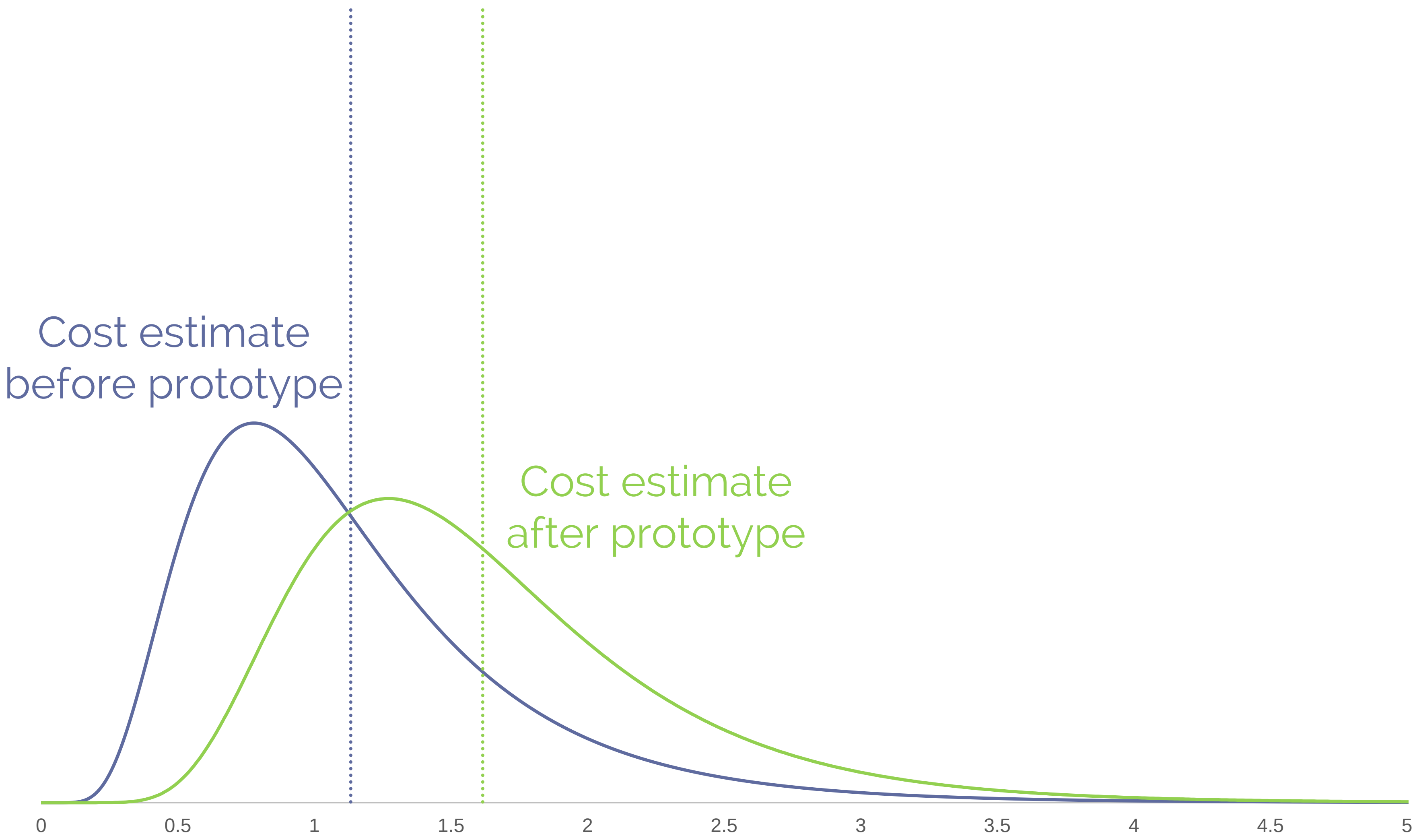

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