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A Systems Complexity-Based Assessment of Risk in Acquisition and Development Programs

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Acquisition Research Program
Naval Postgraduate School

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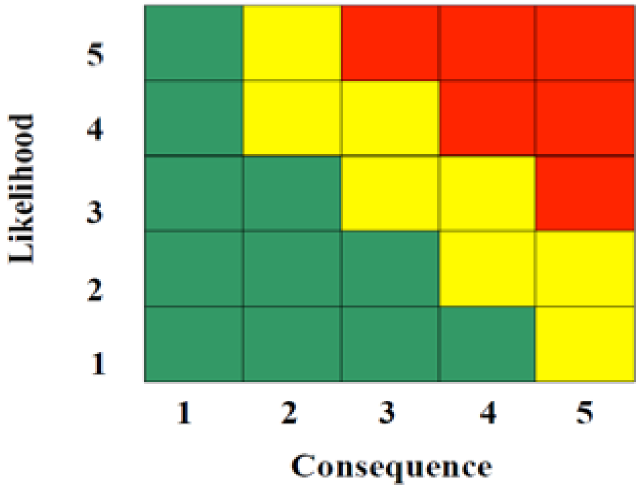
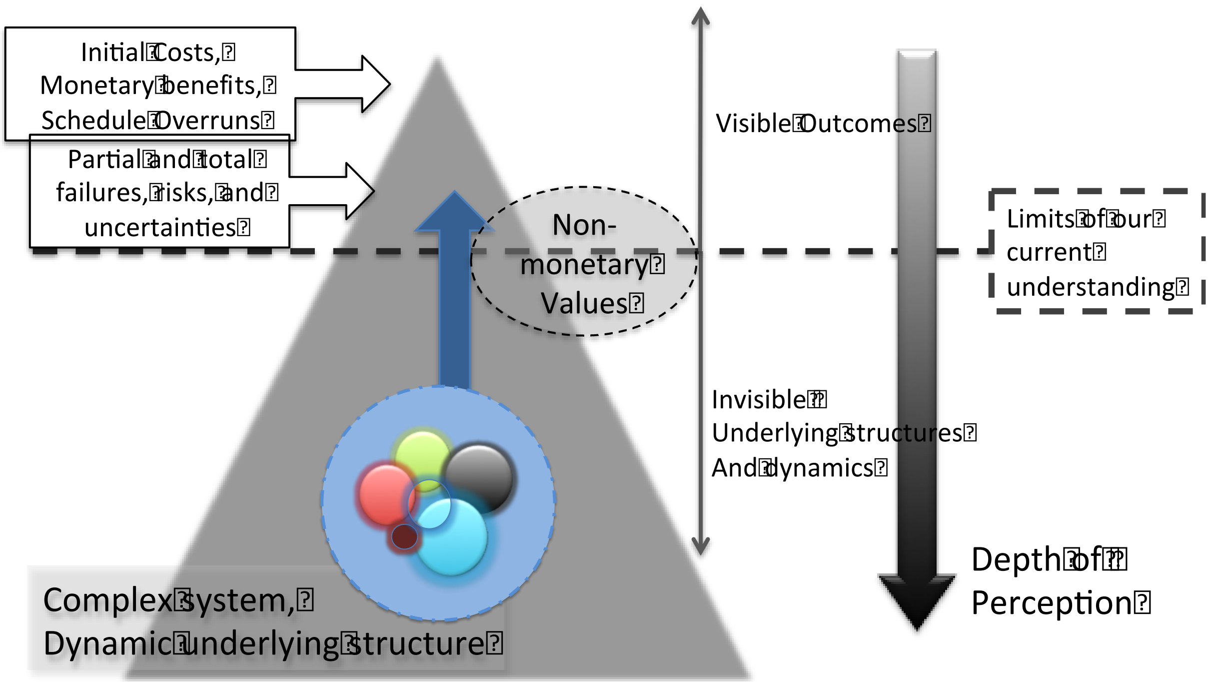
Introduction: Assessing Risk in Various Phases of System Development and Operation



"Risk is a measure of future uncertainties in achieving program performance goals and objectives within defined cost, schedule and performance constraints."
- Office of the Undersecretary of Defense

- The current risk identification method does not inform the decision makers well on the underlying causes of risk and consequences.
- No variation (error bars) around three colors. Abrupt shift from one color to other is possible and is seen in practice.
- Interactions and ordering among risks cannot be shown. Consequences are not presented in tangible forms of potential cost and schedule overruns as well as underperformance
- No typology of risks associated with causes (internal, external), phases of life cycle (certain risks are more common in particular phases), and interconnections among choices.
- Consequences are not presented in tangible forms of potential cost to remedy (a NASA practice) and extent of schedule overruns. PMs cannot use risk matrix to make trades.

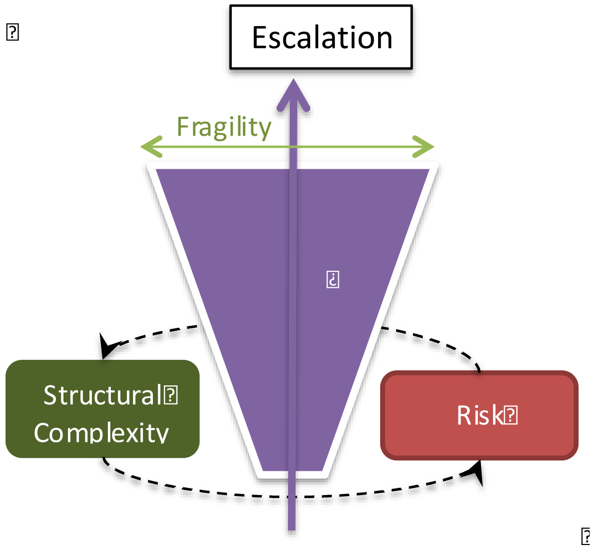
Introduction: Complexity and Risk Relationship



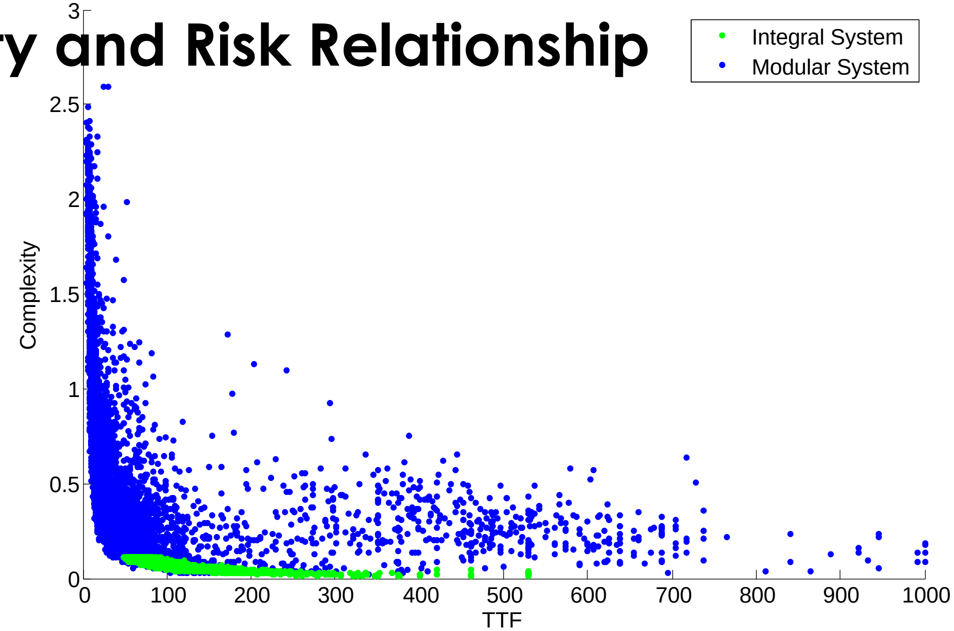
Systems complexity in various phases of development and operation of an engineered system can surface into visible and detectable realm in forms of costs, schedule overruns and partial or catastrophic failure

- Risk and consequences of uncertainty are often symptoms of deeper dynamics that exist in the technical system and the creating/managing organization.
- A portion of the technical risks are often rooted in the system's complexity, and/or the lack of know-how of the managing organization to handle the complexity of the technical system.
- Quantifying the engineered system complexity, can aid PMs to make optimal decisions in design and operation of a technical system

Introduction: Complexity and Risk Relationship



The Complexity-Risk spiral. Insignificant uncertainties and risks in combination with structural complexity escalate into a fragile situation and to a point of no return at which failure is certain.



F6 Simulation results showing that increased structural complexity leads to shorter time to failure in the system.

Research Objective:

To link technical complexity with uncertainty and risk across the stages of the acquisition process or various system development, and based on changes quantify and update risk elements for decision-making on technical choices, project continuation, modification or cancellation.

Introduction: The Need for Complexity Measures in Engineered Systems

The spacecraft was a partially **reusable** human spaceflight vehicle for Low Earth Orbit, which resulted from joint **NASA and US Air Force** efforts after Apollo. “The vehicle consisted of a **spaceplane** for orbit and re-entry, fueled by an expendable liquid hydrogen/liquid oxygen tank, with reusable strap-on solid booster rockets. […] A total of five operational orbiters were built, and of these, **two** were destroyed in **accidents**.”

