NPS-AM-07-034

EXCERPT FROM THE

PROCEEDINGS

OF THE

FOURTH ANNUAL ACQUISITION RESEARCH SYMPOSIUM THURSDAY SESSIONS

Managing Uncertainty and Risk in Public-sector Investments

by

Richard Suter, Information Technology Systems, Inc., RS Consulting

4th Annual Acquisition Research Symposium of the Naval Postgraduate School:

Acquisition Research: Creating Synergy for Informed Change

May 16-17, 2007

Prepared for: Naval Postgraduate School, Monterey, California 93943

Acquisition Research program
GRADUATE SCHOOL OF BUSINESS & PUBLIC POLICY
NAVAL POSTGRADUATE SCHOOL

The research presented at the symposium was supported by the Acquisition Chair of the Graduate School of Business & Public Policy at the Naval Postgraduate School.

To request Defense Acquisition Research or to become a research sponsor, please contact:

NPS Acquisition Research Program
Attn: James B. Greene, RADM, USN, (Ret)
Acquisition Chair
Graduate School of Business and Public Policy
Naval Postgraduate School
555 Dyer Road, Room 332
Monterey, CA 93943-5103
Tel: (831) 656-2092
Fax: (831) 656-2253
E-mail: jbgreene@nps.edu

Copies of the Acquisition Sponsored Research Reports may be printed from our website www.acquisitionresearch.org

Conference Website:
www.researchsymposium.org

Proceedings of the Annual Acquisition Research Program

The following article is taken as an excerpt from the proceedings of the annual Acquisition Research Program. This annual event showcases the research projects funded through the Acquisition Research Program at the Graduate School of Business and Public Policy at the Naval Postgraduate School. Featuring keynote speakers, plenary panels, multiple panel sessions, a student research poster show and social events, the Annual Acquisition Research Symposium offers a candid environment where high-ranking Department of Defense (DoD) officials, industry officials, accomplished faculty and military students are encouraged to collaborate on finding applicable solutions to the challenges facing acquisition policies and processes within the DoD today. By jointly and publicly questioning the norms of industry and academia, the resulting research benefits from myriad perspectives and collaborations which can identify better solutions and practices in acquisition, contract, financial, logistics and program management.

For further information regarding the Acquisition Research Program, electronic copies of additional research, or to learn more about becoming a sponsor, please visit our program website at:

www.acquistionresearch.org

For further information on or to register for the next Acquisition Research Symposium during the third week of May, please visit our conference website at:

www.researchsymposium.org

THIS PAGE INTENTIONALLY LEFT BLANK

Managing Uncertainty and Risk in Public-sector Investments

Presenter: Richard Suter

Richard Suter
Information Technology Systems, Inc., RS Consulting
Phone: (240) 350-7586
E-mail: rsx@ieee.org

Key Terms: Black-Scholes, Capital Asset Pricing, Efficient Frontier, Entropy, Equity Markets, Information Theory, Ito's Lemma, Options Pricing, Portfolio Management, Risk, Uncertainty

Introduction

The Department of Defense (DoD) has an annual budget approaching a half trillion dollars. A significant portion of that budget is either directly or indirectly affected by Information Technology (IT) Infrastructure modernization initiatives. Nationally, investment in IT infrastructure modernization is about $250 Billion a year, spanning approximately 175,000 projects. Unfortunately, various studies indicate that only 28% are completed on time and on budget—with the number dropping to around 9% for larger companies in 1994, including government programs. By 1998, these numbers had improved, with success rates for larger companies, for example, up to 24%. But, only small organizations have managed to implement more than half of their applications into one integrated system (Smith, 2000; Johnson, 1999, December; Keller, 2006, May 29).

The factors driving these numbers are myriad—and include the sheer scope and complexity of infrastructure modernization programs, unstable/undefined requirements, unstable funding, moving target objectives, and evolving threats. As a result, the costs of integration complexity increase exponentially, but yet are almost invariably under estimated. These challenges notwithstanding, one Gartner study asserts that IT asset productivity will drive market capitalization (Gartner, 2002, July).

Given the scope, importance and complexity of these projects, reliable, cost-effective, early warning indicators of problems are essential. Yet, classical investment theory provides little guidance for dealing with public-sector investment. The result is a general absence of computationally efficient, predictive models applicable to the analysis of those investments.

However, there is progress on several fronts. At the micro level, Earned Value Management is gaining acceptance. But, more comprehensive, flexible methods can be developed by viewing a Firm as an engineered artifact whose responses to a range of inputs can be characterized in terms of duration, mass, time, stability, and location. (By way of terminology, a "Firm" refers to both public and private organizations with investment responsibilities.)

How well a Firm executes an investment depends largely on how well it acquires and uses information. The efficiency of that use provides a basis for pricing the value of a Firm, independent of valuations derived from equity markets.

For the private sector, Capital Asset Pricing Models (CAPM) models provide valuation tools, of which the Black-Scholes equation is the most well known and successful example. These models are based on the observation that equity markets have properties that can be analyzed using models based on the "Law of Large Numbers."

Unfortunately, the conditions enabling the quantitative analysis of private-sector investments do not apply in the public sector because there is no market for public-sector goods. Nonetheless, all Firms, regardless of whether they are in the private or public sector, must respond to a range of random (external and internal) perturbations. The responses can be accurately modeled using mathematical processes based on the "Law of Large Numbers." In particular, the operation of a Firm can be modeled as a stochastic feedback system using algorithms from Information Theory and System Control Theory.

IT Infrastructure Investments—in the Public Sector

Private equity markets provide a profit incentive to resolve uncertainty, which has as one of its effects the rapid aggregation of information from large numbers of participants. In general, the larger and more diverse the number of sources, the more accurate is a Firm's valuation. (With the Web and the Internet dramatically lowering the cost of information, similar aggregation effects are occurring in the news media, entertainment, and politics, as evidenced by the impact of Napster, U-Tube, Google, etc.)