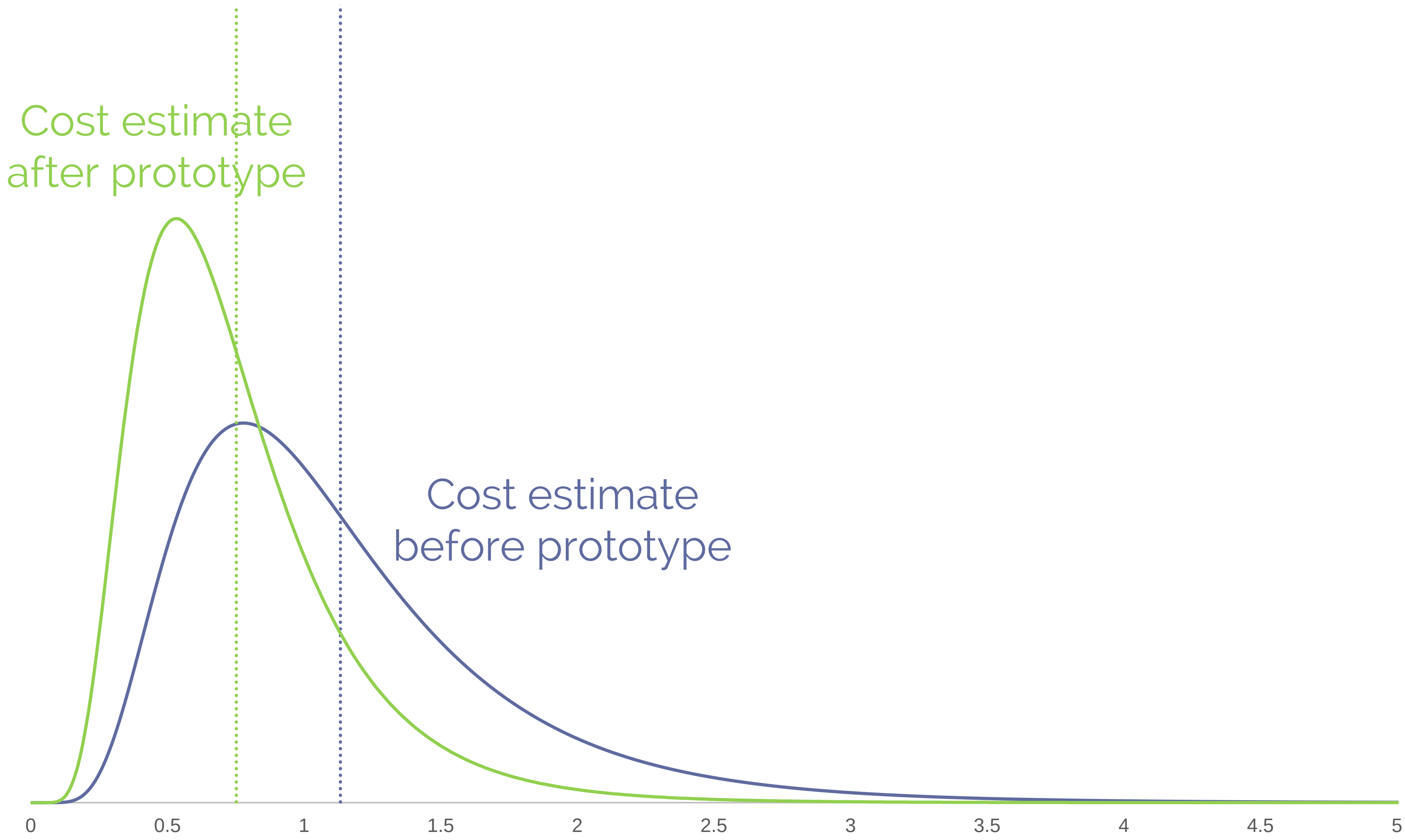
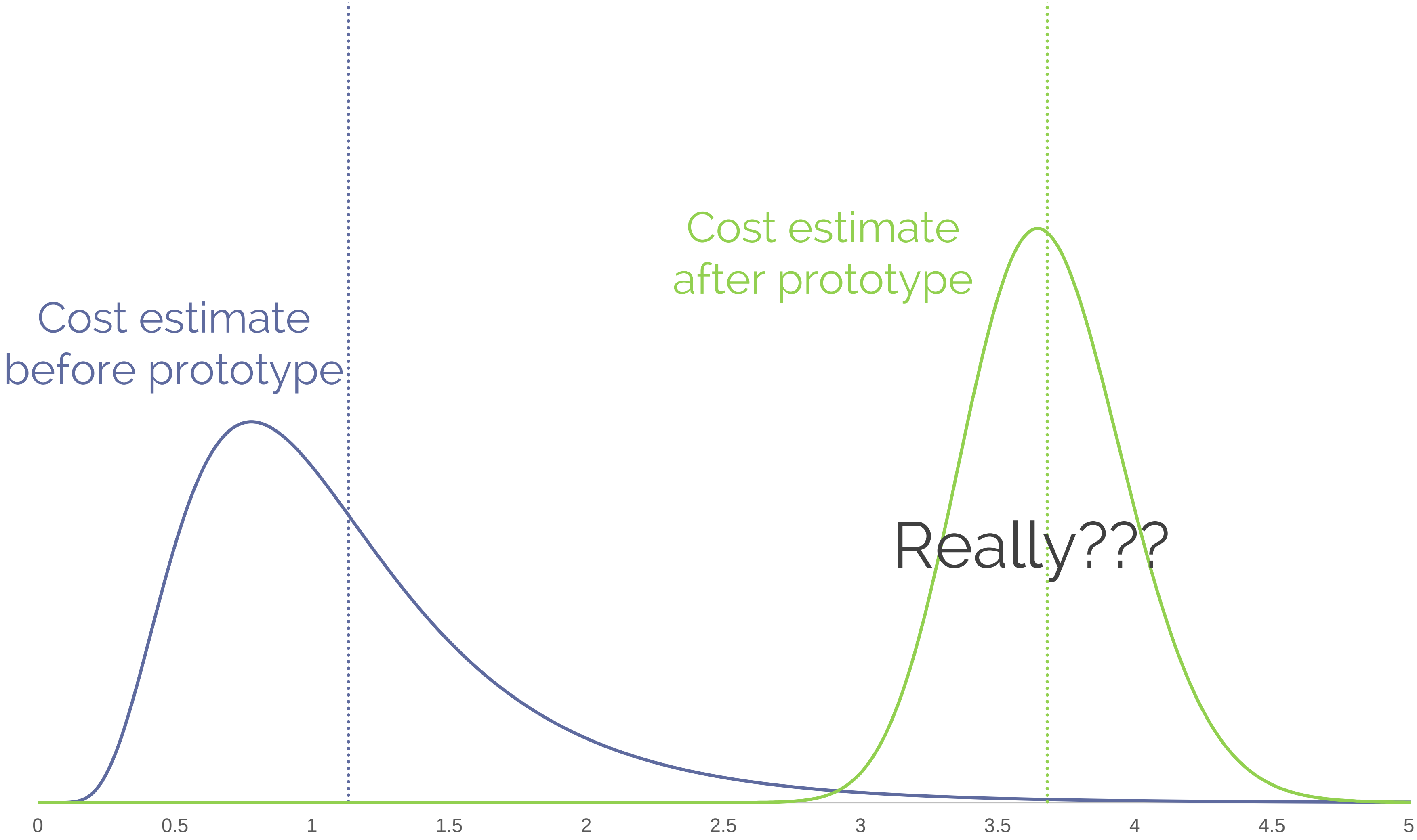


