# FLEXIBLE SHIP OPTIONS:
# Business Case Valuation of Strategic Flexibility in Ship Building

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# Introduction

- Research Goal: Assess total future value of Flexible Ships design features to enable affordable war fighting relevance over a ship's full service life.
  - Affordable War Fighting means a higher cost now but greater ROI over the entire service life and lifecycle of the ship.
  - IRM methodology can be used to support and refine the Future Surface Combatant Analysis of Alternatives.
- Methodology provides a reusable, extensible, adaptable, and comprehensive advanced analytical modeling process.
- Will help the U.S. Navy in quantifying, modeling, valuing, and optimizing a set of ship design options.
- Approach can be used to develop a robust business case for making strategic design decisions, under uncertainty. Not design/engineering.

# The Five Tenets of Flexible Ships

- Payloads Decoupled from Platforms
- Standard Interfaces
- Rapid Reconfiguration
- Planned Access Routes
- Allowance Margins for Modernization

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Strategic real options valuation (ROV) provides the option holder the right, but not the obligation, to hold off on executing a certain decision until a later time when uncertainties are resolved and when better information is available. The option implies that flexibility to execute a certain path exists and was predetermined or predesigned in advance.

# The Benefits of Flexible Ships

- Affordable warfighting relevance over the entire ship service life (higher cost now but greater ROI over the service life and lifecycle of the ship).
- Parallel development of payload vs. platform production (give me the power and space I need and we will bring in the weapon systems later, e.g., directed energy weapons).
- Reduction from lengthy and costly ship production work (make it easier up front for later swapping of technologies without predefining the exact point solutions of future unknown capabilities and timing).
- Increased competition and innovation (helps commoditize systems, without need to sunset).
- Cross platform commonality (LCS missions bay with the proper configuration management).

# The Benefits of Flexible Ships (II)

- Rapid Prototyping of Payloads for rapid acquisition of new capabilities (growth margins and future growth potentials are prebuilt).
- Modular Open Systems increases acquisition agility (put the studs in and do the panels later as needed, whether it be ceramic, Kevlar, high intensity polymers).
- Standard Interfaces provide for common platforms and enclosures, swappable equipment.
- Efficient technology refresh, faster incremental upgrades, and faster development, faster technology adoption and fielding.
- Paces future threats (flexible in meeting unknown future threats, cost, schedule, capability).

# The Challenges

- Flexibility vs. Affordability. Long term value, not immediate gains.
- Strategic Point of View. Flexibility means thinking through the future on where we might want the U.S. Navy to go.
- A Tactical Approach. With any upcoming repairs, implement small modularity capabilities instead of repairing back to the original.
- The research looks at building business case models to justify flexibility. We need to consider the need to “cut steel” during major ship alterations vs. faster implementation. Also, the higher the number of deployments and ships on station we obtain with flexible ships (opportunity costs of not being active in the fleet, back on station faster, faster schedule and lower labor and ship alteration costs in the future).

# Case Examples of Flexible Ships

- AEGIS Ashore. Aegis with SPY-6 Radar where modularity was a result rather than being designed-in up front. The need was for reusability on ships with rapid setup and deployment as well as rapid take down of equipment. MDA working with the Navy and ACE. The relocatable requirement forced the need for modularity.
- Air Missile Defense Radar (AMDR) SPY-6 architecture has enhanced capabilities including longer range and greater number of issues detection, a game changer. The main advancement is its longer range and ability in simultaneous threat assessments, and is integrated with the AEGIS system. AMDR is sensor agnostic with an open architecture, solid state system, and standard interface. Additional data links can be added quickly and cheaply, with simple maintenance and higher efficiency. Can also be integrated with other Electronic Warfare (EW) systems for rapid kill assessments and coordination of soft kill and hard kills.

# Case Examples of Flexible Ships (II)

- Directed Energy Weapons
  - A lot of unknowns such as power density needs (watts/cm), aperture physics, capability of continuous tracking and targeting high speed objects, and other advanced threats (e.g., Hyper-glides, UAV swarms).
  - The idea is to have excess and on-demand power (you have it the instant you need it, and to have more than you will need).
- Hybrid Power Systems and Storage for Directed Energy Weapons
  - Leverage 30X sensor improvement with only 2X power needs.
  - Constraints are the ship's size, weight, cooling, and fire control.
  - Need capabilities to face unknown future advanced threats. Capability gaps are identified with the help and coordination of the intelligence community.
  - The idea is not to have a perfect single point estimate foresight of capability needs but to be prepared to implement a range of future unknown systems to meet a set of future unknown threats.

# Case Examples of Flexible Ships (III)

- High Density Power
  - DDG51 FLT III presents an opportunity to upgrade its power plant for FY2020 to accommodate directed energy weapons (sponsor buy-in and the budget requirement to make sure things get into production on time… clock starts now).
  - Requires fast charge and ready at a moment's notice and instantaneous requirements. Power and energy is the foundation of the kill chain.
  - The uncertainty is that there is a stochastic load demand, which means that if the Navy is using directed energy weapons, they'd better have plenty of it.
  - With a capability to handle large demand loads, advanced solid state circuit protection and robust combat power controls are also required.
  - There needs to be a multifunction energy storage capability with a compact power conversion structure to reduce size and weight.
  - Unknowns: AC vs. DC, 6/12/18 KV system, heat loads and coolant levels, outputs (4MW x 20 buses) for Medium Volt DC, frequency, power conversion, storage area, fit on smaller ships, decoupled buses and needs for rotor alignments…

# Case Examples of Flexible Ships (IV)

- CANES Backfit
  - Started in 2013. Expectation that all surface combatants (~180) would be fully operational by 2022.
  - CANES undergoes a software refresh every two years and hardware upgrades every four years.
  - Given the extra volume that was built into LPD-17 (margin for growth and easier accessibility – wider main pee-way to accommodate larger item) the CANES backfit (replacing SWAN) on LPD-17 been more cost efficient than the CANES backfit on the DDG Flight IIA.
- LCS Missile Module
  - Initial module was funded by the Army for the XM501 NLOS Launch System.
  - The program was canceled in 2010 and the Navy was left without a replacement. Because the LCS was designed for a modular missile payload instead of being designed with a structurally integral missile system, the LCS was able to deploy and meet mission requirements while a new modular missile payload was developed.
  - The Hellfire Longbow was structurally tested in 2017 on USS Detroit.
  - This example highlights the cost savings of modularity. If LCS has been structurally designed with the XM501, replacement would have been costly, with extended yard periods and the ship would not have been able to deploy. With the modular missile bay, LCS was able to deploy while parallel development of a new missile module took place.