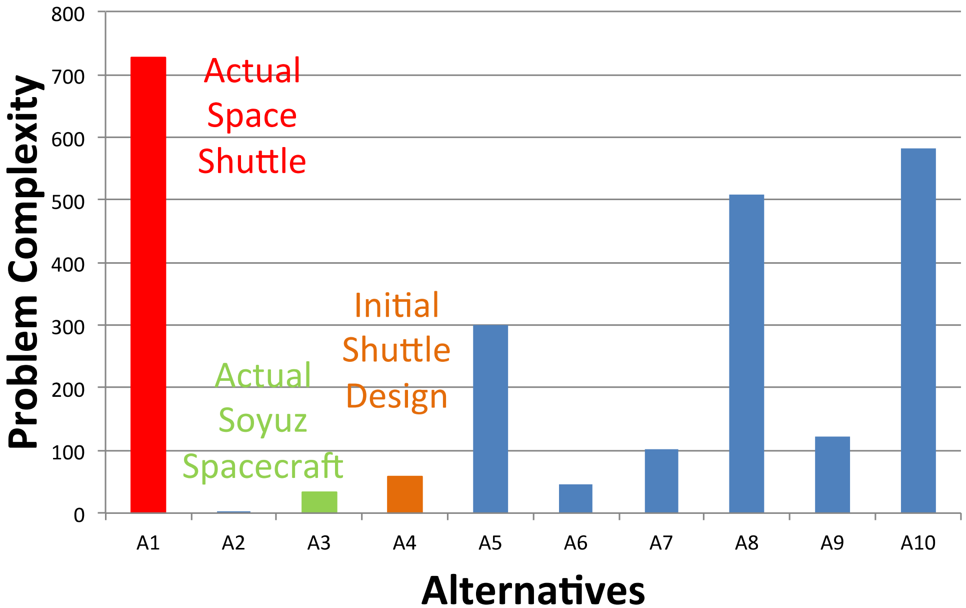
“Soyuz is a series of spacecraft initially designed for the **Soviet space programme** and **still in service today**. […] The Soyuz was originally built as part of the Soviet Manned Lunar programme. […] The Soyuz spacecraft is launched by the Soyuz rocket, the most frequently used and **most reliable** Russian launch vehicle to date.”

Problem Complexity: Shuttle vs. Soyuz

Reference: Salado and Nilchiani 2014

$$C_p = K \cdot \left(\sum_{i=1}^{n} a_i \cdot r_{f_i}\right)^{E} \cdot \prod_{j=1}^{m} H_j^{\,b_j}$$

Salado Problem complexity Equation in Requirements



Introduction: Complexity Measurement in system lifecycle

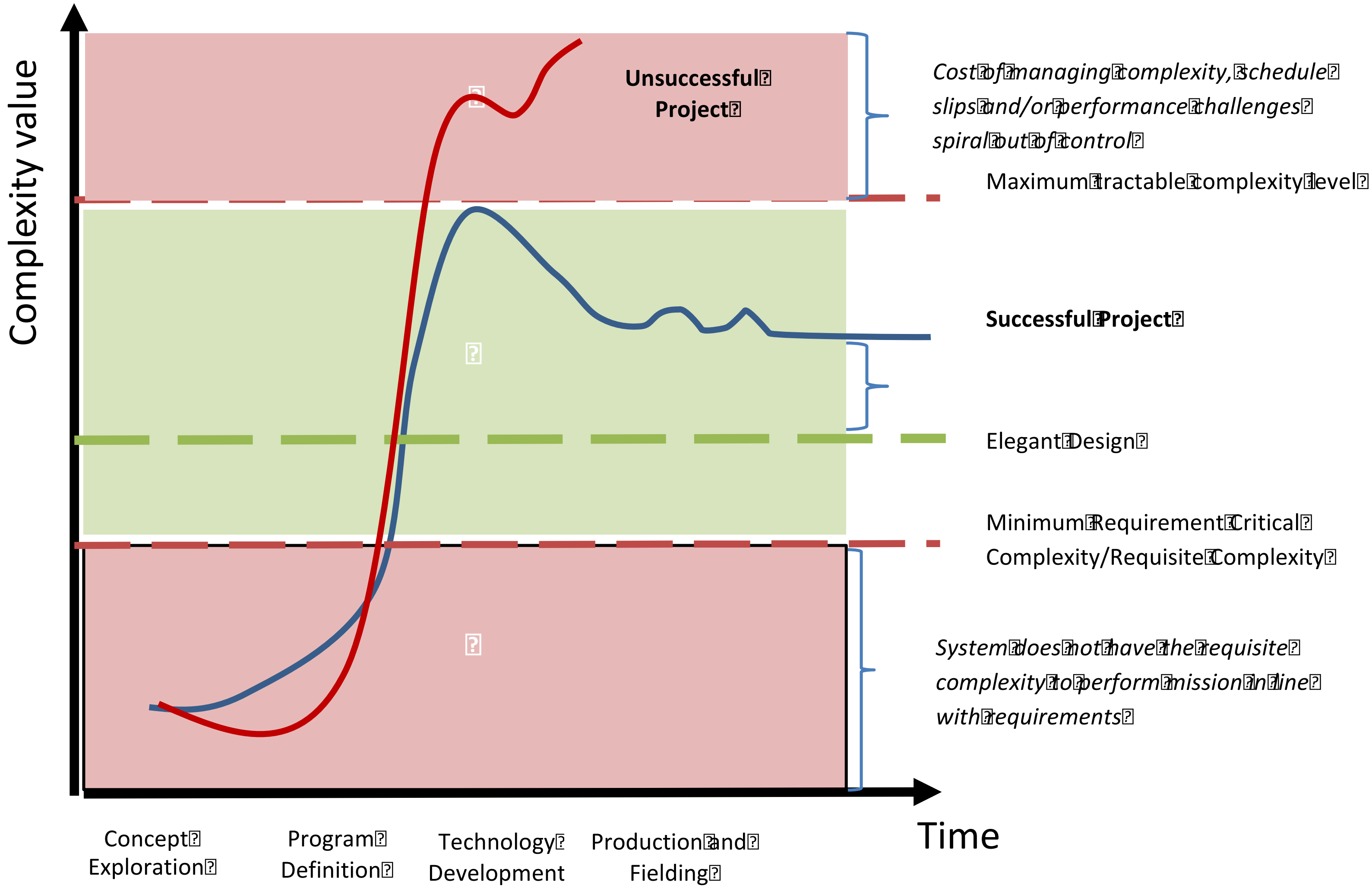


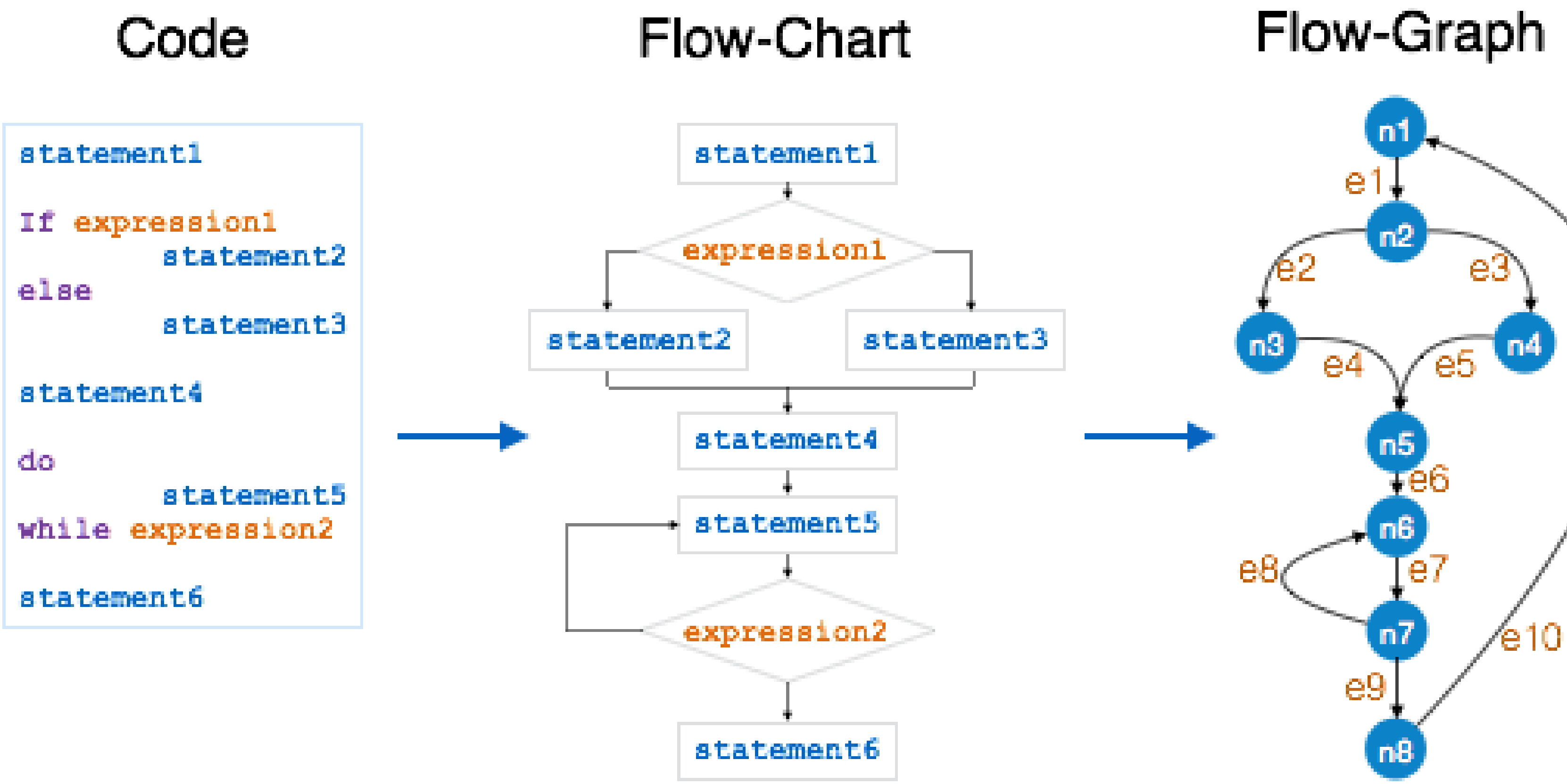
Figure 11. Complexity evolution throughout the systems acquisition lifecycle