Prototypes give us information…but we know a little of what to expect



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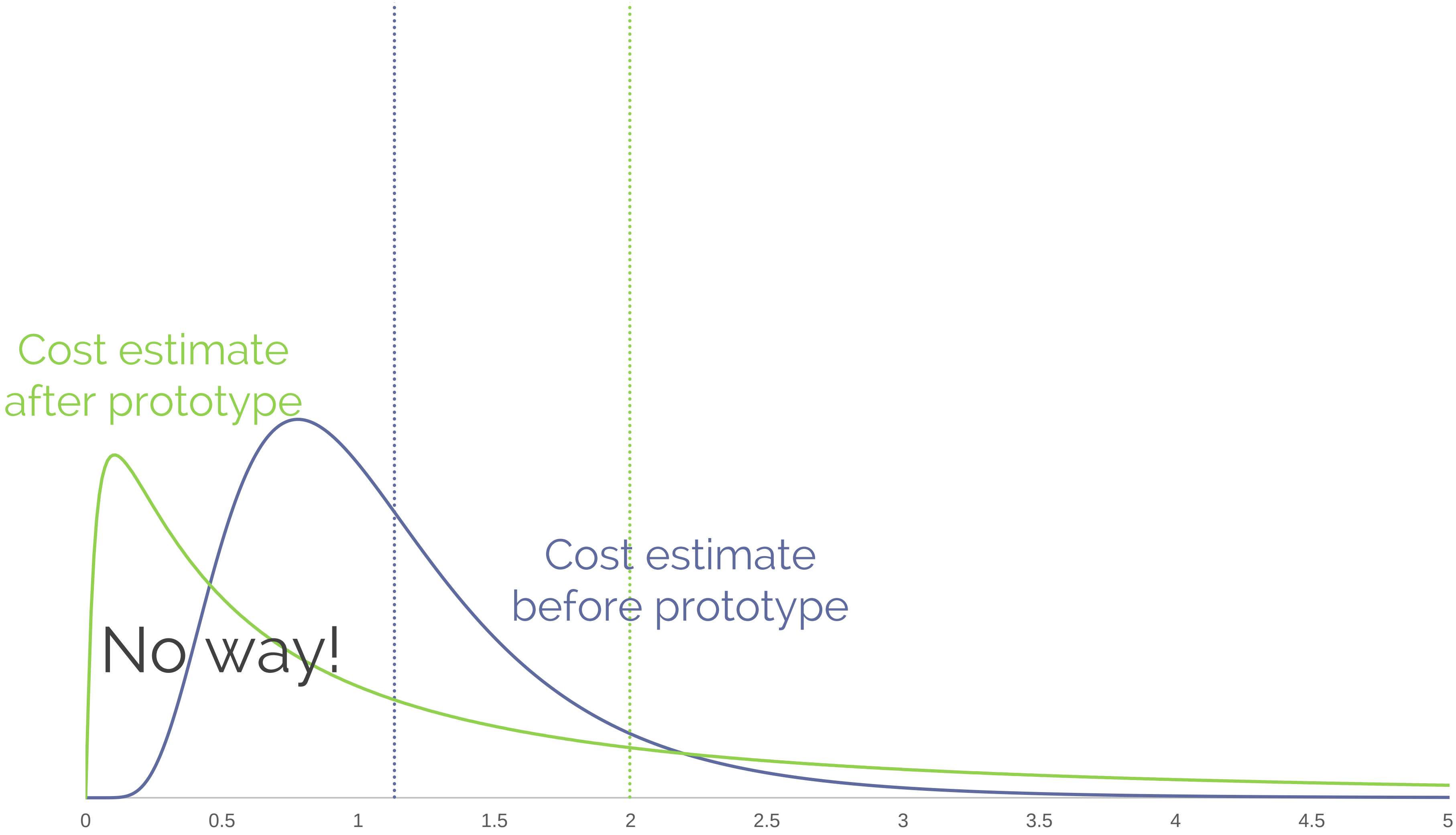

The Preposterior Distribution

The distribution of the means of all possible cost distributions after a prototype outcome

Properties of the Preposterior Distribution

The mean (expected value):

$$\mathrm{E}[preposterior] = \mathrm{E}[prior]$$

The mean of the preposterous is equal to the mean of the prior

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The variance:

$$\mathrm{Var}[preposterior] = \mathrm{Var}[prior] - \mathrm{E}_{d \in posteriors}[\mathrm{Var}[d]]$$

The variance of the preposterous is equal to
the variance of the prior less
the mean of the posterior variances

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The variance:

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The shape:

$$\mathrm{Dist}[preposterior] \rightarrow \mathrm{Dist}[prior] \text{ as } experiments \rightarrow \infty$$

The shape of the preposterous approaches that of the prior with increased experimental data

Properties of the Preposterior Distribution

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The shape:

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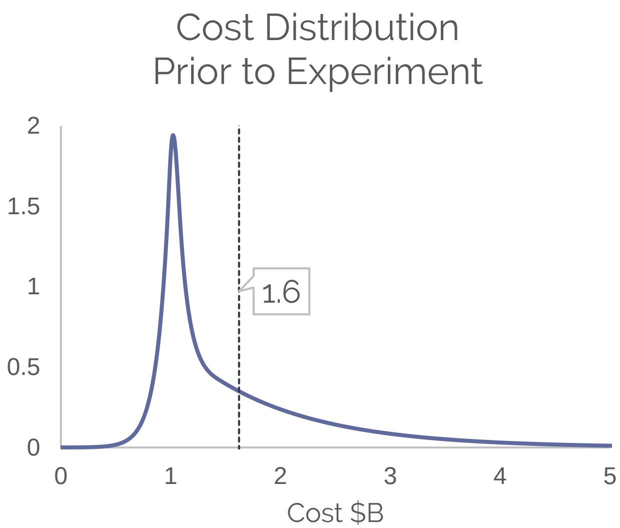
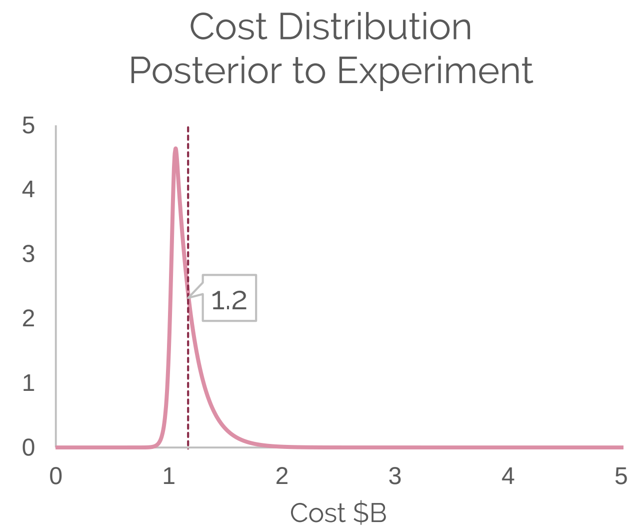
In other words, the preposterior distribution looks a lot like the prior distribution