# Foreign Navies

- Royal Danish Navy
  - Started with the Flyvefisken Class (STANFLEX SF 300) are multi-role vessels (MRV) with standard hulls and modular design
  - STANFLEX design is capitalized on mission modularity by incorporating four interchangeable mission containers, one forward and three aft. These containers house all dedicated machinery and electronic payloads connected by a standard interface panel.
  - The Flyvefisken class demonstrated that a smaller number of MRVs were capable of meeting the same mission demands of a fleet almost twice its size. STANFLEX and modular payload allowed for containers to be pre-staged for mission flexibility while simultaneously reducing downtime for upgrades.
- German Navy
  - The Mehrzweck-Kombination (MEKO), which translates as “multi-purpose combination” uses modular mission payloads: MEKO A-100 Corvette and the MEKO A-200 Frigate.
  - Modules can be rotated for upgrades, and maintenance or between ships, which reduces the number of overall payloads required for the fleet. This simple reduction results in significant cost savings in procurement and maintenance over the life cycle of the ship.
  - MEKO designs rely heavily on modularity that increases the speed at which the ship can be built and facilitates faster upgrades and refits. The F125 will feature weapon modules, electronic modules, mast modules, and a modular combat system with standard interfaces.

# Foreign Navies (II)

- French Navy
  - The Frégate Européenne Multi-Mission (FREMM) was a joint venture between the Italian and French navies, are highly modular frigate designs allowing a choice of equipment with regards to weapons and combat systems.
  - The Aquitaine class FREMM frigates designed for the French Navy features a high-speed data network with an open architecture that will enable future weapon systems to be integrated into the frigates with external communication equipment compliant with NATO standards.
- Royal Australian Navy
  - The Anzac class frigates are long-range escorts with roles that include air defense, anti-submarine warfare and surveillance.
  - The design of the Anzac is based on the Blohm + Voss MEKO 200 modular design which utilizes a basic hull and construction concept to provide flexibility in the choice of command and control, weapons, equipment and sensors.
  - SEA5000 Program is the new Future Frigate initiative launched by the Royal Australian Navy (RAN). And RAN is moving forward with a new class of frigates that will need to incorporate a flexible and adaptable design to meet the growing demand for an efficient, sophisticated, and technologically advanced warship

# Methodology

- Identify which FASO options have a positive ROI (i.e., in which options the benefits outweigh the costs).
- Model Uncertainty and Risks (i.e., Monte Carlo Risk Simulations applied to simulate hundreds of thousands of possible scenarios and outcomes to model the volatility and ever-changing global threat matrix).
- Frame and Value the Ship Design Options (framed in context and valued using cost savings [cost savings due to rapid upgrades at lower costs], costs to obtain these options [costs to design and implement these FASO/MAS options], and potential military benefits).
- Optimize the Portfolio of Options (i.e., a set of FASO design options with different costs, benefits, capabilities, uncertainties; identify which options should be chosen given constraints in budget, schedule, and requirements).