Unfortunately, the Public Sector lacks an incentive mechanism and usually consists of a few service/product providers, and generally only one customer—the government. (Markets with few sellers and buyers are more suited to analysis by Game Theory.)

Despite these differences, private and public sectors share a range of common concerns, especially for large scale IT modernization projects. Among these is a compelling need to answer questions such as:

- What level of uncertainty surrounds cost/schedule estimates, especially at the onset of a major investment in IT infra-structure?
- Under what conditions will risk/uncertainty decline or rise? And at what rate?
- How can risk/uncertainty concerning cost, schedule, and scope be identified in a timely manner, quantified, and mitigated?
- How can a Firm best respond to disruptions to supply, budget and schedule or to the introduction of new technologies by competitors?
- At what point in a project-development cycle will estimates of cost, schedule, and scope become both stable and credible?
- How can confidence levels in cost and schedule estimates be measured?
- Are project requirements under active management sufficiently stable to ensure project completion on time and within budget?
- What (quantitative) models can be used to determine the maximum effective rate of investment for public-sector projects?
- How can an optimal investment portfolio be constructed?
- And, how is such a portfolio optimality measured?

- How do the performance measures such as requirements stability, work package completion, rework rates and resolution rates for major and minor issues correlate with each other?
- Is there a level of disruption or tipping point to the schedule, cost, and resource allocations beyond which a project cannot recover?

Developing a quantitative framework capable of answering these questions is the objective of the paper. The strategy is to employ key parameters governing the efficiency of a Firm's operation in terms of observable, easy-to-compute variables in mathematical models of the underlying process dynamics.

Portfolio Investment Management—An Overview

Large-scale Defense infrastructure modernization programs such as Global Combat Support have complex inter-dependencies and long-time horizons that render fully informed investment decisions difficult to achieve before substantial, and unrecoverable, resources are committed. However complex these decisions, they, nonetheless, can be decomposed along three basic dimensions:

- Uncertainty
- Timing
- Irreversibility

These primary parameters define the value of investment options available to a Firm, regardless of whether it is in the public or private sector. Unfortunately, algorithms capable of modeling the effects of these variables are relatively few, especially for the uncertainty and irreversibility of investment decisions (Dixit & Pyndik, 1994, p. 211). For large-scale Information Technology (IT) modernization programs, there are at least three sources of uncertainty—and, thus, risk

- The technical complexity
- The programmatic complexity of integrating software intensive systems
- The absence of accurate cost information at the onset of major systems/ software programs

Software-intensive systems are particularly sensitive to the systematic under-estimation of risk, primarily because the level of complexity is hard to manage, let alone comprehend. Investment in software-intensive systems tends to be irreversible because it is spent on the labor required to develop the intellectual capital embedded in software.

The outcome of software development is almost invariably unique, a one-of-a-kind artifact—despite the numerous efforts to develop reusable software. Unlike physical assets, the salvage value of software is zero because no benefit is realized until the system is deployed; and that labor, once invested, is unrecoverable. One result is an (implicit) incentive to continue projects that have little chance of success, despite significant cost overruns, schedule delays.

Indeed, an analysis of several hundred NASA projects indicates that accurate estimates at project onset are virtually impossible to achieve, which raises concern for the validity of initial Planned Value estimates since they are the basis of Earned Value calculations (Suter, 2005, p. 261). Thus, measures of uncertainty for cost/schedule estimates and the rate at which that uncertainty declines are a key concern—because, they govern whether and to what extent confidence can be placed in cost and schedule estimates. The key to overcoming initial estimate uncertainty is the capability to harness and to apply information as it becomes available, thus, enabling a Firm to capture the time value of that information.

Indeed, where IT infrastructure modernization projects are supported by a strong quality-assurance, systems-engineering culture (e.g., have institutionalized best-practice regimes such as the CMMI, 6-Sigma, Agile Methods) are likely to quickly reduce estimate errors incurred at project start-up. Firms without that culture tend to have limited information efficiency. (Drawing an analogy to thermo-dynamic systems, such Firms constitute highly dissipative systems in that they exhibit a high degree of entropy, which takes the form of information disorganization).

Unfortunately, traditional methods of discounting investment risk such as Net Present Value (NPV) do not account for irreversibility and uncertainty. In part, this is due to the fact that NPV computes the value of a portfolio of investments as the maximized mean of discounted cash flows on the assumption that the risk to underlying investment options can be replicated by assets in a financial market.

NPV also implicitly assumes that the value of the underlying asset is known and accurate at the time the investment decision is made.

These assumptions seldom apply for large-scale infra-modernization programs, in either the public or the private sector. In addition, NPV investment is undertaken when the value of a unit of capital is at least as large as its purchase and installation costs. But, this can be error prone since opportunity costs are highly sensitive to the uncertainty surrounding the future value of the project due to factors such as the riskiness of future cash flows. These considerations also extend to econometric models, which exclude irreversibility, the incorporation of which transforms investment models into non-linear equations (Dixit & Pindyck, 1994, p. 421). Nonetheless, irreversibility constitutes both a negative opportunity cost and a lost-option value that must be included in the cost of investment.

In addition, the competitive equilibrium of a market is virtually never stationary, even in the long run. Rather, it is a dynamic process in which prices can fluctuate widely, and, thus, contribute to uncertainty. Neither classical investment theory nor discounting methods such as NPV take these factors into account. Yet, for long-term, capital-intensive investments such as oil exploration and IT infrastructure modernization, price fluctuation constitutes significant risk—which must be factored into investment decisions (1994, p. 396).

These difficulties are due to an underlying limitation common to both classical investment theory and valuation methods such as NPV: their reliance on simple equilibrium relationships between rates of investment and risk, which has the practical effect of precluding the effect of uncertainty and irreversibility on investment. The factors that adversely impact NPV also impact the accuracy of Planned Value benchmarks that are the basis of Earned Value Calculations (Suter, 2006, p. 406). For these reasons, ”classical” methods have met with little success in providing accurate valuations of Public investments and qualified success for those in the Private Sector.