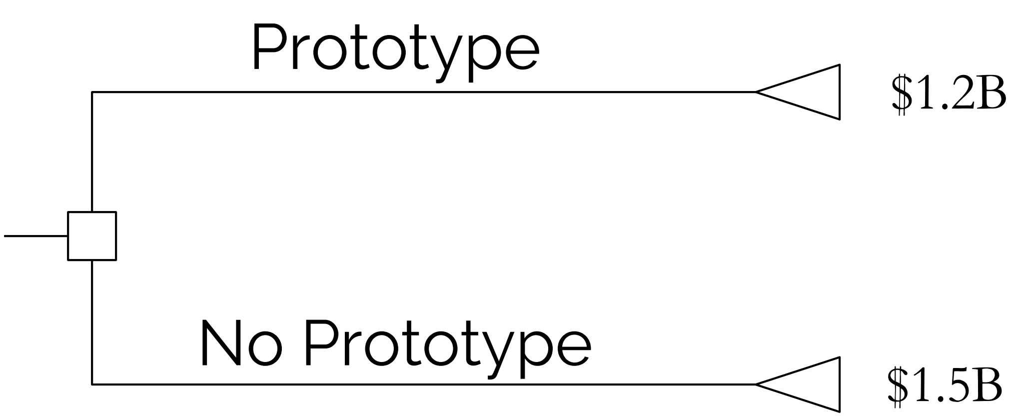
How to use the preposterior distribution