Literature Review

- Cyclomatic Number
- Free Energy Density Rate
- Propagation Cost and Clustered Cost
- Spectral Structural Complexity Metric

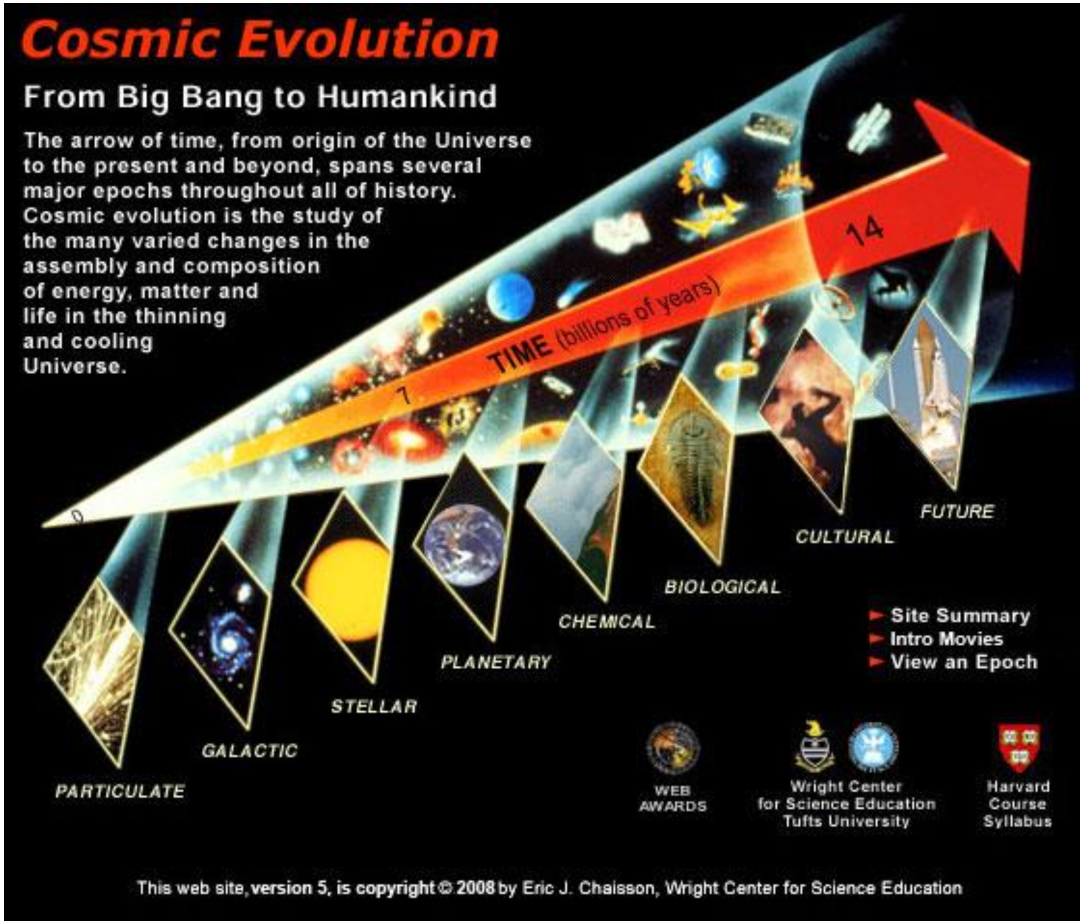
Cyclomatic Number McCabe

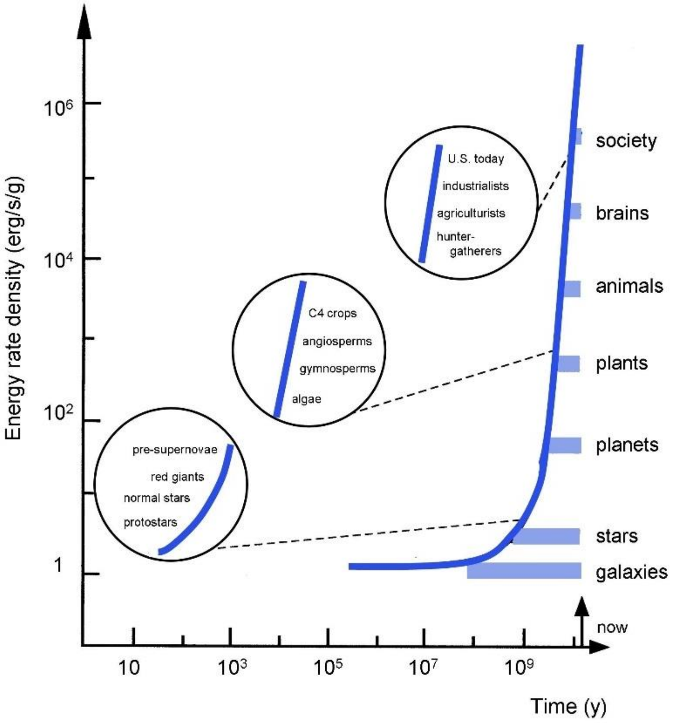
$$v(G) = e - n + p$$



https://www.tutorialspoint.com/software_engineering/software_design_complexity.htm

Free Energy Density Rate Chaisson





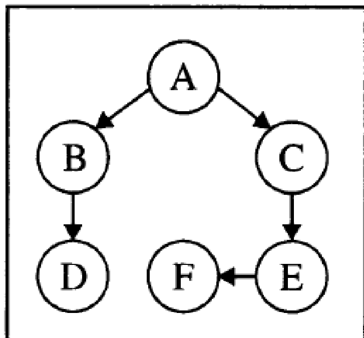
$$Complexity = \frac{Energy}{Time * Mass}$$

http://www.informationphilosopher.com/solutions/scientists/chaisson/
http://www.metanexus.net/essay/we-are-going-cosmic-flow-will-we-float-or-sink

Propagation Cost and Clustered Cost MacCormack, Baldwin, Rusnak

Architectural analysis of software systems

- Files are nodes
- Function calls are edges
- DSM based (adjacency matrix)



	A	B	C	D	E	F
A	0	1	1	0	0	0
B	0	0	0	1	0	0
C	0	0	0	0	1	0
D	0	0	0	0	0	0
E	0	0	0	0	0	1
F	0	0	0	0	0	0

Propagation cost:

- Cost of impact of change in one file on others
- Evaluated through matrix powers
- Average over dependencies

Clustered cost:

- Weighted propagation cost
- Dependencies within cluster are low cost
- Dependencies between clusters are high cost

Spectral Structural Complexity
Sinha, deWeck

Hückel Molecular Orbital (HMO) Theory

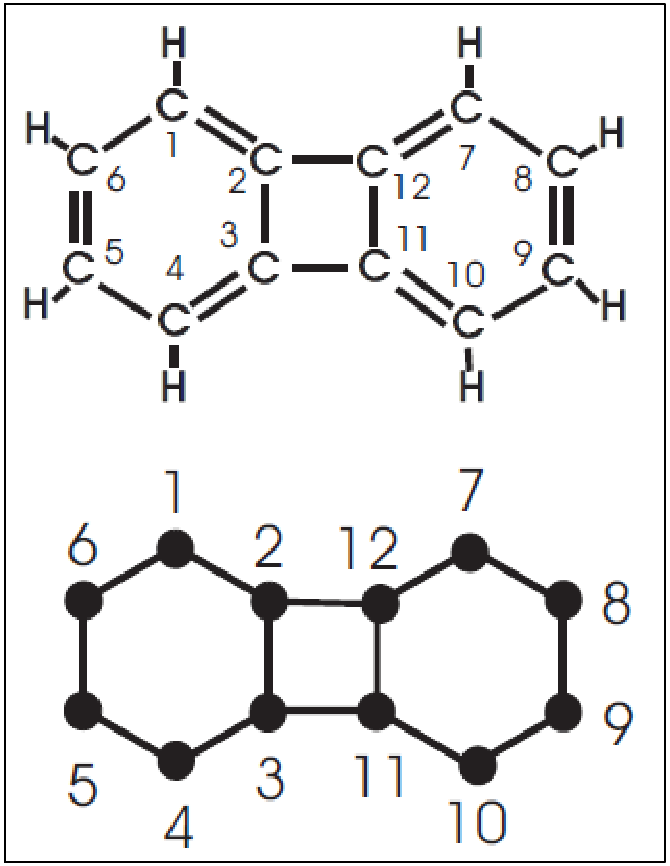
$$H\psi = E\psi$$

Definition of structural complexity

$$C(n,m,A) = \underbrace{\sum_{i=1}^{n} \alpha_i}_{C_1} + \underbrace{\left(\sum_{i=1}^{n}\sum_{j=1}^{n} \beta_{ij} A_{ij}\right)}_{C_2} \underbrace{\gamma E(A)}_{C_3}$$

where

- C_1 is the contribution of the size,
- C_2 is the contribution of the connectivity,
- C_3 is the contribution of the topology.