Capital Asset Pricing models such as Black-Scholes, however, sidestep these problems by transforming the analysis from a deterministic formulation to one based on probability. To handle uncertainty and risk driven by price fluctuations, it uses the *Ito Lemma* to compute valuations. Black-Scholes proceeds from the assumption that there is a true value for a stock that corresponds to its risk, and that value can be used to decide whether the market price for a stock is too high or too low. That is, an option's value equals the value of the information concerning that risk.

Black-Scholes models the price of a stock option as a Market-driven process defined by Eqn [3.1], the fundamental condition of equilibrium (Dixit & Pindyck, 1994, p. 115; Cover & Thomas, 1991, p. 28):

[3.1] $\mu = r + \theta * \sigma * p_{xm}$

Where:

μ = the risk adjusted return

θ = market price of risk

r = riskless rate of return

p_{xm} = coefficient of correlation between returns on the particular asset ("x" subscript) and the entire market portfolio of stocks (denoted by the "m" subscript).

σ = the proportional variance parameter

dz = the increment of the standard Wiener process

The computational efficiency of Black-Scholes model enables Floor Traders (and computer-programmed trading algorithms) to exploit small, short-term price fluctuations in real time and to use new pricing information to continually rebalance portfolios. Investors (as distinguished from Floor Traders by virtue of their longer-term time horizon) find Black-Scholes no less useful because of its ability to link risk to valuation over timeframes ranging into years. More recent refinements such as Levy processes introduce more realism by generalizing Brownian motion processes to include discrete state jumps. (The jumps can be local, global, simultaneous, independent or correlated.)

The significance of Black-Scholes is its computational efficiency for modeling price, interest, and discount rates—using a few readily observable parameters that provide reasonable approximations to the underlying physical processes using methods based on the Law of Large Numbers. In particular, it specifically eschews unobservable/hard-to-measure parameters such as "investor psychology." The Black-Scholes strategy is at variance with efforts to improve the analysis by increasing the dimensionality of the problem via the addition of more parameters (as is often done with econometric models, and on occasion with Balance Scorecard methods). The addition of extra parameters may well provide a more detailed picture of performance, but at additional cost and without necessarily improving the accuracy needed for decision-making (McShea, 2006, November, p. 31). As a result, these refinements have added relatively minimal value and can be relatively expensive to implement.

The key is to recognize that decision-making is not necessarily dependent on a detailed understanding of causality to be effective. Thus, Capital Asset Pricing models focus on those few variables with the most explanatory power. The objective is not to predict which firms are most likely to succeed, but only what they are “worth” as measured against various combinations of risk, uncertainty, interest rates, and competing investment opportunities. The task of this paper is to provide equivalent, computationally efficient methods for estimating valuations in public-sector investments, using the fewest parameters with the most predictive power.

There are, of course, methods other than market-based Capital Asset Pricing for determining asset valuations. But, these also suffer from various limitations. For example, Dynamic Programming (DP) could be applied to public-sector investments and is useful in solving multi-stage optimization problems, but only if a small number of possible choices exist at each stage. Indeed, a small increase in the number of possible choices leads to a combinatorial explosion, thus curtailing overall efficiency of DP.

Another method is the Discrete Binomial Model, which uses a risk-adjusted stochastic process for modeling the underlying asset. The strategy is to approximate uncorrelated investment dynamics using two basic ideas: a change of time scale and a change of the basis of the asset span to approximate uncorrelated geometric Brownian motion. Yet another variant, the Lattice Binomial model, has proved useful for valuing complex option problems when payoff depends on multiple state variables that follow correlated geometric Brownian processes. In this case, the strategy is to approximate a multi-dimensional geometric Brownian motion with a binomial lattice by choosing the size and the probabilities of the jumps so that the characteristic function of the discrete distribution converges to the characteristic function of the continuous distribution.

But, both methods require knowledge of the underlying probability distributions—a requirement that can be difficult to satisfy—and will converge to a solution only in the limit. For practical applications, the time required to acquire sufficient data to identify a convergent solution can preclude widespread application, especially if the time value of information rapidly declines, thus forcing the decision-maker to decide on acting with incomplete information, or on risking being overtaken by events.

Information Theory and “Synthetic Prices”

In competitive markets, a single number—the price of a Firm’s stock—represents risk. Under ideal conditions that price fully captures the Firm’s internal efficiencies, Return-on-Investment (ROI) and earning potential. Those efficiencies determine the Firm’s capability to harness new information as it becomes available. For private-sector firms, prices provide two important types of information:

1. The rate at which information becomes known.

The rate is analogous to the diffusion problem in heat transfer, which means that information diffusion can be modeled as a Brownian motion processes.

2. How information is aggregated.

In the private sector, the efficiency of aggregation indicates market efficiency; yet, while no such aggregation occurs in public-sector markets because there is no incentive.

Hayek was the first to identify these effects and to provide the rationale for defining markets in terms of their information value. The definition, in highly abbreviated form, is as follows:

> Competitive markets provide for the efficient coordination of decisions involving time and uncertainty. The process can be modeled as a Random Walk (in which the limit is approximated by a Brownian motion process). The rationale derives from the fact that where information flows without impediments, stock prices immediately reflect the latest information—so that a price change today will reflect only today's news and is independent of any prior price change (such processes are a weak form of market efficiency and have been modeled with some success using Markov models) (Dixit & Pindyck, 1994, p. 63). And, since news often is unpredictable, price changes are also unpredictable, but fully reflect all known information—thus, justifying the Random Walk interpretation.

However, competitive markets are not the only mechanism for determining investment valuations. A Firm's internal (information) processing efficiencies enable it to reallocate resources based on new information—and, thus, to manage risk. The efficiency of that process can be measured independently of market-based valuation. In this way, it can be applied to public-sector Firms to construct "synthetic prices"—thus linking valuations of its investments to its asset of a Firm: the ability to process information efficiently.