Prototype



Suppose the alternative costs \$1.5B

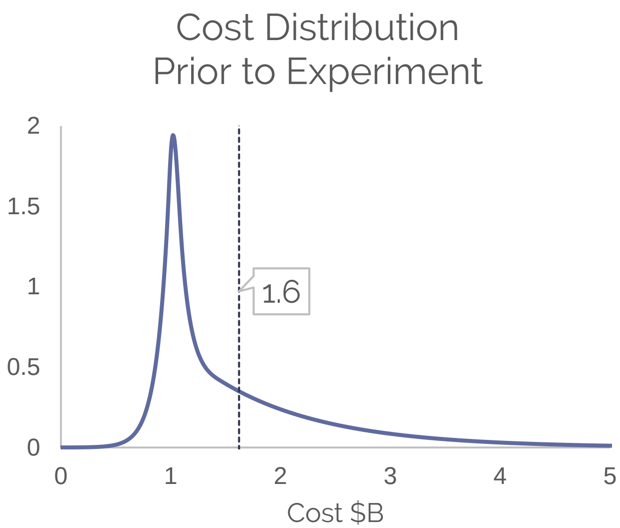


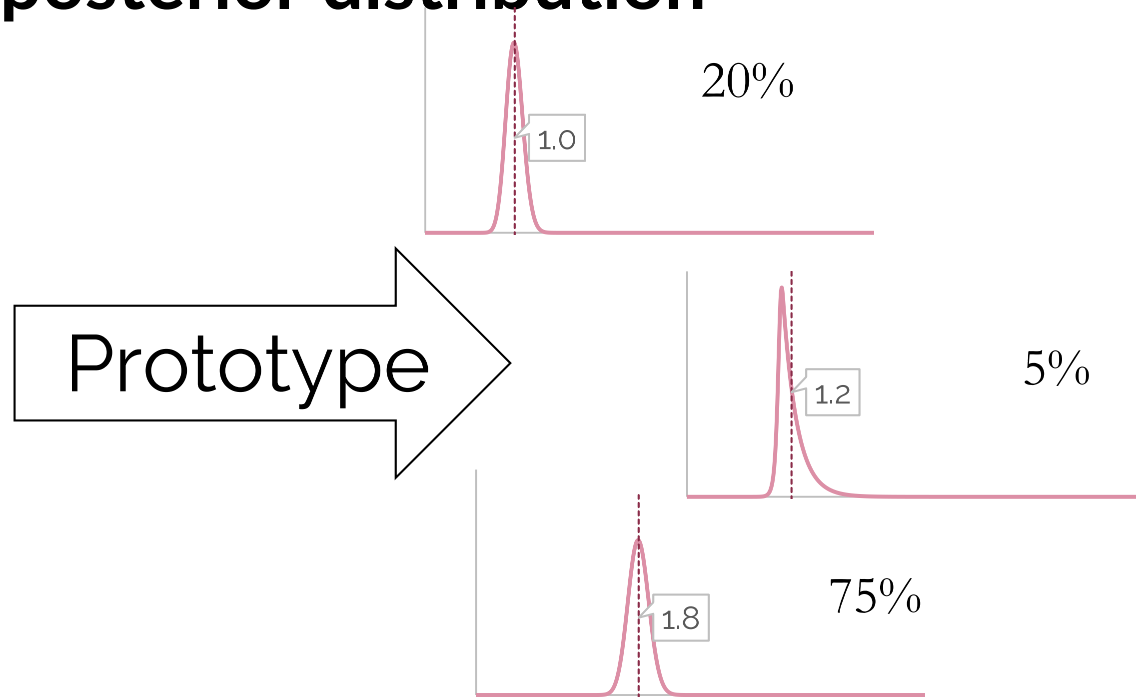
$\Delta = \$0.3\text{B}$

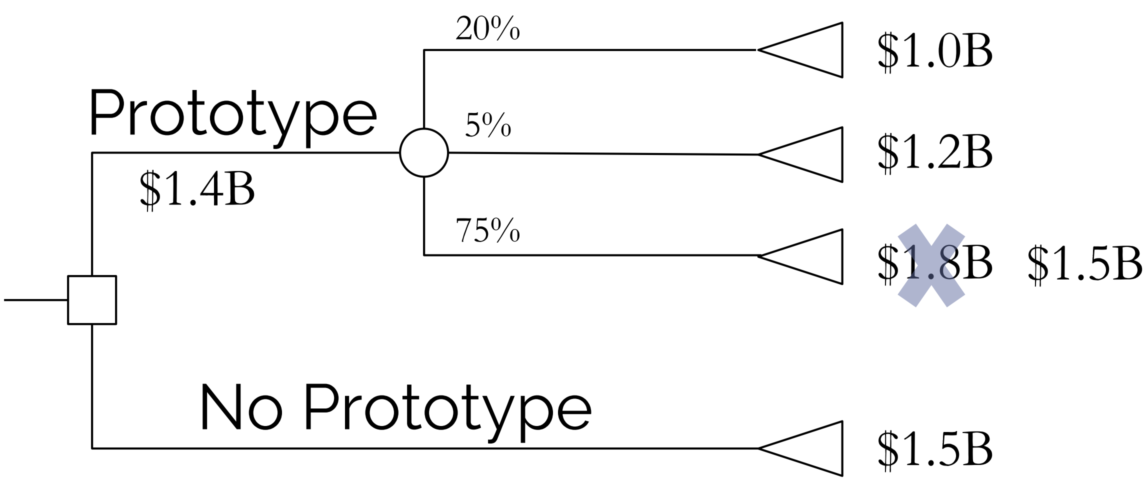
Value of information from prototype

Alternative costs $1.5B

How to use the preposterior distribution





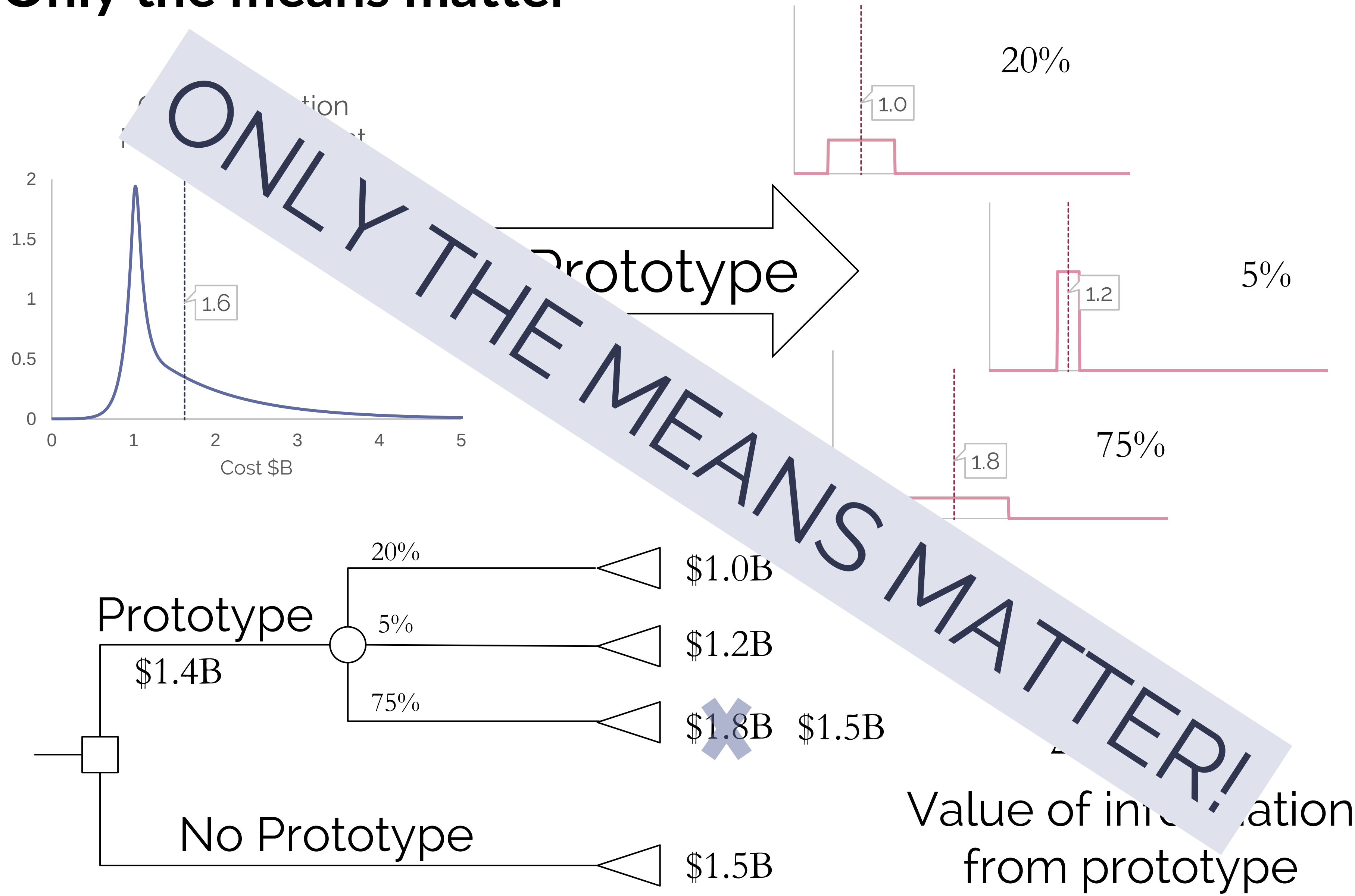


$\Delta = \$0.1B$

Value of information from prototype

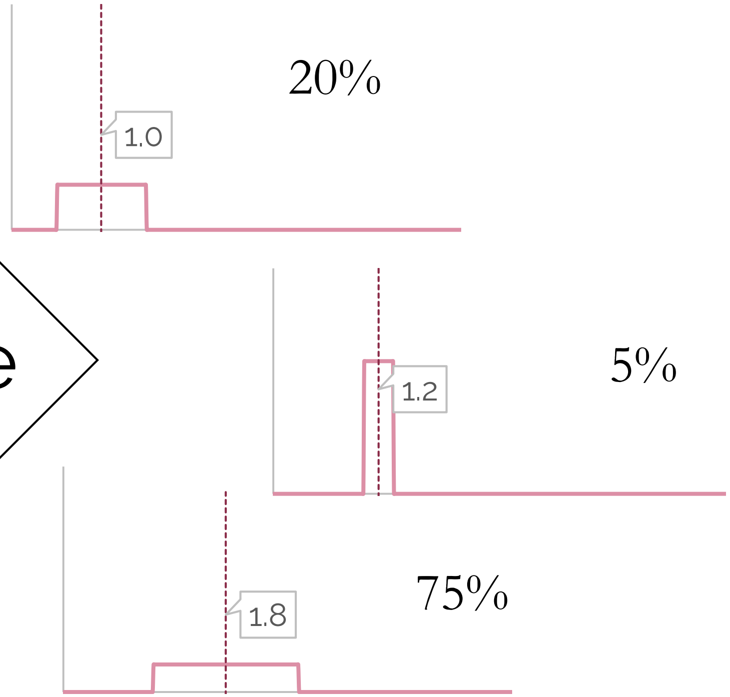
Alternative costs $1.5B

Only the means matter



Prepostior to the rescue

Prototype



Prior distribution

Posterior distributions

We know the mean of these means!

We can even make a decent guess as to the shape of the distribution of these means!

If we can estimate the variance of these distributions, we know the variance of these means!