# What Are Real Options?

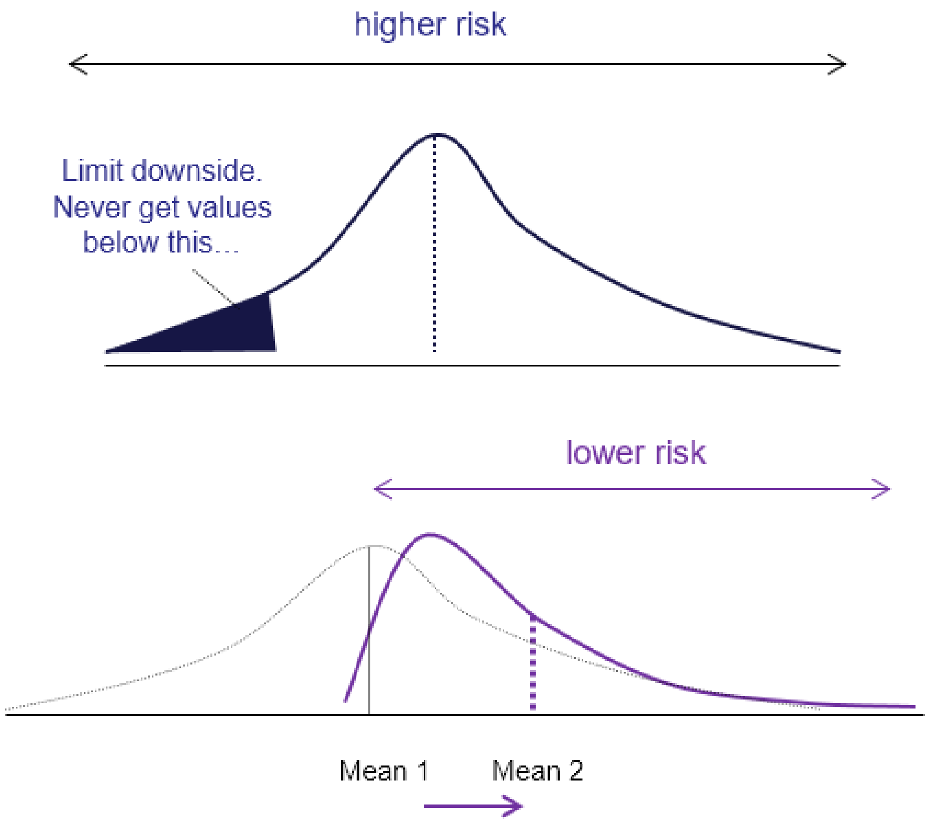

## Traditional decision analysis approach:

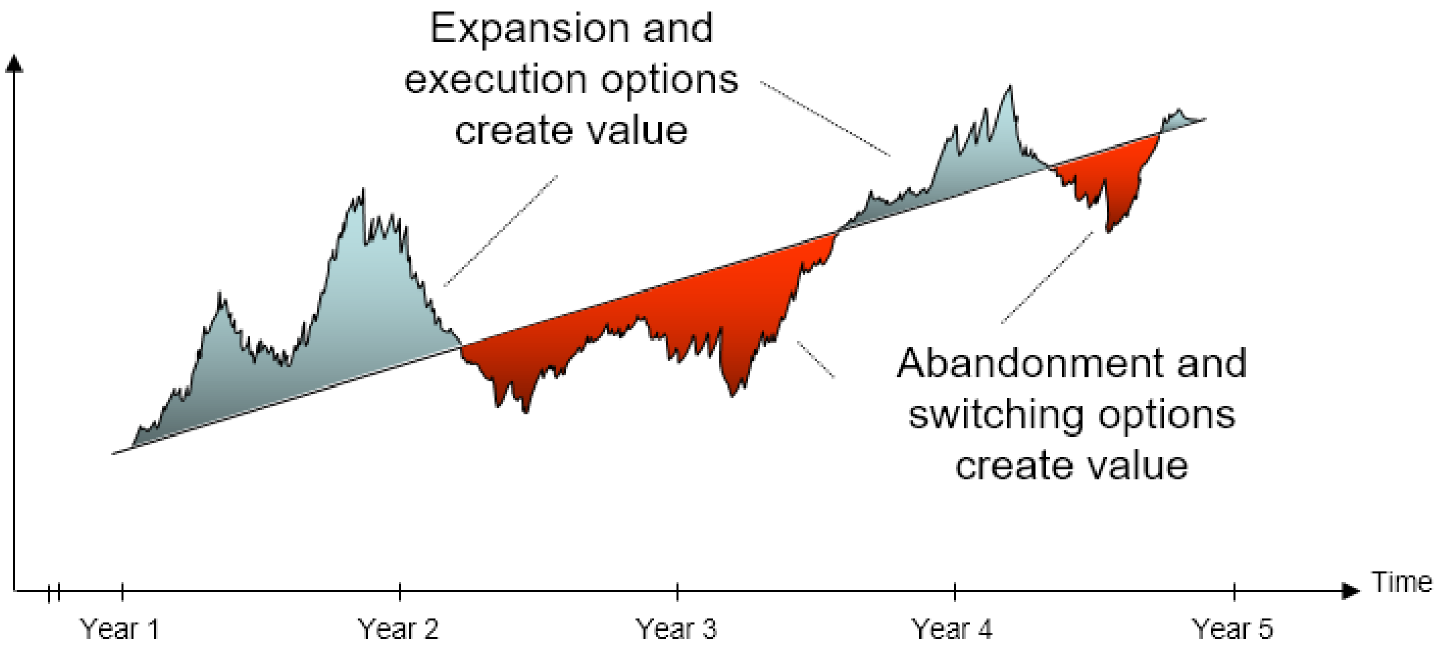
- Provides single decision pathway
- Allows only one future outcome
- Locks in a single risk rate
- All assumptions determined at the outset

## Real Options approach:

- Allows multiple decision pathways
- Maximizes financial flexibility
- Recognizes managerial decision making
- Incorporates new assumptions over time
- Allows variable risk

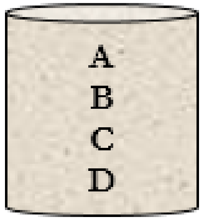
# A Visual Representation of ROV



## 1 QUALITATIVE MANAGEMENT SCREENING

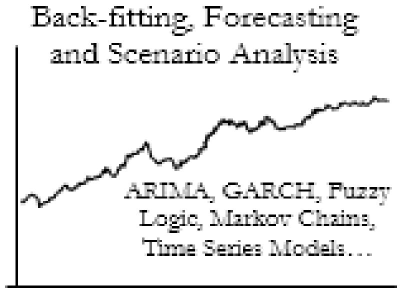
RISK IDENTIFICATION




Start with a list of projects or strategies to be evaluated that have already been through qualitative screening…

## 2 FORECAST PREDICTION MODELING

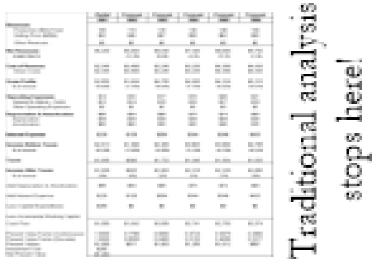
RISK PREDICTION




…with the assistance of forecasting algorithms, future outcomes can be predicted...

## 3 BASE CASE STATIC MODELS

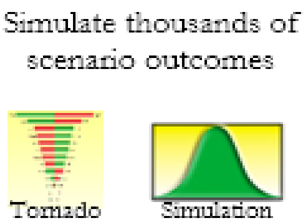
RISK MODELING




…create traditional static base case financial or economic models for each project…

## 4 DYNAMIC MONTE CARLO RISK SIMULATION

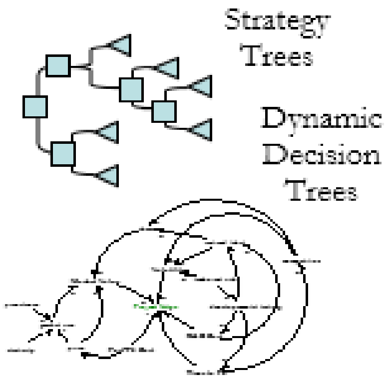
RISK ANALYSIS




…Tornado analysis identifies critical success factors, then dynamic sensitivities and Monte Carlo risk simulations are run…

## 5 REAL OPTIONS PROBLEM FRAMING

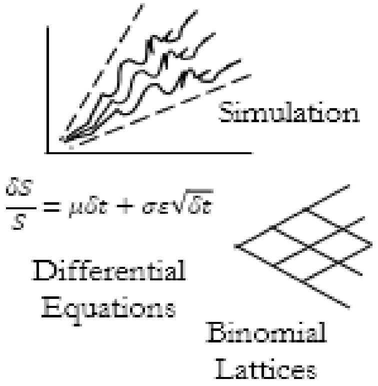
RISK MITIGATION




…strategic real options are framed to hedge and mitigate downside risks and take advantage of upside potential…

## 6 REAL OPTIONS VALUATION AND MODELING

RISK HEDGING

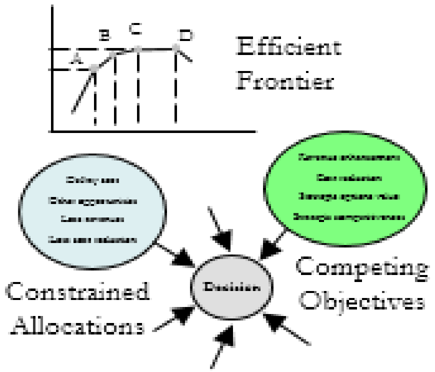



$$\frac{\delta S}{S} = \mu \delta t + \sigma \varepsilon \sqrt{\delta t}$$

…the real options are valued using binomial lattices and closed-form partial-differential models with simulation…