For the private sector, perturbation-based measures should converge in the limit to the market-based valuation, thus providing a basis for testing the validity of using the internal efficiencies to derive a "synthetic price" for the value of a Firm (Hayek, 945, September, p. 35).

In fact, the latter should reflect more accurately the "true" value of a Firm, which is what "Value Investors" and Hedge Funds are constantly trying to identify. Indeed, perturbation-response models offer a means to quantitatively link micro, Firm-level models to macro, policy-level models.

Pricing Public-sector Investments

Measures of investment efficiency for the public sector enable a portfolio manager to keep investments allocated to the most profitable outcomes—despite shocks and perturbations to operations. But, effective Portfolio Management depends on accurate pricing information. When information is limited or uncertain, risk is not efficiently priced. As a result, the marginal social utility of an investment will not equate to its price—thus, leaving no means to identify a "socially optimal" level of investment in either the private or the public sector. The net effect is analogous to Nash Equilibrium: a sup-optimal investment equilibrium condition that occurs when no player has incentive to unilaterally change strategy—because a change by any one of them would lead that player to earn less than would be obtained by remaining with the current strategy (Cover & Thomas, 1991, p. 460, 475; Dixit & Pindyck, 1994, p. 147, 283).

Sub-optimality can be driven by limited information, by a lack of incentives to change (as is the case for the Nash Equilibrium), as well as by uncertainty driven by factors such as market volatility. Higher volatility estimates reflect greater-than-expected fluctuations in underlying price levels and result in higher-option premiums for both puts and calls. With respect to the internal processing efficiencies of a Firm, the Taguchi's common and special sources of variability drive the perturbations of a Firm's workflow (León, Shoemaker, & Kacker, 1987). The efficiency of response determines whether and to what extent a Firm can synthesize information into actionable decisions in a timely manner. The often inconclusive, interwoven, and ambiguous nature of available information can result in time consumed to assess its value, which depreciates the time value of information. The resulting delay propagates uncertainty, which equates to opportunities lost. How much time is required to resolve the ambiguity is a function of a Firm's ability to manage that variability, which depends on its ability to synthesize information.

The impact of information efficiency can be measured in terms of response to random perturbations. The parameters characterizing the perturbations include the amplitude and time lag for changes in valuation and resource allocations, the amount of work/rework completed, requirements churn, the ratio of assets invested to return on those assets, output price/cost, fluctuations in labor, resource availability, the time to close major issues, the time to identify and to minimize both special and common causes of variability, schedule, and price, etc. Note that while these parameters indicate a Firm's internal information processing efficiencies with respect to work progress, much as Earned Value does, they say nothing of business value—i.e., whether the product provides anything useful to a customer.

The perturbation-responses are governed by structure of a Firm, which can be represented as a quasi-deterministic State Variable model. The combination of these techniques also can be used to identify the boundaries beyond which the magnitude of the perturbations would result in unstable responses and project failure (Ford & Taylor, 2006, pp. 337-369).[1] Information Theory benefits the analysis through the provision of algorithms that reduce the number of (perturbation-response) states to be considered in evaluating a process. The larger the number of states, the greater the lack of specificity or uncertainty (entropy) of the system. The following, highly over-simplified example illustrates the point.

The number of states that a single dice can assume is 6. But, if we know that the dice is in some sense biased toward either even or odd numbers (i.e., we have additional or "side" information), then the number of states that need to be considered is effectively reduced to 3, thus changing the outcome probabilities but not the probability distribution model. Alternatively stated, the entropy (disorganization) of a system does not depend on the actual values taken by the random variables describing it, but only on the associated probabilities.

Changes in the probability space, driven by changes to the efficiency of information flows within a Firm, provide a quantitative basis for linking the efficiency of information/knowledge management to well-defined mathematical processes, thus providing quantitative measures of risk that correspond to the underlying physical processes.

Information Theory can enable decision-makers to reduce risk, often drastically, by providing a quantitative framework to address issues such as:

1. The quantitative determination of changes in the uncertainty levels associated with cost/schedule/resource estimates as a project proceeds through its lifecycle.

[1] Ford and Taylor's text provides detailed discussion of project/program stability.

2. Discounting the anticipated benefits from a project—by measuring risk as a function of the time lags and amplitude of a Firm's response to various types of perturbations.
3. Identifying and reducing (both normal and special) sources of variability that adversely impact work progress and service/product quality
4. Constructing rules to achieve optimal investments—in terms of ensuring that a Firm's decision-making/information is efficient
5. The use-state variable models to estimate whether, and at what rate, investment management is improving

   Models capable of answering these questions we consider next.

Computational Models

The Firm's responses to the perturbations provide the raw data from which (indirect) measures of that efficiency can be expressed in terms of factors such as the time lag and amplitude of responses, and the variability of both. The perturbation/response processes are modeled using state variable regulator/ controller design methods. One of the best known and widely applied is the Kalman Filter, which can be used to measure how uncertainty propagates over time, and, thus, calculate the information-carrying capacity of a Firm.

The output of the Kalman Filter model is the amount and rate of *information gain* produced by various organizational structures. The magnitude and rate of correction serve as measures of a Firm's information processing capability, and (by implication) its level of entropy or level of internal-information organizational efficiency. The more efficient the Firm, the more quickly it will respond to perturbations (random shocks) regardless of source, internal or external. (The situation is analogous to determining the bandwidth of a communications system).

Schematically, the Kalman Filter information flow/computation cycle is illustrated in Figure 1, below.