Steps to estimate Value of Information for prototype

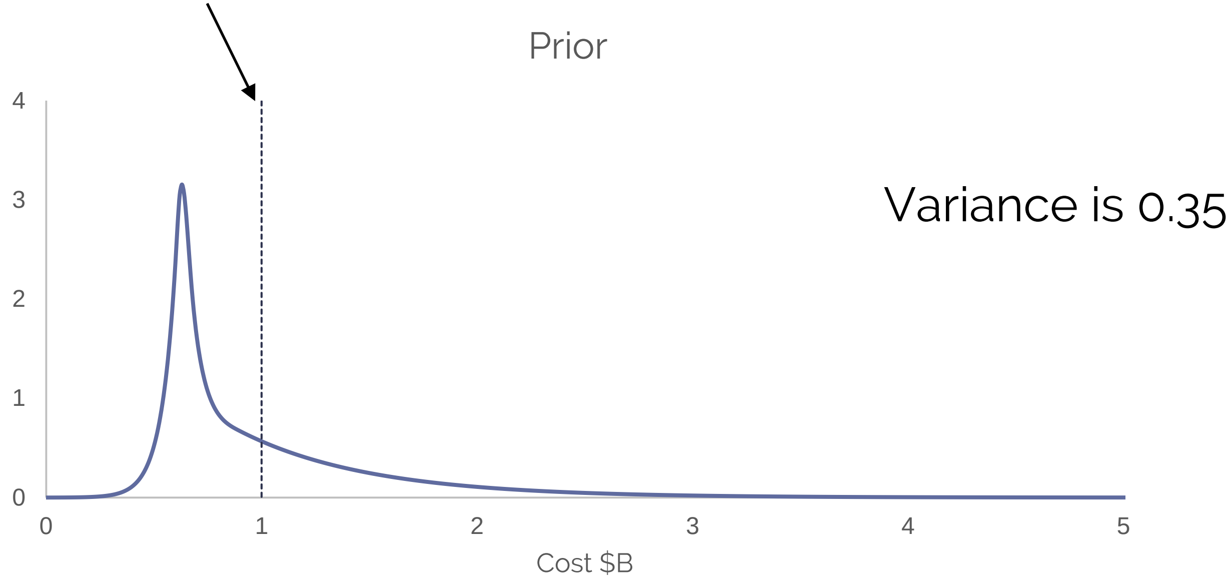
1. Estimate distribution of costs (prior)
2. Estimate reduction in variance of cost due to prototype (posteriors)
3. Model preposterior distribution as prior distribution with mean held constant (from 1) and variance equal to reduction in variance (from 2)
4. Perform decision tree on preposterior distribution with some given alternative cost

Example

From *Prototyping Defense Systems*

Derived prior distribution from percent cost overruns of systems without prototypes

Normalized mean to $1B

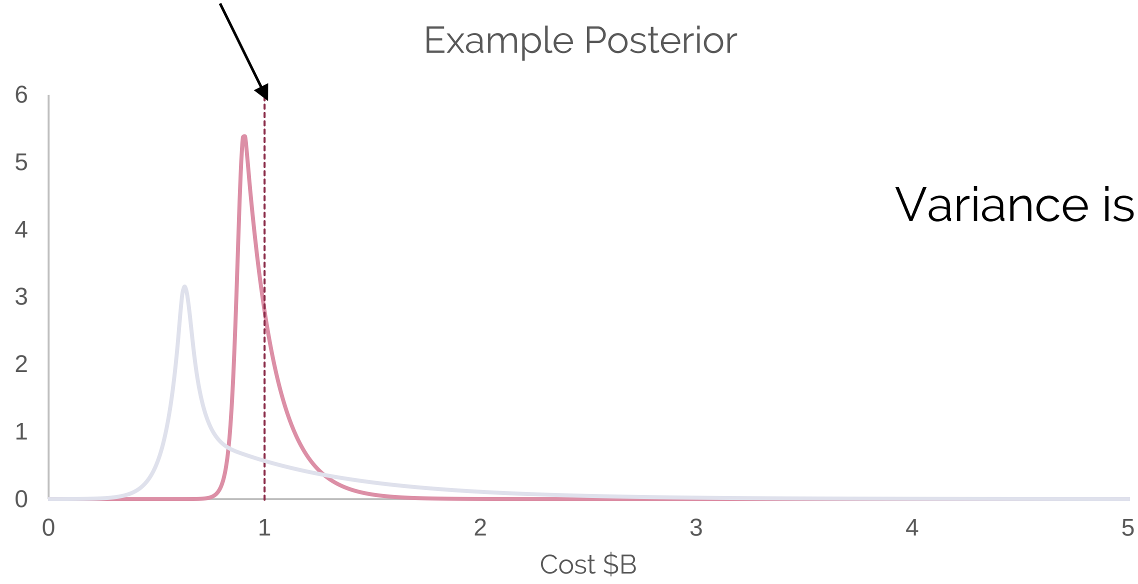


Example

From *Prototyping Defense Systems*

Derived a posterior variance from percent cost overruns of systems with prototypes