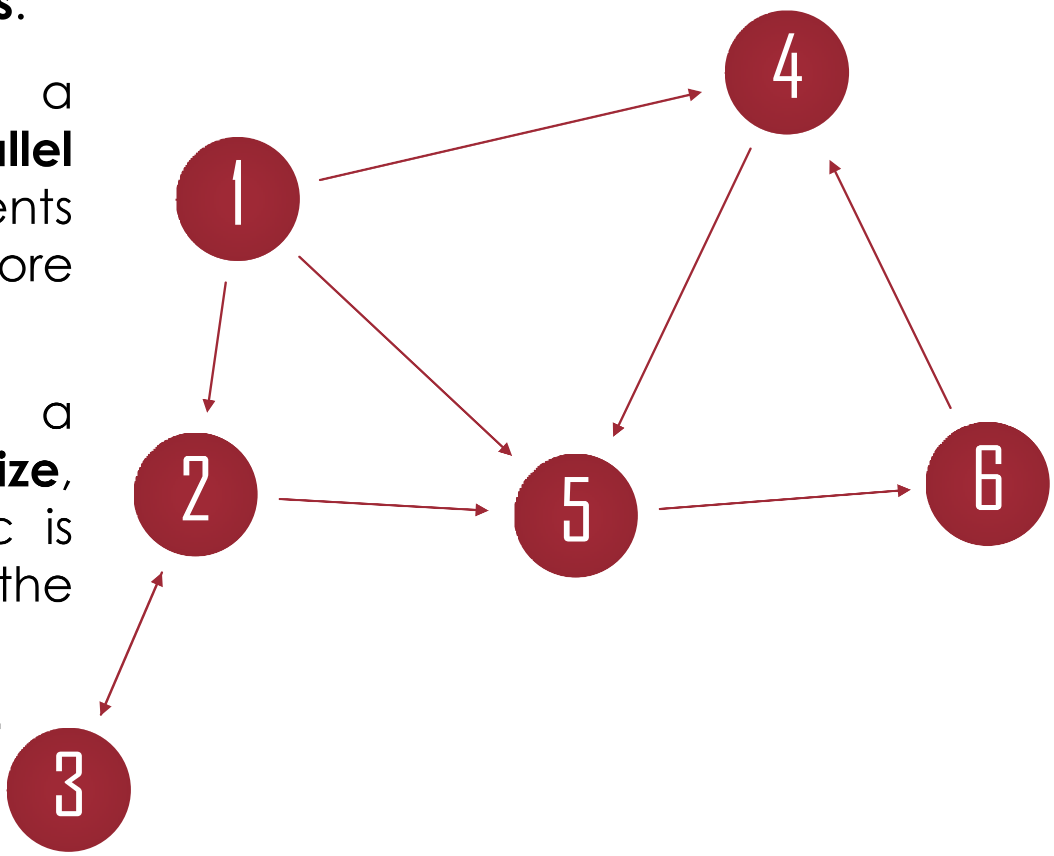


Methodology

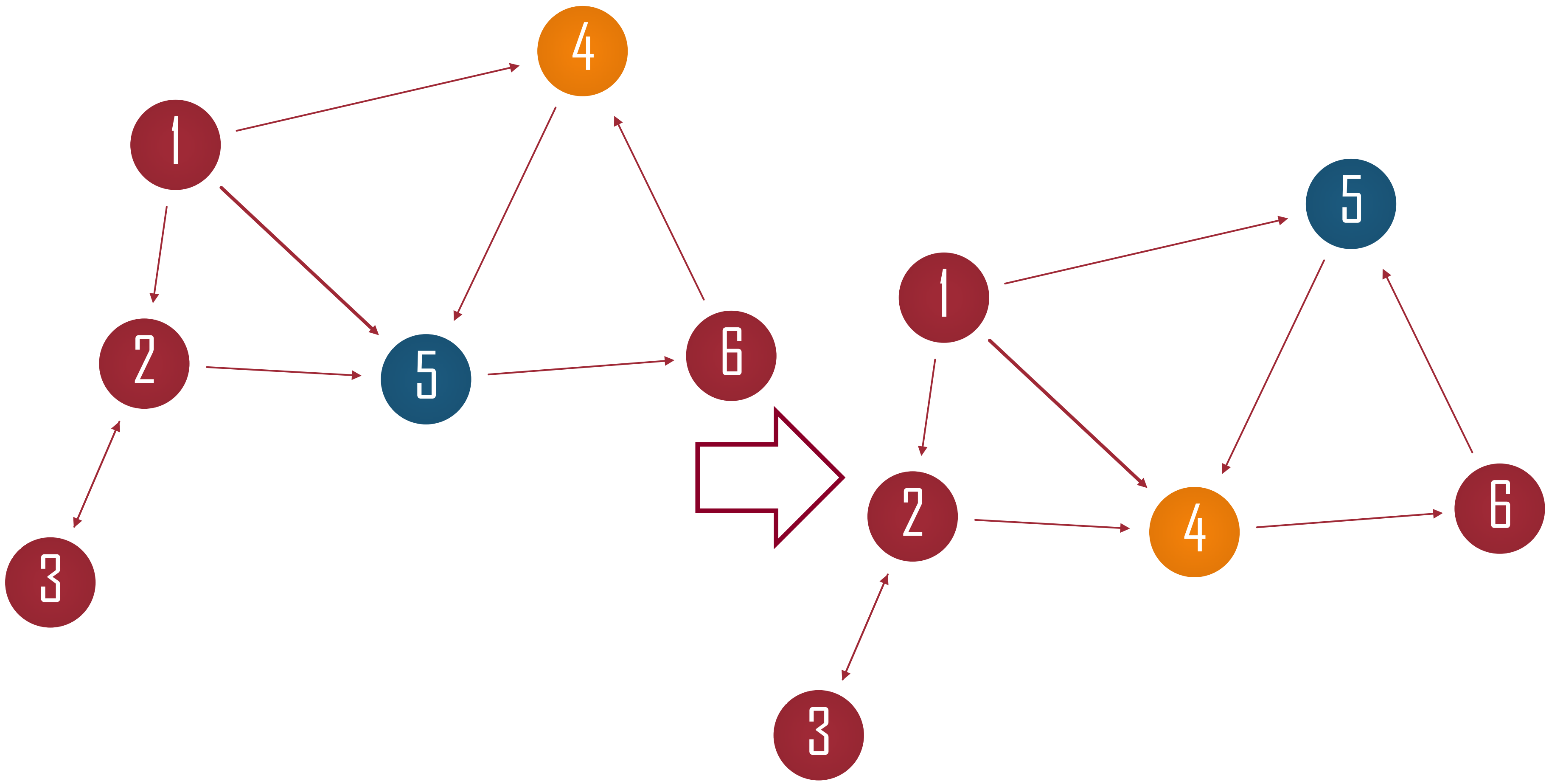
- Requirements for the new metric
- Component swap test
- Interface swap test