## 7 PORTFOLIO AND RESOURCE OPTIMIZATION

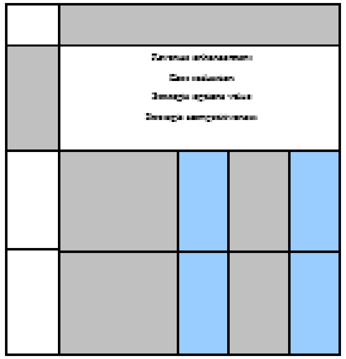
RISK DIVERSIFICATION




…stochastic optimization on multiple projects for efficient asset allocation subject to resource constraints…

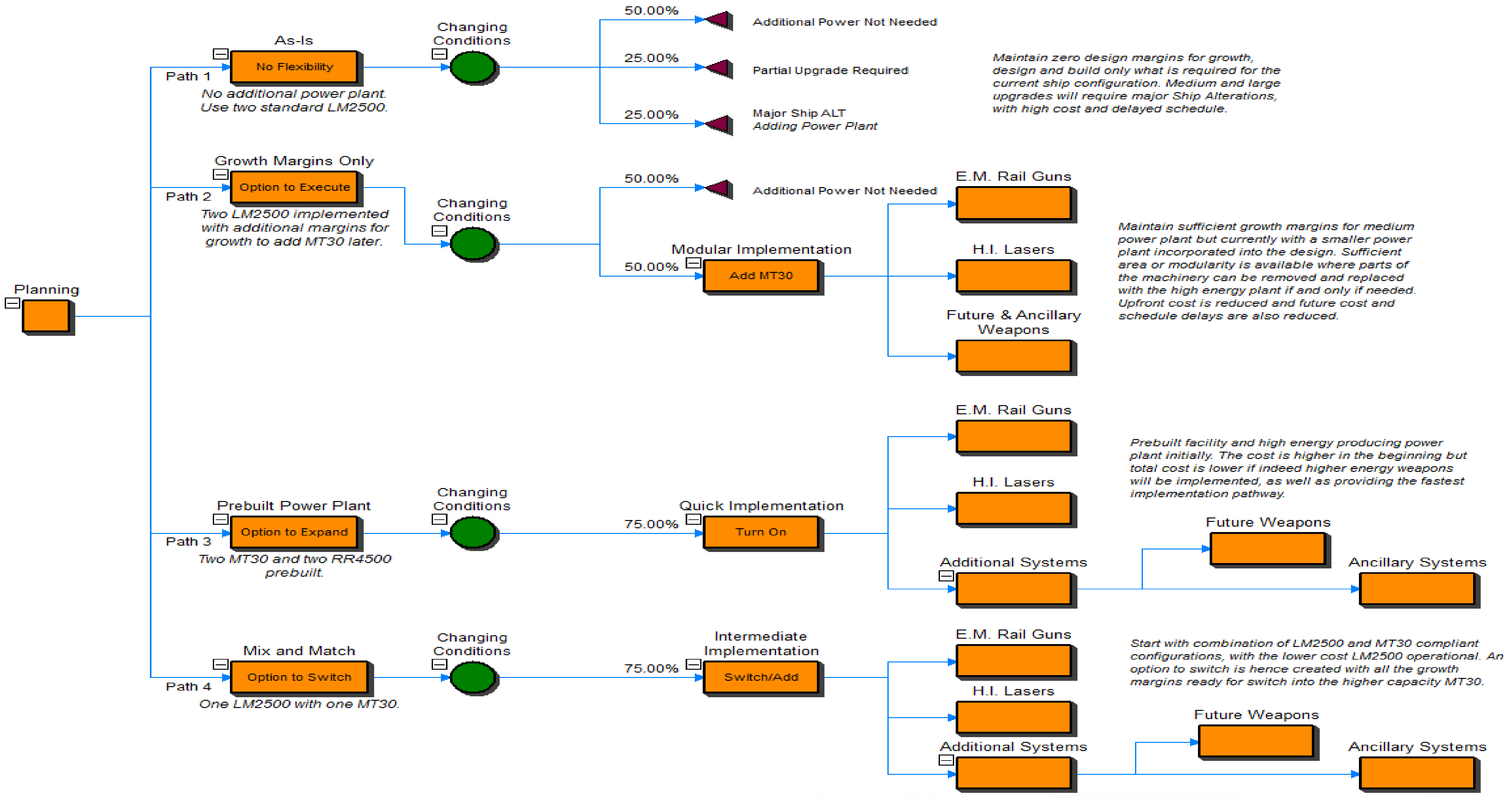
## 8 REPORTS, PRESENTATION, AND UPDATES

RISK MANAGEMENT



…create reports, make decisions, and update analysis iteratively when uncertainty is resolved over time…