Mean $1B

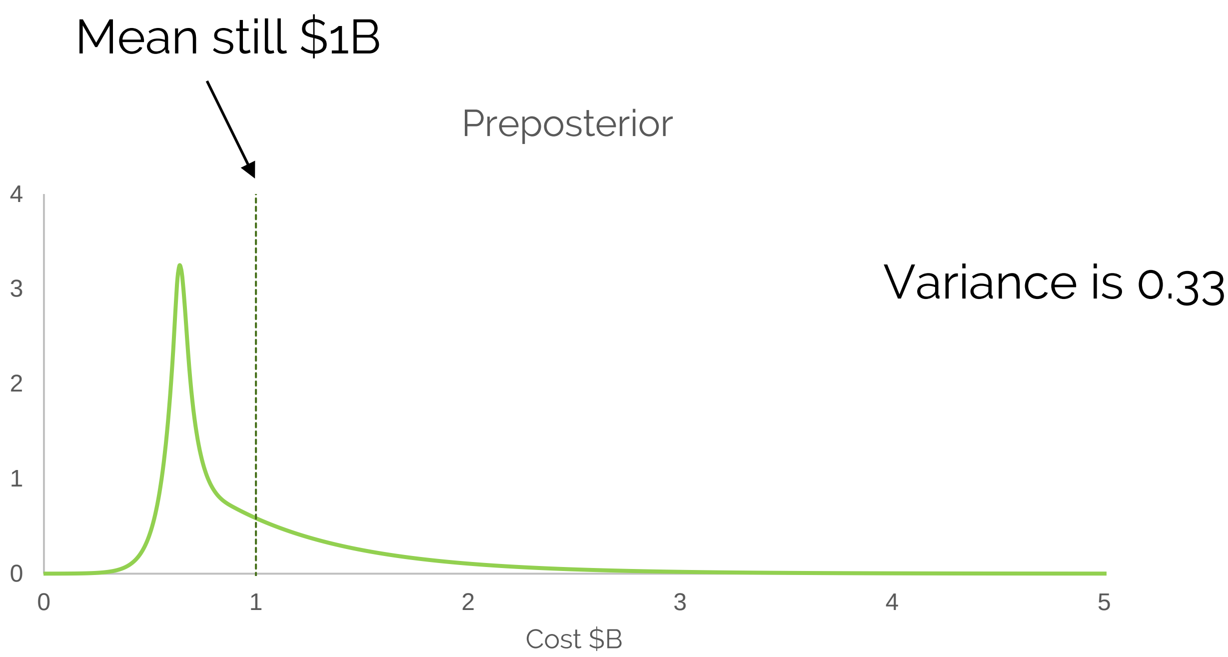


Variance is 0.02

Example

From *Prototyping Defense Systems*

Derived a preposterior distribution as prior with variance decreased by 0.02



Example

Suppose alternate cost is $0.7B

For costs less than $0.7B, find partial expected value $\int_0^{0.7} xp(x)dx$

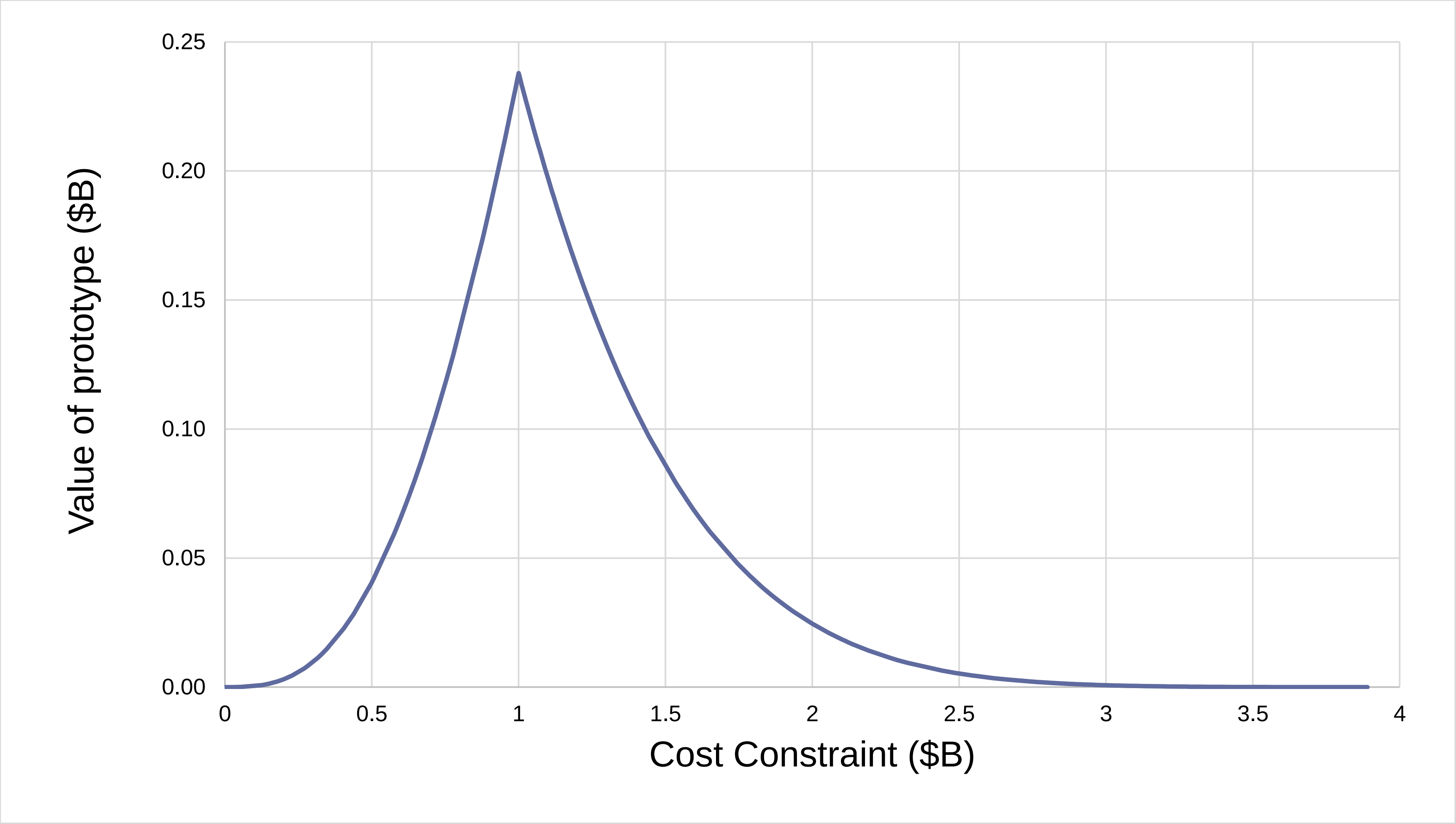
Sum is cost after prototype ($0.665B)