Requirements for a Structural Complexity Metric

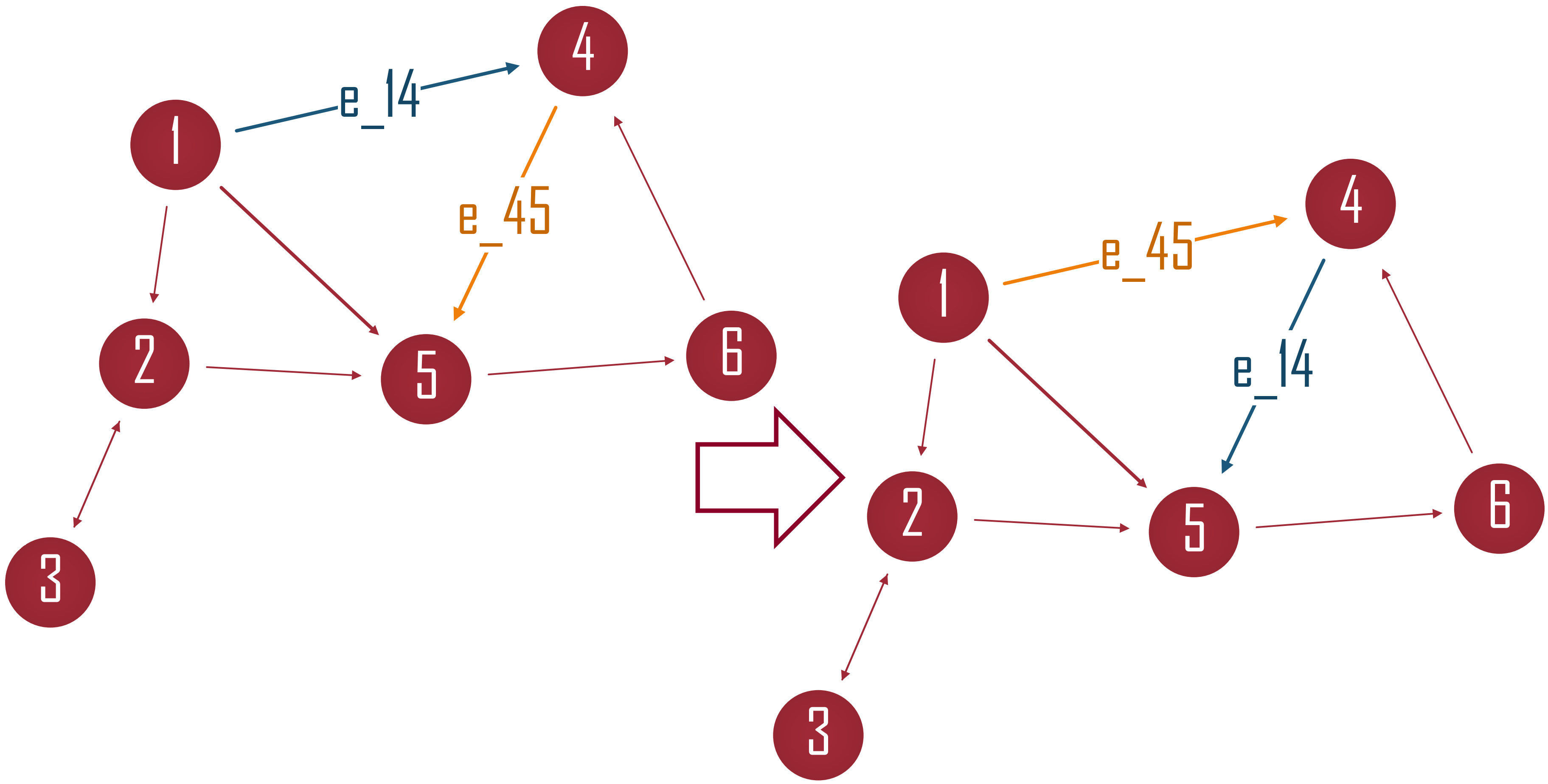
1. Measure the complexity of a system with **directed interfaces**.
2. Measure the complexity of a system with **multiple parallel edges**, in which two components can be connected via more than one edge.
3. Measure the complexity of a system **with respect to its size**, where the complexity metric is normalized with respect to the extension of the system.
4. Pass the **component swap test**
5. Pass the **interface swap test**



Component Swap Test



Interface Swap Test



Spectral Theory of Systems Complexity

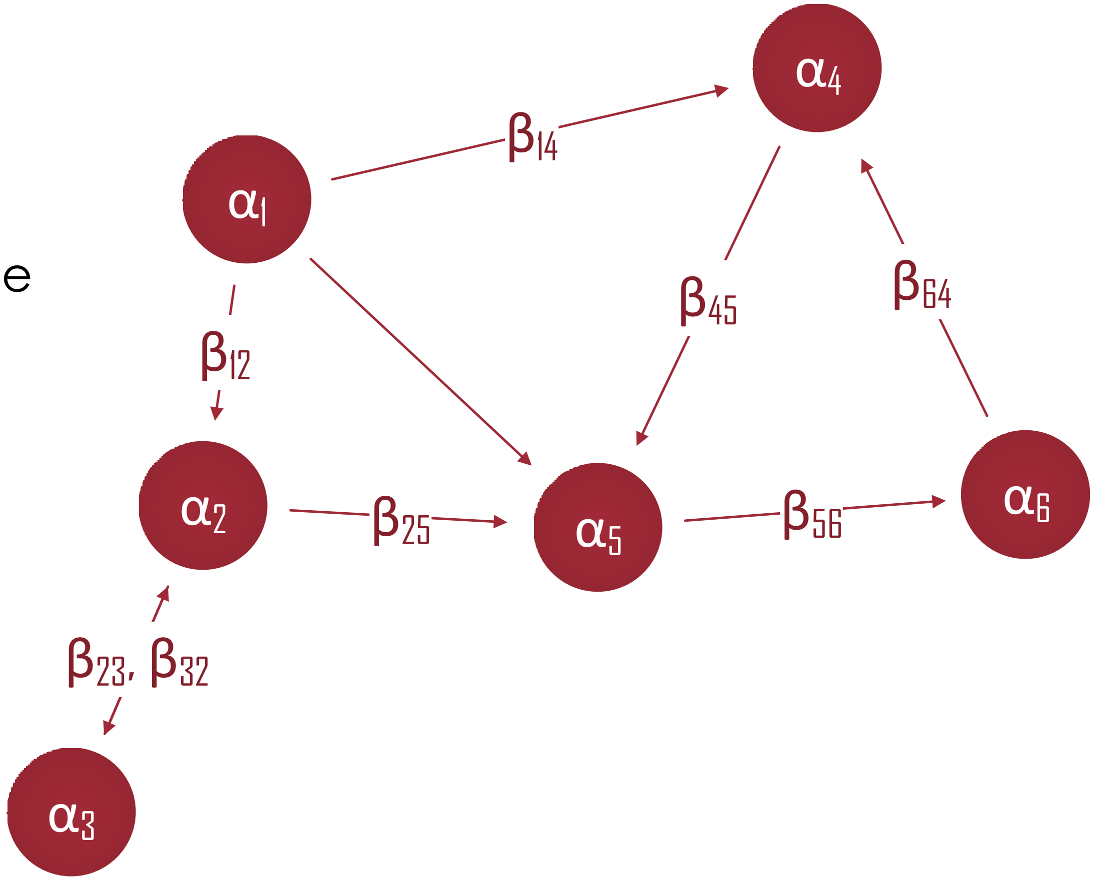
- Spectral Complexity Metric
- Adjacency Matrix
- Laplacian Matrix
- Normalized Laplacian Matrix

Spectral Complexity Metric

Knowing

- α_i complexity of the i^{th} component
- β_{ij} complexity of the interface between the i^{th} and j^{th} component

In this presentation we are assuming $\alpha_i = 1$ and $\beta_{ij} = 1$, therefore the weighted and unweighted cases will be equivalent.



Graph Energy
Matrix Energy

The Graph Energy (Gutman 1978) is evaluated using the eigenvalues of the adjacency matrix, as

$$E_A(G) = \sum_{i=1}^{n} |\lambda_i|$$

The Laplacian Energy of a Graph (Gutman 2005) is defined as

$$E_L(G) = \sum_{i=1}^{n} \left|\mu_i - \frac{2m}{n}\right|$$

The generalization to any matrix (Cavers 2010) is

$$E_M(G) = \sum_{i=1}^{n} \left|\lambda_i(M) - \frac{tr(M)}{n}\right|$$

Three Candidates for a Spectral Complexity Metric

Similar to the approach by Sinha, but with weighted graph

$$C_A = \frac{E_A(G)}{n} = \frac{1}{n}\sum_{i=1}^{n}|\lambda_i|$$

Laplacian approach

$$C_L = \frac{E_L(G)}{n} = \frac{1}{n}\sum_{i=1}^{n}\left|\mu_i - \frac{2m}{n}\right|$$

Normalized Laplacian approach

$$C_{\mathcal{L}} = E_{\mathcal{L}}(G) = \sum_{i=1}^{n}|\nu_i - 1|$$

Are these metrics **computable?**

Adjacency Matrix

$$A(u,v) = \begin{cases} 1 & if\ u\ and\ v\ are\ adjacent, \\ 0 & otherwise. \end{cases}$$

In case of weighted edges

$$A(u,v) = \begin{cases} w(u,v) & if\ u\ and\ v\ are\ adjacent, \\ 0 & otherwise. \end{cases}$$

The eigenvalues in the case of symmetric matrix are

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$$

and the following is true

$$\sum_{i=1}^{n} \lambda_i = 0, \qquad \sum_{i=1}^{n} \lambda_i^2 = 2m$$

Adjacency Matrix
Directed Graphs

Directed edges create an **asymmetry** in the adjacency matrix representation of the graph.

This leads to **complex eigenvalues**.