# Analysis of Alternatives and Decision Analysis



# Financial and Economic Analysis

Note: These are only Notional Values used to illustrate the methodology

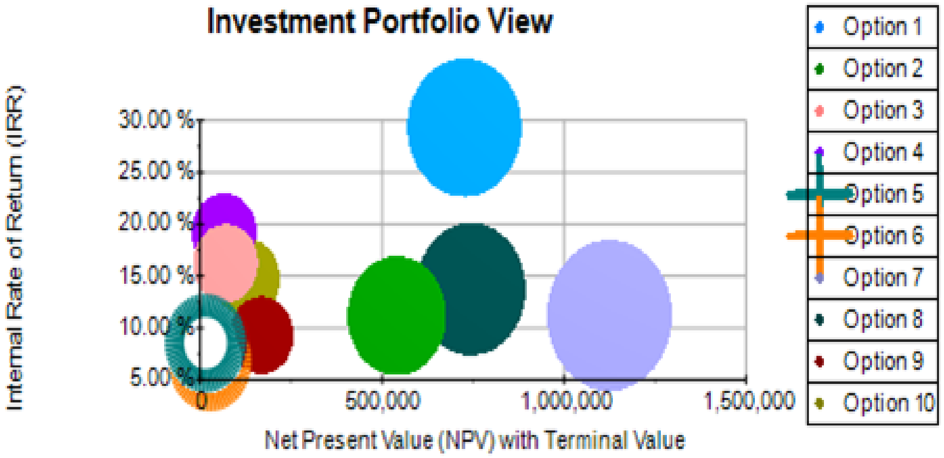
| Year | 2016 | 2017 | 2018 | 2019 | 2020 | 2021 | 2022 | 2023 | ... | 2041 | 2042 | 2043 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ***Revenues*** | $1,742.51 | $11,737.14 | $225,850.13 | $225,850.13 | $225,850.13 | $225,850.13 | $225,850.13 | $225,850.13 | ... | $235,437.44 | $235,437.44 | $235,437.44 |
| Cost Savings (Future Upgrades and Insertion) | $1,132.63 | $7,629.14 | $146,802.58 | $146,802.58 | $146,802.58 | $146,802.58 | $146,802.58 | $146,802.58 | ... | $153,034.34 | $153,034.34 | $153,034.34 |
| Cost Mitigated (Alternative Equipment) | $522.75 | $3,521.14 | $67,755.04 | $67,755.04 | $67,755.04 | $67,755.04 | $67,755.04 | $67,755.04 | ... | $70,631.23 | $70,631.23 | $70,631.23 |
| Cost Deferred (Maintenance and Operations) | $87.13 | $586.86 | $11,292.51 | $11,292.51 | $11,292.51 | $11,292.51 | $11,292.51 | $11,292.51 | ... | $11,771.87 | $11,771.87 | $11,771.87 |
| ***Direct Costs*** | $1,141.09 | $1,141.09 | $26,392.75 | $26,392.75 | $26,392.75 | $26,456.81 | $27,888.82 | $27,888.82 | ... | $32,021.41 | $32,021.41 | $32,021.41 |
| Direct Expenses | $1,110.26 | $1,110.26 | $24,896.68 | $24,896.68 | $24,896.68 | $24,896.68 | $24,896.68 | $24,896.68 | ... | $25,961.75 | $25,961.75 | $25,961.75 |
| Operational Costs | $18.50 | $18.50 | $414.95 | $414.95 | $414.95 | $453.38 | $829.89 | $829.89 | ... | $1,730.79 | $1,730.79 | $1,730.79 |
| Maintenance | $12.33 | $12.33 | $25.62 | $25.62 | $25.62 | $51.25 | $51.25 | $51.25 | ... | $106.87 | $106.87 | $106.87 |
| Direct Expenses | $0.00 | $0.00 | $1,055.50 | $1,055.50 | $1,055.50 | $1,055.50 | $2,111.00 | $2,111.00 | ... | $4,222.00 | $4,222.00 | $4,222.00 |
| ***Gross Profit (Operating Income)*** | $601.42 | $10,596.05 | $199,457.38 | $199,457.38 | $199,457.38 | $199,393.32 | $197,961.31 | $197,961.31 | ... | $203,416.03 | $203,416.03 | $203,416.03 |
| ***Indirect Expenses (General & Administrative)*** | $799.42 | $3,073.28 | $9,212.61 | $9,212.61 | $9,212.61 | $9,212.61 | $9,212.61 | $10,877.49 | ... | $12,259.92 | $9,465.41 | $9,465.41 |
| Training and Administrative | $0.00 | $31.00 | $703.00 | $703.00 | $703.00 | $703.00 | $703.00 | $703.00 | ... | $733.00 | $733.00 | $733.00 |
| Contracts and Bidding | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | ... | $0.00 | $0.00 | $0.00 |
| Operations | $0.00 | $0.00 | $1,248.07 | $1,248.07 | $1,248.07 | $1,248.07 | $1,248.07 | $1,248.07 | ... | $1,248.07 | $1,248.07 | $1,248.07 |
| Maintenance | $799.42 | $2,997.82 | $4,758.48 | $4,758.48 | $4,758.48 | $4,758.48 | $4,758.48 | $6,423.36 | ... | $7,733.14 | $4,938.63 | $4,938.63 |
| Parts and Service | $0.00 | $0.00 | $1,506.00 | $1,506.00 | $1,506.00 | $1,506.00 | $1,506.00 | $1,506.00 | ... | $1,506.00 | $1,506.00 | $1,506.00 |
| Miscellaneous | $0.00 | $44.46 | $997.06 | $997.06 | $997.06 | $997.06 | $997.06 | $997.06 | ... | $1,039.71 | $1,039.71 | $1,039.71 |
| ***EBITDA: Earnings Before Interest, Taxes, Depreciation, and Amortization*** | ($198.00) | $7,522.77 | $190,244.77 | $190,244.77 | $190,244.77 | $190,180.71 | $188,748.70 | $187,083.82 | ... | $191,156.11 | $193,950.62 | $193,950.62 |
| Depreciation | $0.00 | $9,874.00 | $39,827.00 | $39,074.00 | $38,161.00 | $37,206.00 | $36,172.00 | $35,223.00 | ... | $24,502.00 | $23,977.00 | $23,444.00 |
| Amortization | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | ... | $0.00 | $0.00 | $0.00 |
| ***EBIT: Earnings Before Interest and Taxes*** | ($198.00) | ($2,351.23) | $150,417.77 | $151,170.77 | $152,083.77 | $152,974.71 | $152,576.70 | $151,860.82 | ... | $166,654.11 | $169,973.62 | $170,506.62 |
| Interest | $0.00 | $6,779.32 | $25,892.66 | $22,767.15 | $19,224.35 | $15,842.53 | $13,062.00 | $12,303.79 | ... | $653.99 | $666.90 | $667.48 |
| ***EBT: Earnings Before Taxes*** | ($198.00) | ($9,130.55) | $124,525.11 | $128,403.62 | $132,859.42 | $137,132.18 | $139,514.70 | $139,557.03 | ... | $166,000.12 | $169,306.72 | $169,839.14 |
| ***Corporate Taxes*** | ($56.43) | ($2,602.21) | $35,489.66 | $36,595.03 | $37,864.93 | $39,082.67 | $39,761.69 | $39,773.75 | ... | $47,310.03 | $48,252.42 | $48,404.15 |
| ***NET INCOME*** | ($141.57) | ($6,528.34) | $89,035.45 | $91,808.59 | $94,994.49 | $98,049.51 | $99,753.01 | $99,783.28 | ... | $118,690.09 | $121,054.30 | $121,434.99 |
| ***Total Noncash Expense Items*** | $0.00 | $9,874.00 | $39,827.00 | $39,074.00 | $38,161.00 | $37,206.00 | $36,172.00 | $35,223.00 | ... | $24,502.00 | $23,977.00 | $23,444.00 |
| Change in Net Working Capital | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | ... | $0.00 | $0.00 | $0.00 |
| Capital Expenditures | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | ... | $0.00 | $0.00 | $0.00 |
| Other Noncash Expenses | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | ... | $0.00 | $0.00 | $0.00 |
| Total Gross Invested Operating Capital | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | ... | $0.00 | $0.00 | $0.00 |
| CAPITAL INVESTMENTS | $250,000.00 | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | $0.00 | ... | $0.00 | $0.00 | $0.00 |
| ***NET OPERATING PROFIT AFTER TAXES (NOPAT)*** | ($141.57) | ($1,681.13) | $107,548.71 | $108,087.10 | $108,739.90 | $109,376.92 | $109,092.34 | $108,580.49 | ... | $119,157.69 | $121,531.14 | $121,912.23 |
| ***NET CASH FLOW (NCF)*** | ($141.57) | $3,345.66 | $128,862.45 | $130,882.59 | $133,155.49 | $135,255.51 | $135,925.01 | $135,006.28 | ... | $143,192.09 | $145,031.30 | $144,878.99 |
| ***OPERATING CASH FLOW (OCF)*** | ($141.57) | $8,192.87 | $147,375.71 | $147,161.10 | $146,900.90 | $146,582.92 | $145,264.34 | $143,803.49 | ... | $143,659.69 | $145,508.14 | $145,356.23 |
| ***FREE CASH FLOW (FCF)*** | ($141.57) | $8,192.87 | $147,375.71 | $147,161.10 | $146,900.90 | $146,582.92 | $145,264.34 | $143,803.49 | ... | $143,659.69 | $145,508.14 | $145,356.23 |