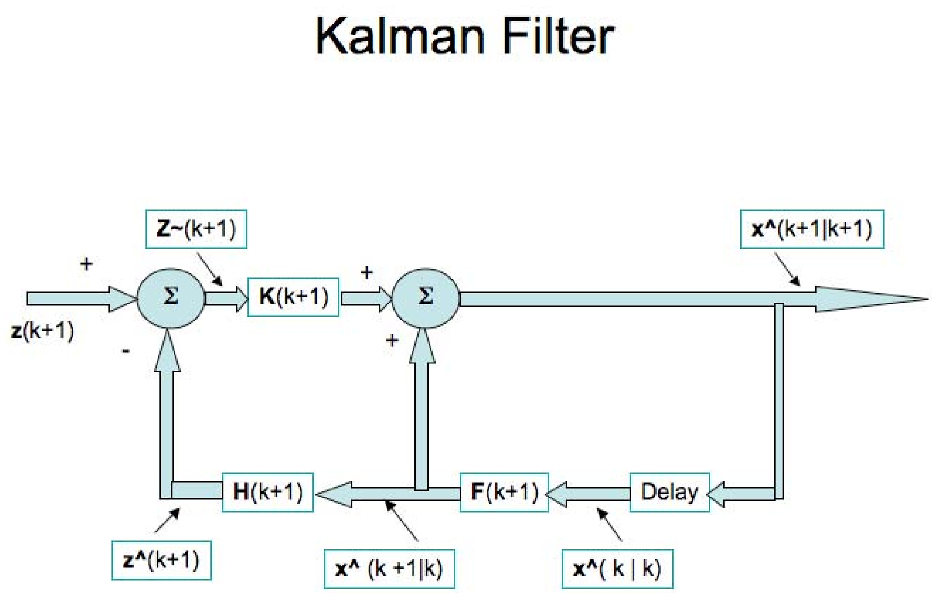

Figure 1. Kalman Filter

Kalman Filter Computation Steps

Legend:

x is the vector of variables comprising the system, whose state is to be estimated over the successive time (e.g., multi-stage investment) periods k =0,1,2,…

x ^ (k+1|k) is the predicted estimate of x for time "k+1" based on measures taken at time "k."

z is the actual, uncorrected measurement of x.

z ~ is estimate of x when corrected errors are introduced by the measurement process.

z ^ is the estimate of x ^ as filtered by H.

H is the measurement transformation matrix that relates the system state vector, x, to its measure, z.

Delay is the lag in system response to a stimulus. Delay is inherent to an organization because information cannot be gathered, analyzed, or transmitted instantaneously. Thus, changes in the environment or slips in schedule may or may not be recognized when they occur. For example, decreases in data quality typically generate increased disruption in operation. As more resources are shifted to fixing and correcting data records, the rate at which information is processed decreases. The resulting inefficiency generates increased correction and rework rates, along with increase delays in task completion.

F is the coefficient matrix of the state variable vector x. It describes the input/output efficiencies of a Firm and its investments. If these change overtime, then F (k) =/= F (k+1), k=0, 1, 2… and the system described by Eqn (1), (2), below, would be non-linear in time "t."

The steps presented in Figure 1 are as follows:

1. Begin with x^(0 |0) =0, for k = 0, 1, 2… Use x^(k | k) to iteratively compute x^ (k +1,k +1) given z(k+1).

   Where:

   x^ (k | k) Is the estimate propagated forward by pre-multiplying it by **F**(k+1, k) to give the predicted estimate **x**^(k+1 |k).

2. Pre-multiply x^(k+1 | k) by H (m +1) to compute is z^(k+1 | k), which is then subtracted from the actual measurement z (k+1).

   - The result is **z**~(k+1 | k), which is the measurement residual—that is, the difference between actual and predicted estimates at time (k+1).

3. The residual, z ~, is pre-multiplied by K (k+1), the Kalman Filter coefficient matrix, which is added to x^(k+1 | k).

   - The result is x^(k+1 | k +1), which is the estimate of x at time (k+1), given the measures updated at time (k+1).

4. x^ (k+1 | k +1), the optimal filtered estimate, is stored until the next measurement is made—at which time the cycle is repeated. (This is the only data that needs to be stored between measures, thus saving considerable computer storage and memory.)

Note that the product **K** (k+1)*__z__~(k+1 | k) is the correction that is added to the predicted estimate **x**^(k+1 |k) to determine the filtered estimate.

The optimal filter consists of the model of the dynamic process which performs the function of a prediction and feedback correction scheme in which the gain-times-residual, **K** (k+1) *__z__~(k+1 |k], per Eqn [3] below enters the model as the forcing function **u**(k) (Meditch, 1969, p. 182).

How all this works is illustrated by the following state-variable model for which the following assumptions are made.

- Time delay decreases the volume of work accomplished per unit of time.
- Time delay is driven by unstable/poorly managed requirements, funding instability, etc., thus acting to increase the amount of rework.
- In this example, model parameters are assumed to be Gaussian (Normally) Distributed, with side information (e.g., feedback) entering into the system as a sequence of predict-correct actions.

x(t) is the vector of state variables, which consists of the two elements:

x_1 (t) =Volume of work/unit of time,

x_2 (t) = Volume of rework/unit of time.

The state variable model is:

[1] **x**' = **F*** **x** (t) + **u**(t)

(where **x'** = d**x** (t)/dt) = [x_1', x_2'] = [dx_1 (t)/dt, x_2' = dx_2 (t)/dt]

$$[2] \quad \begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

Verbally, Eqn [2] states that the rate of change in work package completion is the sum of the current workload (x1) + rework (x2) + the arrival of new work (u1). The rework rate (x2') is equal to the sum of current rework (x2) + new rework (u2), where the vector u(t) is governed by the corrections provided by Eqn [3].

The control vector **u**(t) = [u_1(t), u_2(t)] is defined in terms of predictor/corrector parameters as:

[3] **u**(t) = **K**(t)*[**z** – **H*** **x^**]

Eqn [3] computes the correction, given **x^** from the Kalman Filter Eqn [9], below.

[4] **e**(t) = **K** (t)*[**v** – **H** *(**x** – **x** $_P$)]

Eqn [4] computes the error estimate *prior to* measurement.