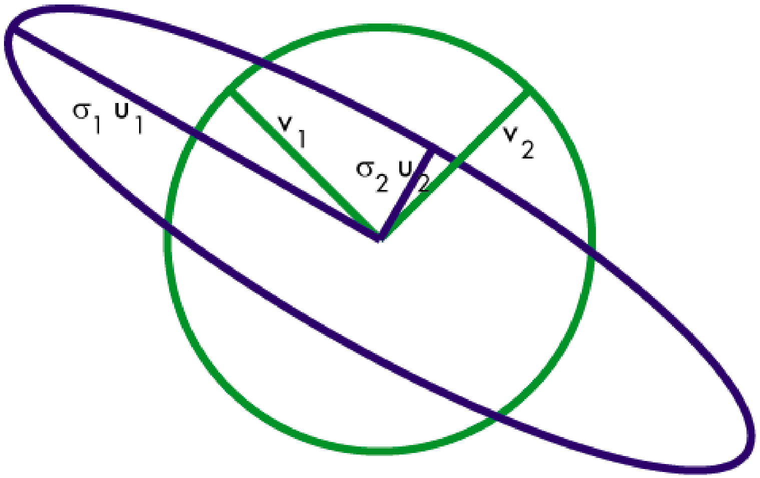
In this case **singular value decomposition** is an alternative to eigenvalue decomposition.

The adjacency matrix is decomposed as

$$A = U\Sigma V^T$$

where U and V are unitary matrices and Σ is a diagonal matrix containing the singular values

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n$$



Eigenvalue and singular value decomposition of a symmetric matrix

$$A = U\Lambda U^T$$

$$A = U\Sigma V^T$$

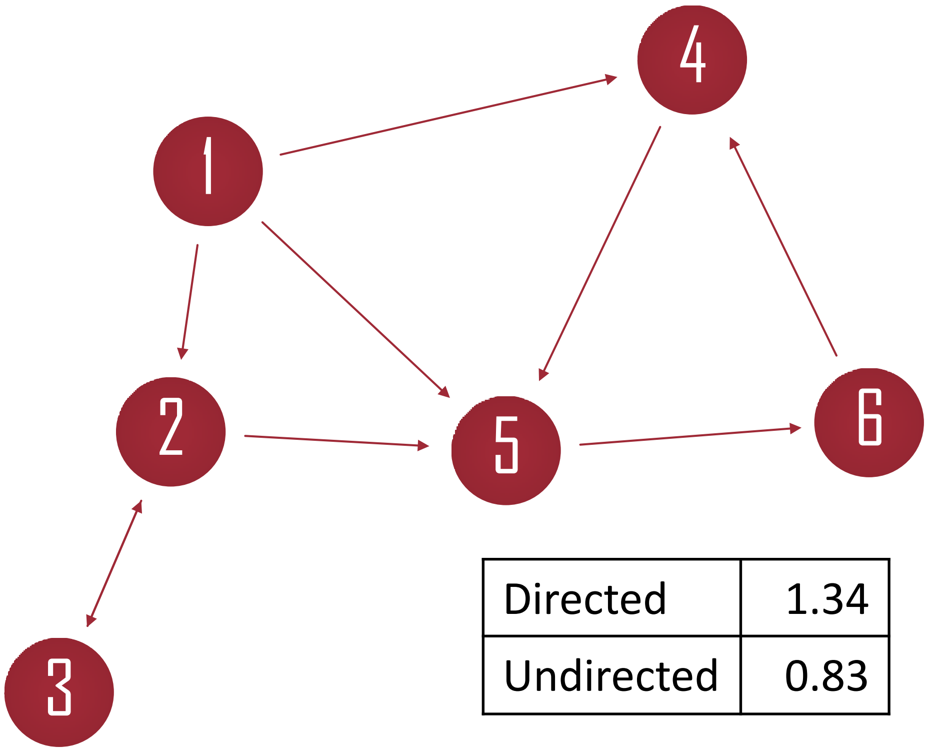
$$\sigma_i = |\lambda_i|$$

https://www.mathworks.com/company/newsletters/articles/professor-svd.html

Adjacency Matrix

$$A_{dir} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$A_{undir} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$



Directed	1.34
Undirected	0.83

Laplacian Matrix Undirected Graphs

$$L(u,v) = D(u,v) - A(u,v) = \begin{cases} d_v & if\ u = v, \\ -1 & if\ u\ and\ v\ are\ adjacent, \\ 0 & otherwise. \end{cases}$$

In case of weighted edges

$$L(u,v) = D(u,v) - A(u,v) = \begin{cases} d_v - w(u,v) & if\ u = v, \\ -w(u,v) & if\ u\ and\ v\ are\ adjacent, \\ 0 & otherwise. \end{cases}$$

The eigenvalues are

$$0 = \mu_1 \leq \mu_2 \leq \cdots \leq \mu_n$$

and the following is true

$$\sum_{i=1}^{n} \mu_i = 2m, \qquad \sum_{i=1}^{n} \mu_i^2 = 2m + \sum_{i=1}^{n} d_i^2$$

Laplacian Matrix
Directed Graphs

Laplacian matrix for directed graphs

$$L = \Phi - \frac{\Phi P + P^* \Phi}{2}$$

Where P is the walk matrix

$$P(u,v) = \begin{cases} \frac{1}{d_u} & if\ (u,v)\ is\ an\ edge, \\ 0 & otherwise. \end{cases}$$

For weighted graphs

$$P(u,v) = \frac{w(u,v)}{d_{out}(u)}$$

And Φ is the diagonal matrix of the Perron vector of P: $\phi(v) > 0$

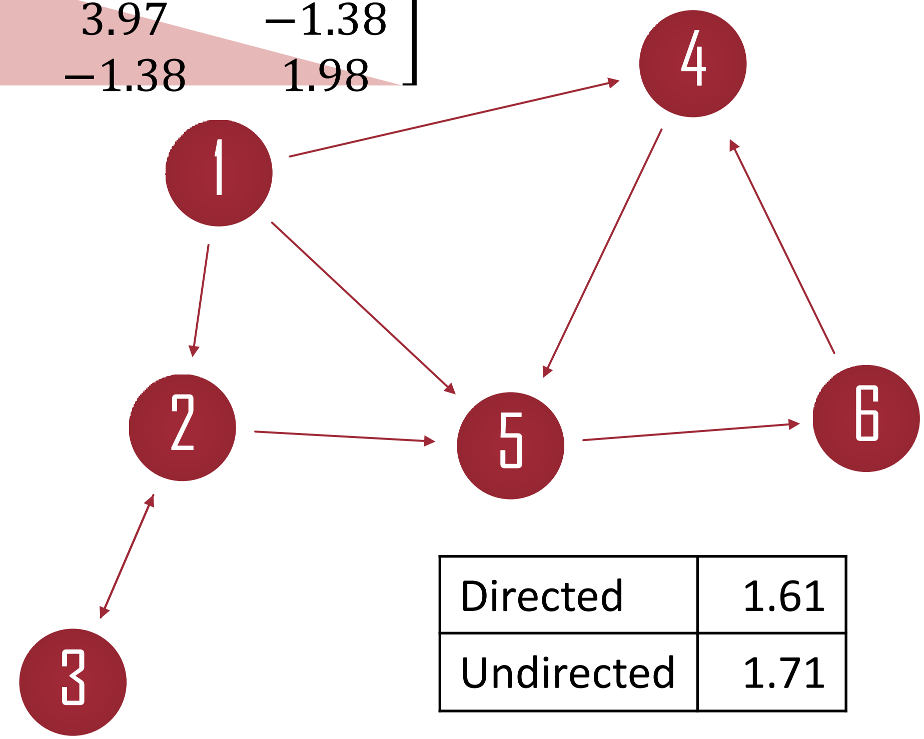
$$\phi P = \rho \phi$$

[Fan Chung – Laplacians and the Cheeger Inequality for Directed Graphs – 2005]

Laplacian Matrix
Graph Energy

$$L_{dir} = \begin{bmatrix} 2.97 & -0.26 & -0.017 & -0.16 & -0.18 & -0.064 \\ -0.26 & 2.97 & -1.20 & -0.041 & -0.32 & -0.034 \\ -0.017 & -1.20 & 0.99 & -0.027 & -0.032 & -0.022 \\ -0.16 & -0.041 & -0.027 & 2.97 & -1.64 & -1.19 \\ -0.18 & -0.32 & -0.032 & -1.64 & 3.97 & -1.38 \\ -0.64 & -0.034 & -0.022 & -1.19 & -1.38 & 1.98 \end{bmatrix}$$

$$L_{undir} = \begin{bmatrix} 3 & -1 & 0 & -1 & -1 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 3 & -1 & -1 \\ -1 & -1 & 0 & -1 & 4 & -1 \\ 0 & 0 & 0 & -1 & -1 & 2 \end{bmatrix}$$



Directed	1.61
Undirected	1.71

Normalized Laplacian Matrix Undirected Graphs

$$\mathcal{L}(u,v) = D^{-1/2} L D^{-1/2} = \begin{cases} 1 & if\ u = v, \\ -\dfrac{1}{\sqrt{d_u d_v}} & if\ u\ and\ v\ are\ adjacent, \\ 0 & otherwise. \end{cases}$$

In case of weighted edges

$$\mathcal{L}(u,v) = D^{-1/2} L D^{-1/2} = \begin{cases} 1 - \dfrac{w(u,v)}{d_u} & if\ u = v, \\ -\dfrac{w(u,v)}{\sqrt{d_u d_v}} & if\ u\ and\ v\ are\ adjacent, \\ 0 & otherwise. \end{cases}$$