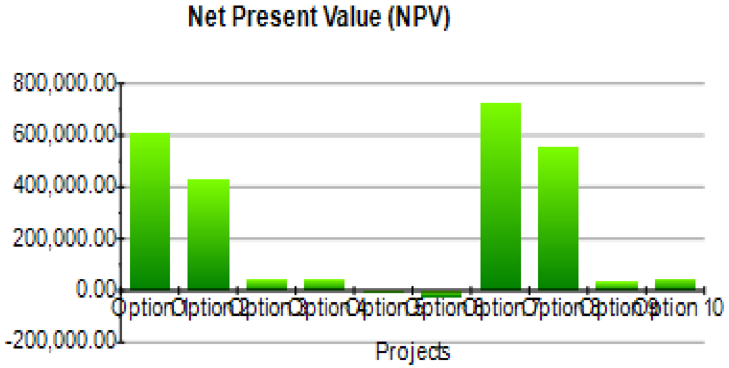
NAVAL POSTGRADUATE SCHOOL

# Analysis of Alternatives

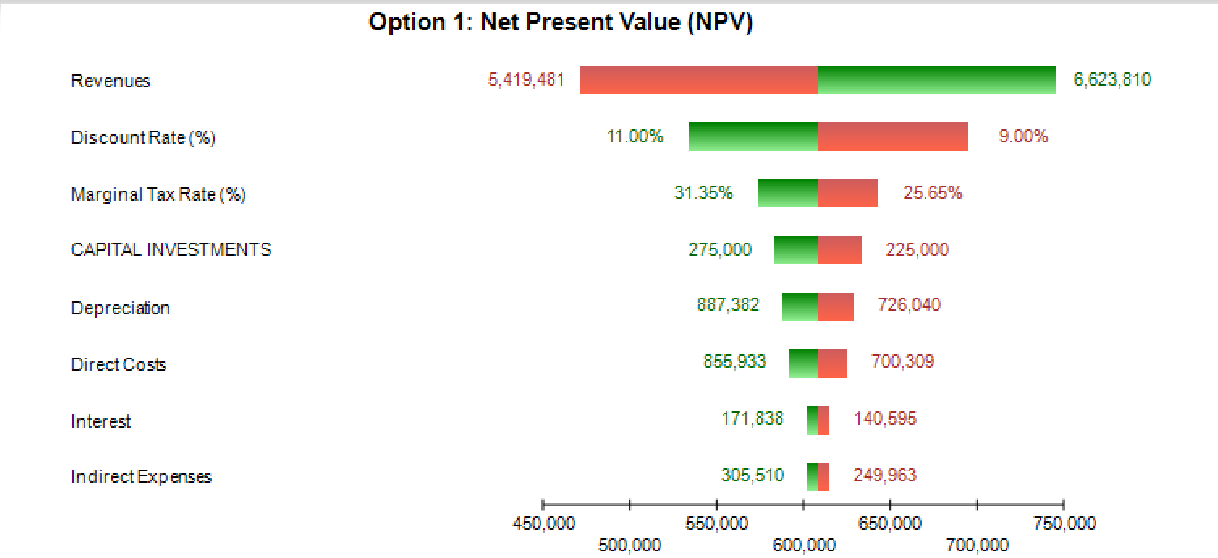
Analysis of Alternatives and Strategic Implementation Pathways provide
a side by side comparison of value and ROI justification