Eqns [3], [4] derive from the linear measurement Model:

[5] **z** = **H*** **x** + **B** * **v**

To keep the computation simple, **H**, **B** are defined as identity matrices:

$$\mathbf{H} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

v is a vector of random measurement errors which are independent of the state **x. (v** is implemented with Monte Carlo simulation input.)

Eqn [1] and Eqn [2] can be cast into Kalman Filter format by defining the following parameters:

X $_P$ is the Estimate of the system state *prior to* measurement, as defined in [7-P], below.

$\mathbf{P_P}$ is Covariance of the system state *prior to* measurement, as defined in [7-P], below.

$\mathbf{x_A}$ is the Estimate of the system state *after* measurement, as defined in [7-A].

$\mathbf{P_A}$ is Covariance of the system state *after* measurement, as defined in [7-A].

[6] $\mathbf{R} = E[\mathbf{v}^*\mathbf{v}']$ Is the mean of the measurement error vector.

[7-P] $\mathbf{P_P} = E[(\mathbf{x_P} - \mathbf{x})^*(\mathbf{x_P} - \mathbf{x})^T]$ Is the error covariance matrix *prior* to measurement.

[7-A] $\mathbf{P_A} = E[(\mathbf{x_A} - \mathbf{x})^*(\mathbf{x_A} - \mathbf{x})^T]$ Is the error covariance matrix *after* measurement.

[8] $\mathbf{x_A} = \mathbf{x_P} + \mathbf{P_P}\ \mathbf{^*H_P}^{T}\mathbf{^*R_P}^{-1}(\mathbf{z} - \mathbf{H^*}\ \mathbf{x_P})$ is the optimal estimate.

Eqn [8] is a model of Eqn [1] with a correction term that is proportional to the difference between the actual measurement $\mathbf{z}_i$ and the predicted measurement $\mathbf{H_i}^*\mathbf{x_i}$

To minimize the subscript clutter used to denote the *before* and *after* calculations, $\mathbf{P}_A$ is changed to **P, for after,** and $\mathbf{P_p}$ to **M, for prior.** Thus, indexing Eqn [8] for measures over successive discrete time periods becomes Eqn [8'].

[8'] $\mathbf{x\hat{}_i} = \mathbf{x}^{-}_i + \mathbf{K_i}\ (\mathbf{z_i} - \mathbf{H} * \mathbf{x}_i^{-})$, where i =m, m+1 ….

for multi-stage investments.

The Gain Matrix for the Kalman Filter, for time periods i = 1, 2… is

[9] $\mathbf{K_i} = \mathbf{P_i} * \mathbf{H_i}^{T} * \mathbf{R_i}^{-1}$

Eqn [9] can be interpreted as the proportionality matrix or the ratio between uncertainty in the covariance matrix $\mathbf{P_i}$, after measurement at time "i" and uncertainty in the measurements $\mathbf{R_i}$ Eqn [6] (which can be interpreted as the effectiveness of a management reporting system).

The propagation of uncertainty in the *discrete* time system, Eqn [11], below, is based on the computation of:

[10] $\mathbf{P_i} = (\mathbf{M_i}^{-1} + \mathbf{H_i}^{T} * \mathbf{R_i}^{-1} * \mathbf{H_i})^{-1}$

[10'] $\mathbf{P_i} = \mathbf{M_i} - \mathbf{M_i} * \mathbf{H_i}^{T} * (\mathbf{H_i} * \mathbf{M_i} * \mathbf{H_i}^{T} + \mathbf{R_i})^{-1}\ \mathbf{H_i} * \mathbf{M_i}$

[11] $\mathbf{M_{i+1}} = \mathbf{F_i} * \mathbf{P_i} * \mathbf{F_i}^{T} + \mathbf{G_i} * \mathbf{Q_i} * \mathbf{G_i}^{T}$

Eqn [11] reflects the balance between the new information-forcing function,

$G_i * Q_i * G_i^T$; and, information processing efficiency, $F_i * P_i * F_i^T$. (In a traditional physical system, F_i would represent the damping efficiency of the system. Collectively, Eqn [10], [10'], and [11] describe the propagation of the covariance of the error estimate, which are independent of the measurements z_i.

Eqn [9]-[11] Is the Kalman Filter for a multi-stage process.

Prediction beyond the last measurement for states indexed as i = m +1, m+2, … is given by Eqn [12].

[12] $x_{(i+1)} = F_{(i)} * x_{(i)} + G_{(i)} * U_{(i)}$

Figure 2, below, provides a heuristic illustration (via the use of "canned" data) of the damping out effects of the Kalman Filter (illustrated by the magenta colored line), the retardation of that effect induced by response delay (as denoted by the green dashed line), with the black line illustrating the impact of a special cause of variability, such as the failure of an integration test, a reduction in funding, a major equipment failure, etc. The blue dashed line (highly exaggerated) illustrates "normal" variability causes, which might include ambiguous governance, the impact on products and services due to aging equipment, but which shows gradual improvement overtime.