And the eigenvalues are

$$0 = v_1 \leq v_2 \leq \cdots \leq v_n \leq 2$$

Normalized Laplacian Matrix Directed Graphs

Normalized Laplacian matrix

$$\mathcal{L} = \Phi^{-1/2} L \Phi^{-1/2} = I - \frac{\Phi^{1/2} P \Phi^{-1/2} + \Phi^{-1/2} P^* \Phi^{1/2}}{2}$$

Where P is the walk matrix

$$P(u,v) = \begin{cases} \frac{1}{d_u} & if\ (u,v)\ is\ an\ edge, \\ 0 & otherwise. \end{cases}$$

For weighted graphs

$$P(u,v) = \frac{w(u,v)}{d_{out}(u)}$$

And Φ is the diagonal matrix of the Perron vector of P: $\phi(v) > 0$

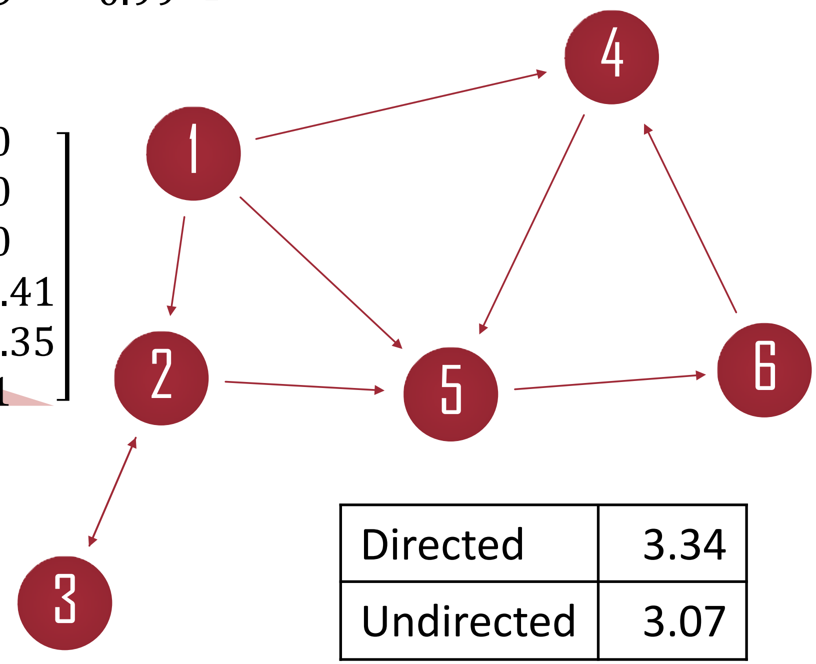
$$\phi P = \rho \phi$$

[Fan Chung – Laplacians and the Cheeger Inequality for Directed Graphs – 2005]

Normalized Laplacian Matrix

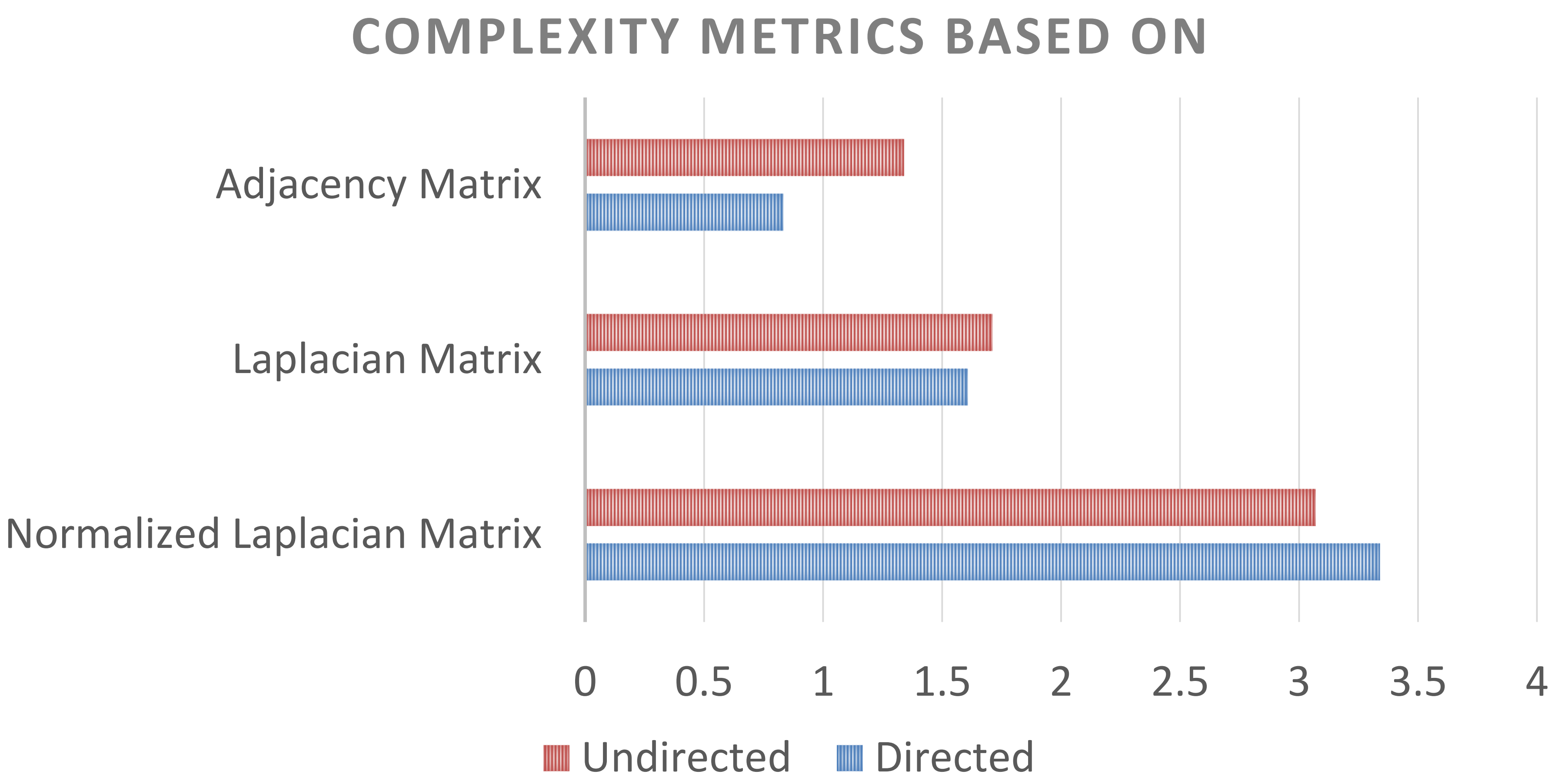
$$\mathcal{L}_{dir} = \begin{bmatrix} 0.99 & -0.088 & -0.0096 & -0.052 & -0.052 & -0.026 \\ -0.088 & 0.99 & -0.69 & -0.014 & -0.092 & -0.014 \\ -0.0096 & -0.69 & 0.99 & -0.016 & -0.016 & -0.016 \\ -0.052 & -0.014 & -0.016 & 0.99 & -0.47 & -0.49 \\ -0.052 & -0.092 & -0.016 & -0.47 & 0.99 & -0.49 \\ -0.026 & -0.014 & -0.016 & -0.49 & -0.49 & 0.99 \end{bmatrix}$$

$$\mathcal{L}_{undir} = \begin{bmatrix} 1 & -0.33 & 0 & -0.33 & -0.29 & 0 \\ -0.33 & 1 & -0.58 & 0 & -0.29 & 0 \\ 0 & -0.58 & 1 & 0 & 0 & 0 \\ -0.33 & 0 & 0 & 1 & -0.29 & -0.41 \\ -0.29 & -0.29 & 0 & -0.28 & 1 & -0.35 \\ 0 & 0 & 0 & -0.41 & -0.35 & 1 \end{bmatrix}$$



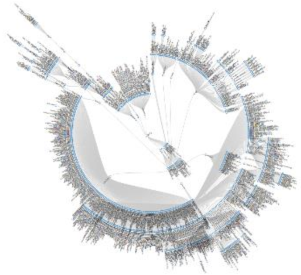
Directed	3.34
Undirected	3.07

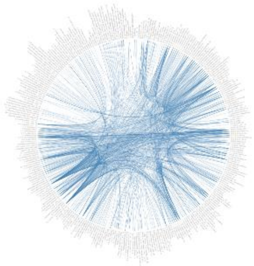
Computation of Metrics



Conclusion

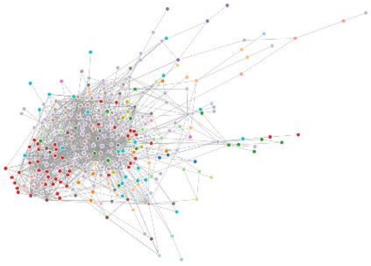
- Identification of features and limitations in existing structural complexity metrics
- Overcoming of limitations with creation of new metrics
- Verification of the computability of the new metrics