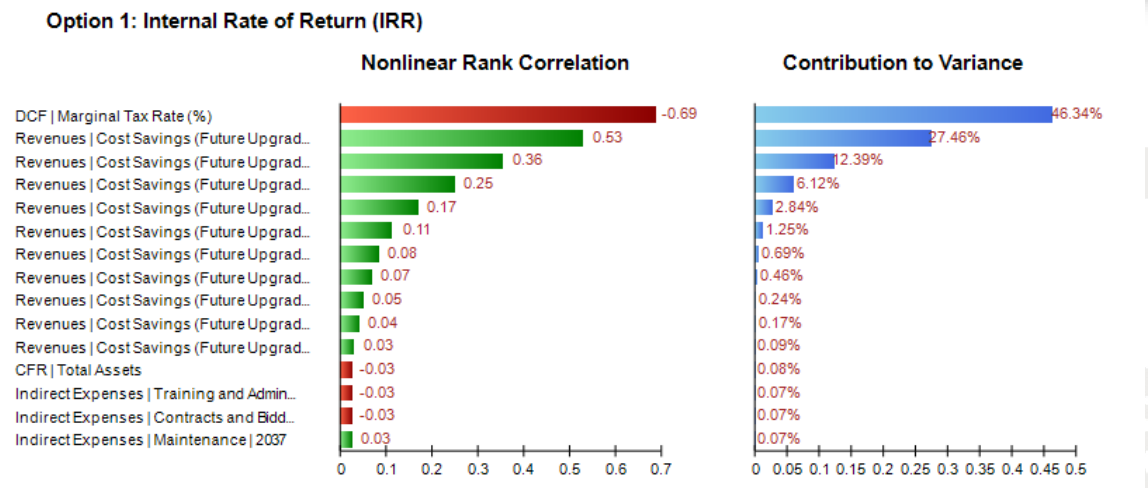
| Economic Results | Option 1 | Option 2 | Option 3 | Option 4 | Option 5 | Option 6 | Option 7 | Option 8 | Option 9 | Option 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Net Present Value (NPV) | $608,388.34 | $427,132.76 | $40,765.22 | $41,613.74 | ($10,610.44) | ($23,774.85) | $728,339.38 | $554,258.99 | $31,837.41 | $46,377.25 |
| Net Present Value (NPV) with Terminal Value | 72648877.00% | 54046710.00% | 7033594.00% | 6615455.00% | 1525722.00% | 2718633.00% | 112457959.00% | 74402419.00% | 16876439.00% | 12973950.00% |
| Internal Rate of Return (IRR) | 29.31% | 11.17% | 16.21% | 19.21% | 8.55% | 6.76% | 11.20% | 13.74% | 9.29% | 14.77% |
| Modified Internal Rate of Return (MIRR) | 0.15 | 0.10 | 0.12 | 0.13 | 0.09 | 0.08 | 0.10 | 0.11 | 0.07 | 0.10 |
| Profitability Index (PI) | 343.00% | 114.00% | 137.00% | 154.00% | 90.00% | 86.00% | 109.00% | 129.00% | 134.00% | 176.00% |
| Return on Investment (ROI) | 2.43 | 0.14 | 0.37 | 0.54 | -0.10 | -0.14 | 0.09 | 0.29 | 0.34 | 0.76 |
| Payback Period (PP) | 3.80 | 10.82 | 6.38 | 5.45 | 10.32 | 9.78 | 9.98 | 7.87 | 9.19 | 7.30 |
| Discounted Payback Period (DPP) | $4.80 | $22.81 | $9.80 | $7.84 | | | $22.35 | $13.79 | $12.18 | $9.21 |
| Show on Charts | | | | | | | | | | |