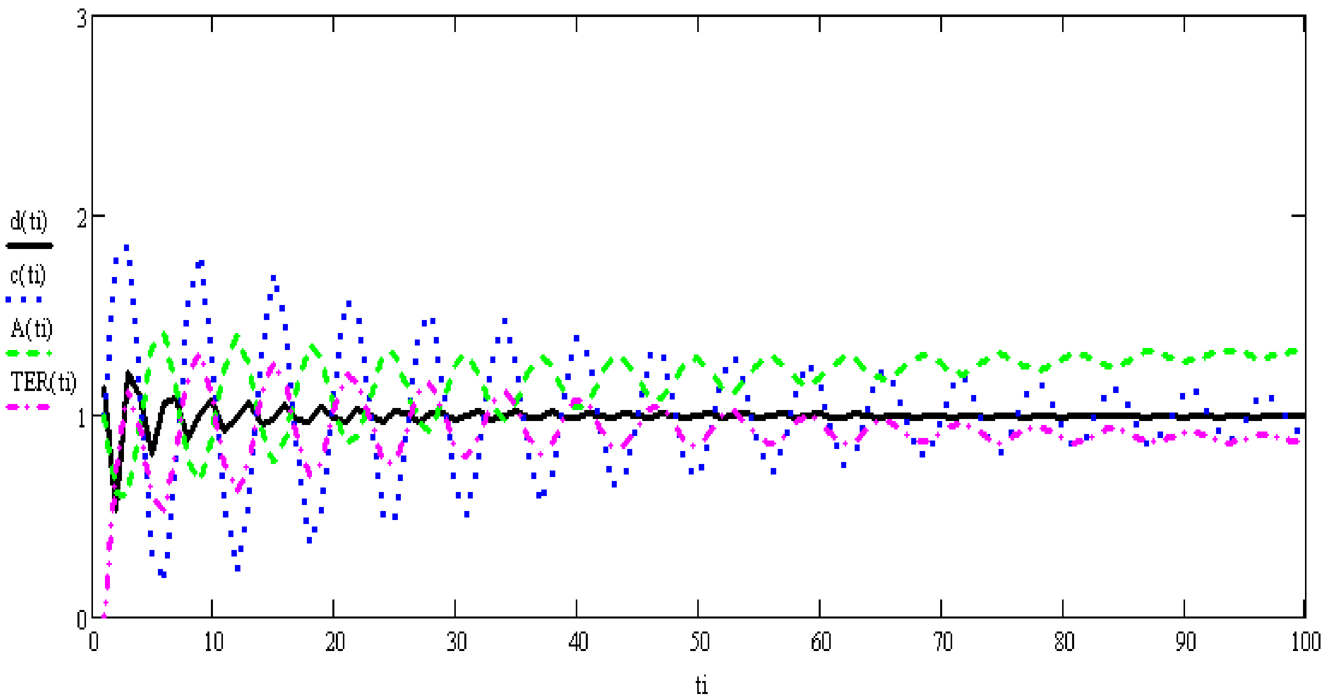

Figure 2. Perturbation-response Output from Kalman Filter

Summary/Next Research Steps

The strategy and models outlined in this brief paper indicate how Information Theory can be used to quantify the value of information and translate that value into a “price” for public-sector investments that, in principle, is comparable in a competitive market.

The next steps include:

- Fully defining the conditions under which the synthetic prices for public-sector goods are comparable in competitive markets.
- Developing methods to normalize comparisons across portfolios of diverse investment projects.
- Applying and evaluating the models using “real” data from public- and private-sector IT infrastructure investment projects.
- Using the model output to evaluate “synthetic prices” based on information gains defined using Kalman Filters.
- Comparing model results across a range of scenarios.
- Identifying algorithm improvements that accelerate convergence to specific price solutions.

Glossary

Term	Description
Black-Scholes model	A mathematical model used to calculate the value of a project or an investment as derived from market-place dynamics, based on approximations to Brownian motion processes
Brownian Motion	The random motion of particles in a liquid. The mathematical model of describing this motion is the Wiener process. A continuous time process that forms the basis of many important mathematical models in thermodynamics and Finance
Binomial Lattice Methods	An algorithm for valuing complex option problems whose payoff depends on multiple state variables following correlated geometric Brownian processes
Capital Asset Pricing (CAP)	The concept that there is a true value for a stock corresponding to its risk, for which various computational models can be used to determine risk-adjusted discount rates for investments, and to decide whether a stock price is too high or too low.
Complexity	There are at least two basic types of complexity: Descriptive complexity of an object—Kolmogorov complexity. While not directly computable, it can be bound between computable measures to describe the complexity of a sequence of symbols; Computational complexity—measures the time or space required for a computation
Derivative	A financial instrument that derives its value from the value of some other financial instrument or variable. For example, a stock option is a derivative because it derives its value from the value of a stock. An interest-rate swap is a derivative

	because it derives its value from one or more interest rate indices
Dynamic Hedging	The purchasing/selling of financial instruments to reduce or cancel out the risk in another investment as required by changing market conditions
Earned Value	Measures the dollar-value work completed per unit of time. It is a measure of progress against an objective, from which schedule (SC) and cost variances (CV) can be computed using Planned Value (PV) and Actual Cost (AC). SV = EV – PV CV = EV – AC. But, realistic estimates of Planned Value are seldom available—especially at the on-set of a large, complex project. Hence, the value of Information Theory lies in determining when and to what extent confidence can be placed in a benchmark such as Planned Value
Efficient Frontier	A concept that there is a true value for a stock corresponding to its risk; this theory of stock price is called Capital Assets Pricing Model and is used to decide whether a stock price is too high or too low.
Entropy	A measure of the disorganization of a physical system/The uncertainty of a single random variable/The minimum descriptive complexity of a random variable. An irreducible level of complexity below which a signal cannot be compressed
Equity Markets	A (competitive) stock market that efficiently coordinates decisions involving time and uncertainty
Exogenous	Refers to variables whose values are driven by factors external to the firm, or processes, of interest
Firm	In this paper, the “Firm” refers to an organization in either the private or public sector tasked with investing in and developing new products and services
Game Theory	The branch of applied mathematics and economics that studies situations where players choose different actions in an attempt to maximize their returns. It provides a formal, quantitative modeling approach to social situations in which decision-makers interact
Information Theory	A discipline spanning mathematics, economics, physics, communication theory, statistics, involving the quantification of data. For communications, the goal is to enable as much data as possible to be reliably stored or communicated over a channel.
Ito’s Lemma	Is used to integrate and differentiate stochastic processes. An *Ito process* can represent the dynamics of the value of a project which does not have a time derivative in the conventional sense- because its fluctuations over (short) periods of time do not have derivatives
Kalman Filter	A mathematical algorithm that operates in a predict-correct fashion that uses feedback to maintain a system (e.g., a rocket, or an investment portfolio) on a desired trajectory
Levy Process	A generalization of Brownian Motion processes to include discrete state jumps. The jumps can be local, global,