For costs more than $0.7B, find $0.7 * P(x > 0.7)$

Prototype is worth $0.035B

Example

Letting cost of alternative vary



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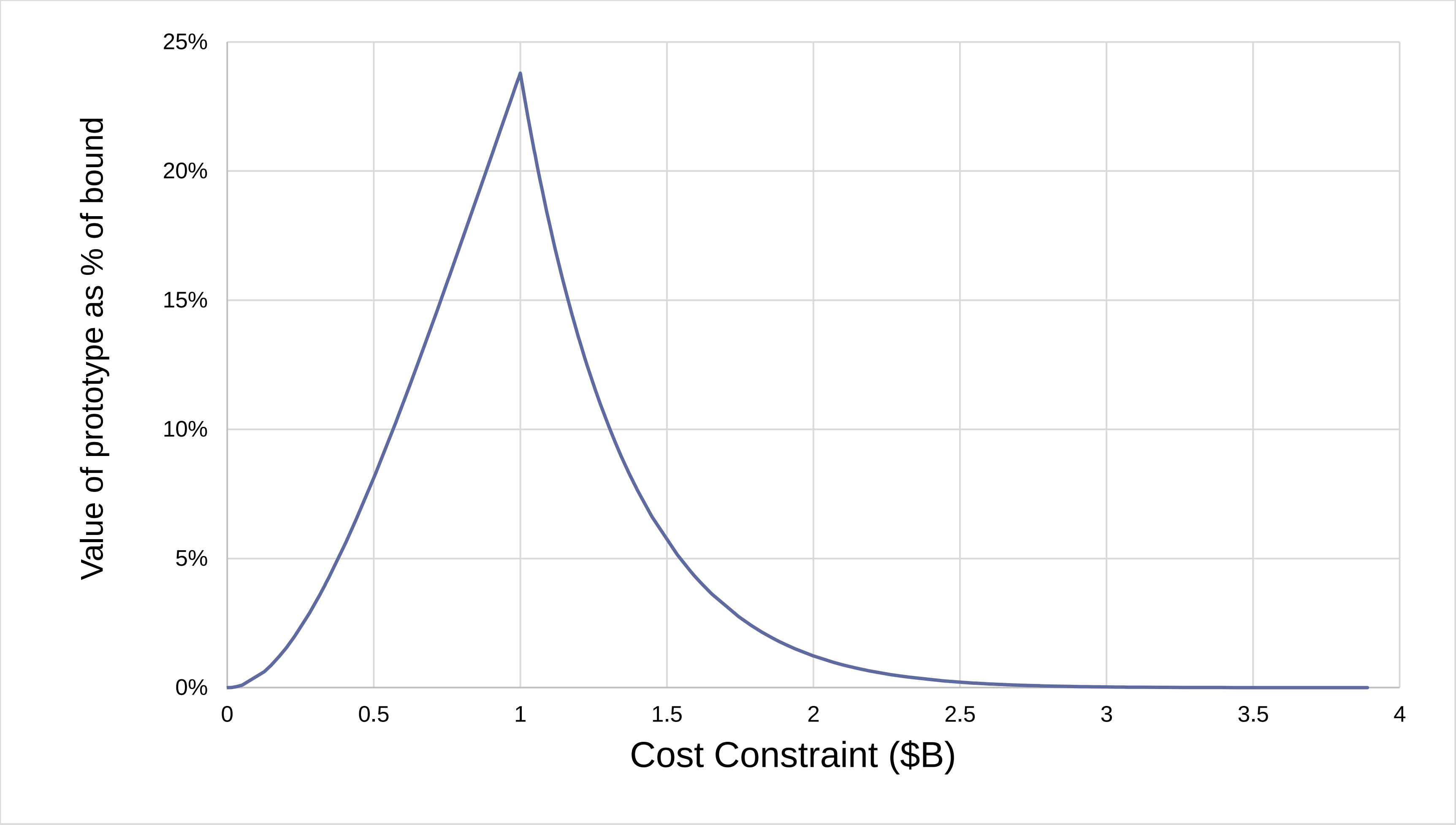

Thanks to the Air Force Research Laboratory for their inspiration and help in this project.

And thanks to you!

Backup

Bayes tells us something about what we can expect

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

We want the distribution of
- the cost *given* some prototype outcome

We need the distribution of
- the prototype *given* cost
- the cost
- ~~the prototypes~~

??????

Just a scaler!

So, not great.
But what if we look at attributes that incorporate all possible prototype outcomes?