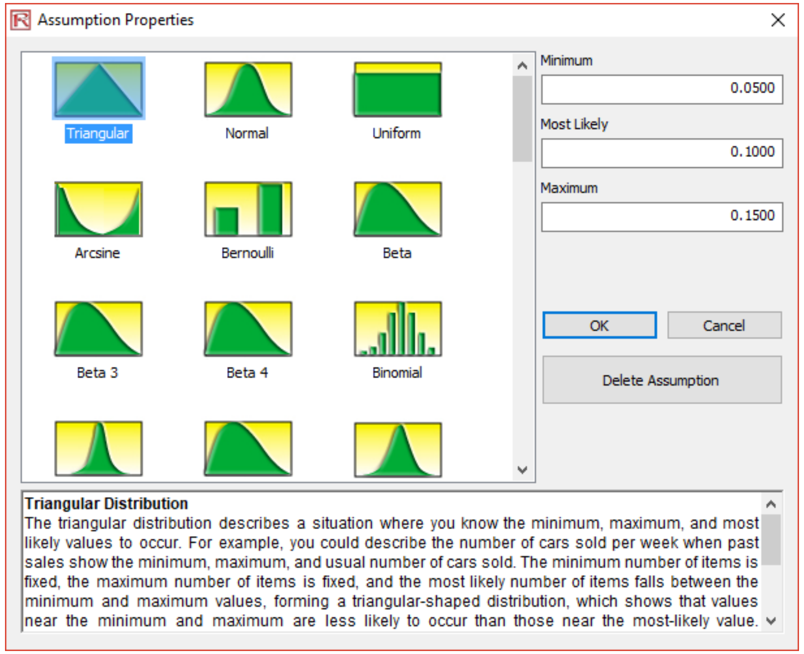


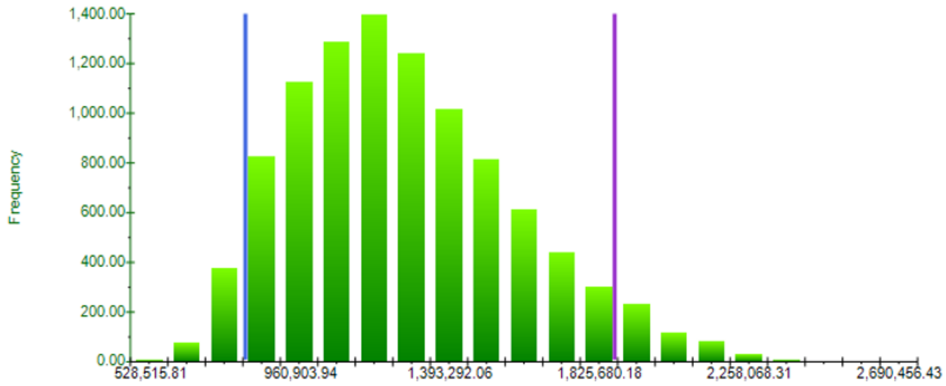
# Scenario, Sensitivity, Risk Analytics



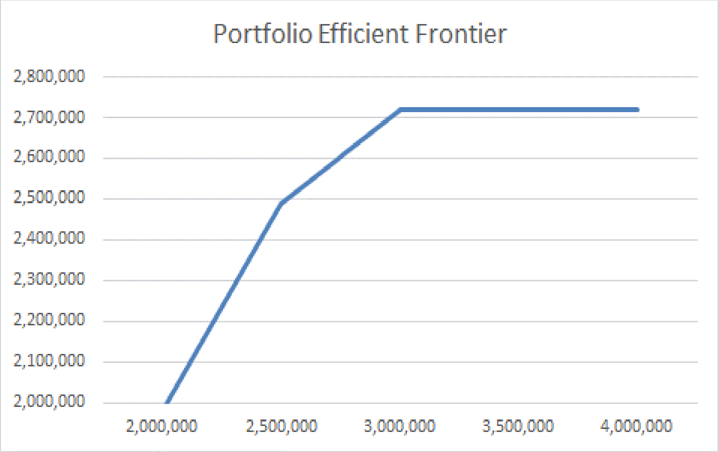

| Statistics/Percentiles | Value |
|---|---|
| Trials | 10,000.00 |
| Mean | 1,264,569.20 |
| Median | 1,223,025.65 |
| Stdev | 323,440.89 |
| CV | 0.26 |
| Skew | 0.59 |
| Kurtosis | 0.07 |
| Minimum | 528,515.81 |
| Maximum | 2,690,456.43 |
| Range | 2,161,940.62 |
| 0.00% | 528,515.81 |
| 5.00% | 806,158.76 |
| 10.00% | 873,477.43 |
| 20% | $975,191 |
| **30%** | **1,066,998.73** |
| 40.00% | 1,146,536 |
| 50.00% | 1,223,025.65 |
| 60.00% | 1,310,131.64 |
| 70.00% | 1,408,675.46 |
| 80.00% | 152992680.50% |
| 90.00% | 1,711,752.31 |
| 95.00% | 1,871,099.08 |
| 100.00% | 2,690,456.43 |



# Portfolio Optimization

Efficient Frontier: Obtains the optimal portfolio combination of all flexible options within various levels of budgetary and other constraints

| Objective Function | 6.1286 | 6.7465 | 6.9478 | 6.9478 | 6.9478 |
|---|---|---|---|---|---|
| Frontier Variable | 2000000.0000 | 2500000.0000 | 3000000.0000 | 3500000.0000 | 4000000.0000 |
| Optimized Constraint | 1978818.0000 | 2487042.0000 | 2718646.0000 | 2718646.0000 | 2718646.0000 |
| Option1 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Option2 | 0.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Option3 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Option4 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Option5 | 1.00 | 0.00 | 1.00 | 1.00 | 1.00 |
| Option6 | 0.00 | 0.00 | 1.00 | 1.00 | 1.00 |
| Option7 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| Option8 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Option9 | 0.00 | 0.00 | 1.00 | 1.00 | 1.00 |
| Option10 | 0.00 | 1.00 | 1.00 | 1.00 | 1.00 |




Multi-Objective Portfolio Matrix: Optimizes the portfolio of options from different stakeholders' points of view (e.g., OPNAV Requirements, Lethality, Future Weapons Upgradability, SME Military Value, Financial Metrics, and any other noneconomic qualitative variables)

| Model | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Count |
|---|---|---|---|---|---|---|
| Objective | 1,408,735.73 | 51.16 | 53.56 | 48.10 | 53.56 | |
| Budget Constraint | 3,800,000 | 4,000,000 | 4,000,000 | 3,750,000 | 4,000,000 | |
| Program Constraint | 6 | 7 | 7 | 6 | 7 | |
| Option 1 | 1.00 | 1.00 | 1.00 | 0.00 | 1.00 | 4 |
| Option 2 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0 |
| Option 3 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 5 |
| Option 4 | 0.00 | 1.00 | 1.00 | 0.00 | 1.00 | 3 |
| Option 5 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 5 |
| Option 6 | 0.00 | 1.00 | 1.00 | 1.00 | 1.00 | 4 |
| Option 7 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1 |
| Option 8 | 0.00 | 1.00 | 1.00 | 1.00 | 1.00 | 4 |
| Option 9 | 1.00 | 0.00 | 0.00 | 1.00 | 0.00 | 2 |
| Option 10 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 5 |

# Yes, there's some math involved…

For instance, we first start by solving for the critical value of $I$, an iterative component in the model using:

$$X_2 = Ie^{-q(T_2-t_1)}\Phi\left(\frac{\ln(I/X_1)+(r-q+\sigma^2/2)(T_2-t_1)}{\sigma\sqrt{(T_2-t_1)}}\right)$$

$$-X_1e^{-r(T_2-t_1)}\Phi\left(\frac{\ln(I/X_1)+(r-q-\sigma^2/2)(T_2-t_1)}{\sigma\sqrt{(T_2-t_1)}}\right)$$

Then, solve recursively for the value $I$ above and input it into the

$$Compound\ Option = Se^{-qT_2}\Omega\left[\begin{array}{l}\dfrac{\ln(S/X_1)+(r-q+\sigma^2/2)T_2}{\sigma\sqrt{T_2}};\\ \dfrac{\ln(S/I)+(r-q+\sigma^2/2)t_1}{\sigma\sqrt{t_1}};\sqrt{t_1/T_2}\end{array}\right]$$

$$-X_1e^{-rT_2}\Omega\left[\begin{array}{l}\dfrac{\ln(S/X_1)+(r-q+\sigma^2/2)T_2}{\sigma\sqrt{T_2}}-\sigma\sqrt{T_2};\\ \dfrac{\ln(S/I)+(r-q+\sigma^2/2)t_1}{\sigma\sqrt{t_1}}-\sigma\sqrt{t_1};\sqrt{t_1/T_2}\end{array}\right]$$

$$-X_2e^{-rt_1}\Phi\left[\frac{\ln(S/I)+(r-q+\sigma^2/2)t_1}{\sigma\sqrt{t_1}}-\sigma\sqrt{t_1}\right]$$

The preceding closed-form differential equation models are then verified using the risk-neutral market-replicating portfolio approach assuming a sequential compound option. In solving the market-replicating approach, we use the following functional forms (Mun, 2006):

- Hedge ratio *(h)*:

  $$h_{i-1} = \frac{C_{up} - C_{down}}{S_{up} - S_{down}}$$

- Debt load *(D)*:

  $$D_{i-1} = S_i(h_{i-1}) - C_i$$

- Call value *(C)* at node $i$:

  $$C_i = S_i(h_i) - D_i e^{-rf(\delta t)}$$

- Risk-adjusted probability *(q)*:

  $$q_i = \frac{S_{i-1} - S_{down}}{S_{up} - S_{down}}$$ obtained assuming

  $$S_{i-1} = q_i S_{up} + (1-q_i)S_{down}$$

- This means that

  $$S_{i-1} = q_i S_{up} + S_{down} - q_i S_{down}$$ and $$q_i[S_{up} - S_{down}] = S_{i-1} - S_{down},$$

  so we get $$q_i = \frac{S_{i-1} - S_{down}}{S_{up} - S_{down}}$$