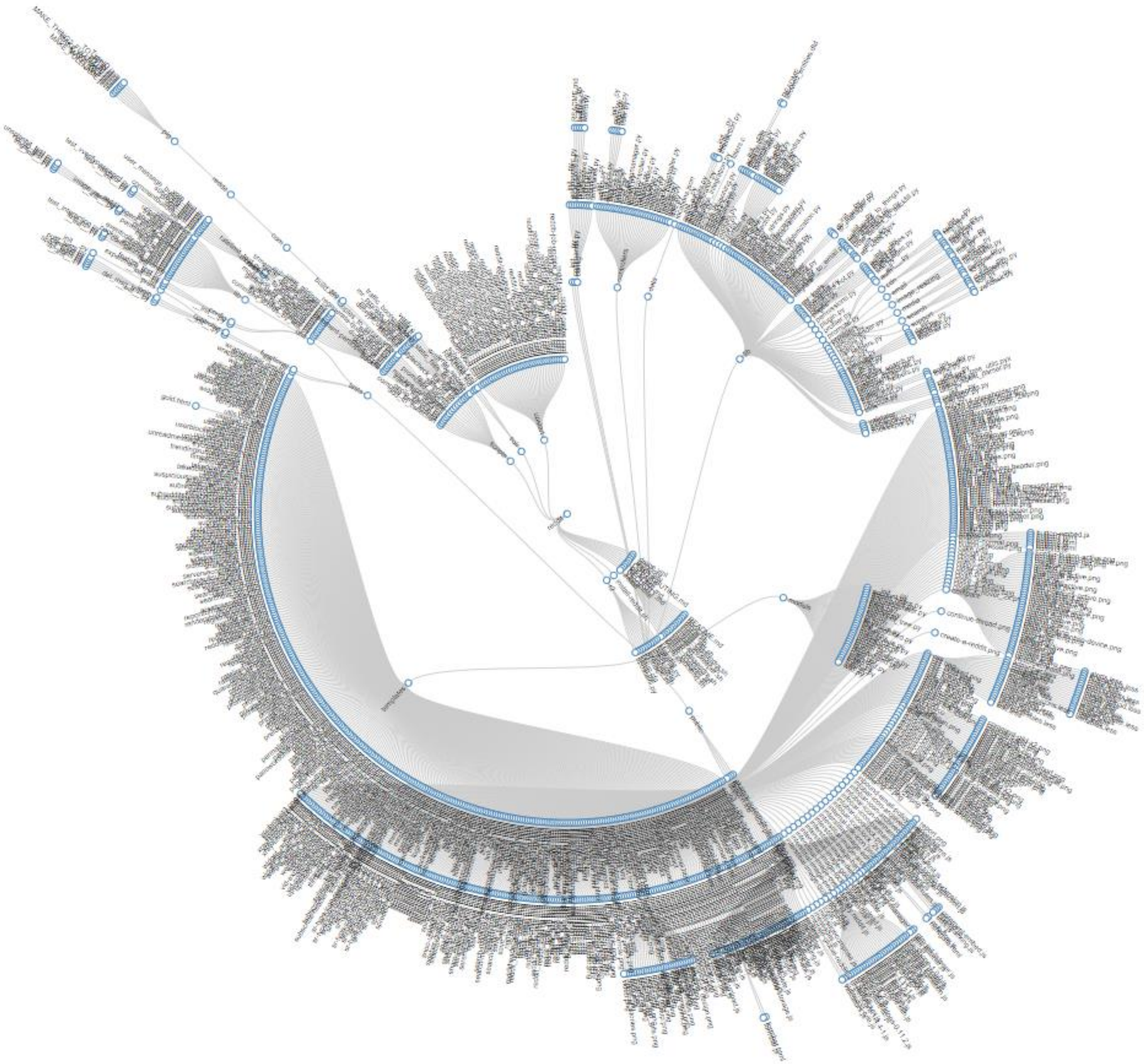


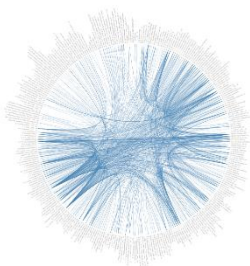
Future work

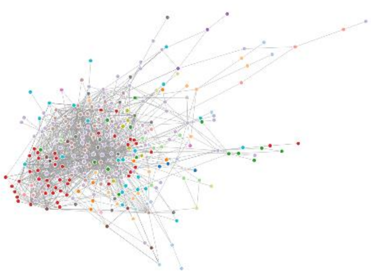
- Validation of the new metrics
- Application to real world cases



Future Work







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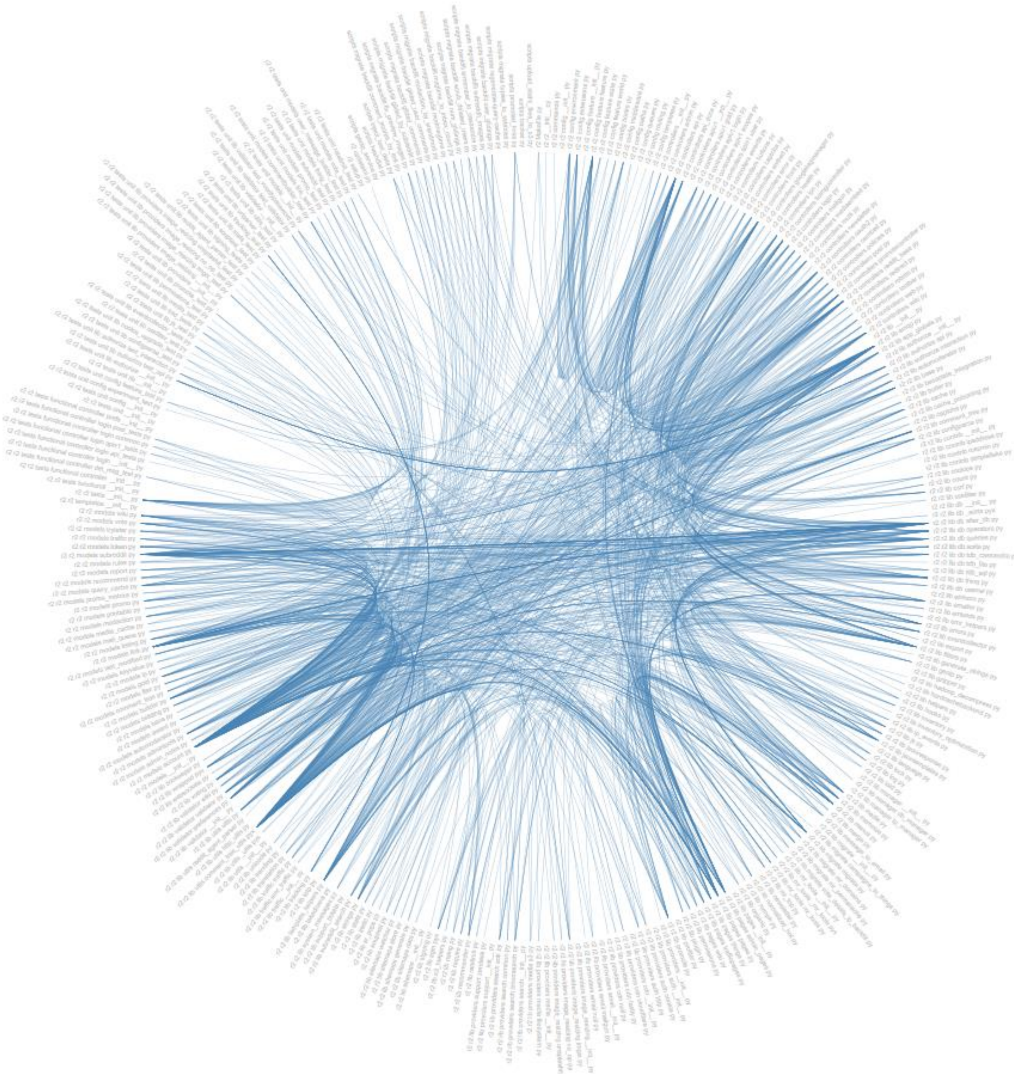
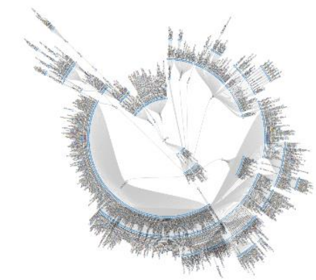
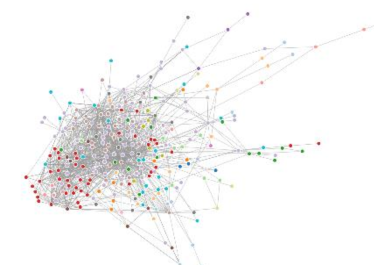
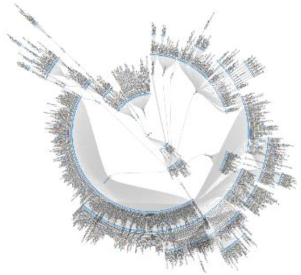
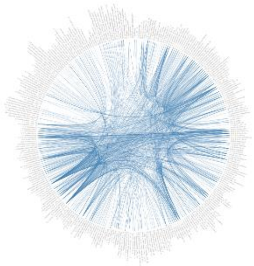






Future Work





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