	simultaneous, independent or correlated
Law of Large Numbers	The sum of independent, identically distributed random variables that can be approximated arbitrarily closely to the expected value of the random variables
Markov Property	The property of a process that current information is useful for forecasting the future path of a process. Applied to Stock processes on the premise that public information is quickly incorporated into the current price /D-63/
Mutual information	The communication rate (efficiency) in the presence of noise. It is a measure of the amount of information that one random variable contains about another random variable. It is the reduction in the uncertainty of one random variable induced by knowledge of the other
Nash Equilibrium	A condition in Game Theory in which no player has incentive to unilaterally change her action. Players are in equilibrium if a change in strategies by any one of them would lead that player to earn less than if she remained with her current strategy
Net Present Value (NPV)	NPV is a standard method for the financial evaluation of a long-term project. Used for capital budgeting, and widely throughout economics, it measures the excess or shortfall of cash flows (in present value (PV) terms) once financing charges are met. By definition, NPV = Present value of net cash flows
Options Pricing	Is a contract between a buyer and a seller, or a provision of a contract, that gives one party (the option holder) the right, but not the obligation, to perform a specified transaction with another party (the option issuer or option writer) according to specified terms. Option contracts are a form of derivative instrument
Portfolio Management	The discipline of managing a portfolio of investments with the objective of maximizing the value of the entire portfolio by reallocating resources among the investments comprising the portfolio
Public-sector Firms	Firms such as public health or security tasked with providing goods and services whose valuations are not saleable in private equity (i.e., stock) markets, but which benefit society
Random Walk	A process that takes a discrete move in a specific direction according to a specified probability distribution. The Brownian Motion (Weiner process) is the limit for the discrete random walk process
Rate Distortion Theory	A major branch of information theory; it addresses the problem of determining the minimal amount of entropy (or information) that can be communicated over a channel, so that the source (input signal) can be approximately reconstructed at the receiver (output signal) without exceeding a given distortion level.
Risk	“Risk" is randomness with knowable probabilities; "uncertainty" is randomness with unknowable probabilities. Frank Knight (1921): An engineering definition of risk is “the (probability of an adverse event) x (loss per event)”
ROI—Return on Investment	A measure of the net income a firm is able to earn with its total assets. Return on investment is calculated by dividing net

	profits after taxes by total assets. The Rate of Return (ROR) or Return on Investment (ROI), or sometimes just return, is the ratio of money gained or lost on an investment relative to the amount of money invested. The amount of money gained or lost may be referred to as interest, profit/loss, gain/loss, or net income/loss. The money invested may be referred to as the asset, capital, principal, or the cost basis of the investment.
Side information	Information that is relevant to the outcome of an event such as a coin toss, or a horse race
System Control Theory	The discipline of controlling complex machines such as aircraft, computer networks, financial and manufacturing systems
Uncertainty	A characteristic of a random variable that is measured as Entropy. It is the number of bits required to describe a Random Variable. The larger the number of values that the random variable can take, the larger the uncertainty.
Variability: Special and common sources	A common source of variability: The quality level of an item created by a machine tool that is wearing out. A special source: an unlikely event such as the breaking of an artifact being processed by machine tool that fails unexpectedly

References:

Black, A., & Scholes, M. (1973). Capital asset pricing. *Journal of Political Economy*.

Bryson, A., & Ho, Y. (1975). *Applied optimal control*. Hemisphere Publishing.

Baum, C. (2005, March 2). *US Government needs a uniform strategic planning process for IT projects* (Gartner Publication G001269012).

Cesa-Bianchi. (2006). *Prediction, learning and games.* Cambridge.

Cover, T., & Thomas, J. (1991). *Elements of information theory.* Wiley Inter-science.

Dixit, A., & Pindyck, R. (1994). *Investment under uncertainty.* Princeton University Press.

Financial Risk Dictionary. Retrieved from http://www.riskglossary.com

Ford, D., & Taylor, T. (2006). Managing tipping point dynamics in single development. In *Proceedings, 3rd Annual Acquisition Research Symposium* (pp. 337-369). Monterey, CA: Naval Postgraduate School.

Gartner. (2002, July). *Return on enterprise architecture: Measure it in asset productivity* (RPT-0702-0119).

Gamba, A., & Trigeorgis, L. (2004, January). *An improved binomial lattice method for multi-dimensional option problems*.

Hayek, F.A. (1945, September). The use of knowledge in society. *The American Economic Review*, 35(4).

Johnson, J. (1999, December). Turning chaos into success. *Software Magazine*. Retrieved from www.SoftwareMag.com

Keller, E., (2005, May 29). *The death of packaged apps*? Retrieved from www.sandhill.com/opinion/editorial.php?id=83.

Knight, F. (1921). *Risk, uncertainty, and profit.*

León, R.V., Shoemaker, A.C., & Kacker, R.N. (1987). Performance measures independent of adjustment: An explanation and extension of Taguchi's signal-to-noise ratios (with discussion). *Technometrics,* 29, 253-285.

McShea, M. (2006, November). IT value management. *IT Professional*, November, p. 31.

Melsa, & Schultz. (1969). *Linear control systems.* Mc-Graw-Hill.

Meditch, J. (1969). *Stochastic optimal linear estimation and control.* McGraw-Hill.

Schweppe, F. (1973). *Uncertain dynamic systems.* Prentice-Hall.

Smith, T. (2000). *IT project management research findings.* Louisville, KY: Tech Republic.

Spohrer, J. et. al. (2007, January). Steps toward a science of service systems*. IEEE, Computer*.

Suter, R. (2005). A framework for calculating indirect costs and earned value for IT infrastructure modernization programs. In *Proceedings of the 2nd Acquisition Research Symposium.* Monterey, CA: Naval Postgraduate School.

Suter, R. (2006). Measuring uncertainty in earned value. In *Proceedings of the 3rd Acquisition Research Symposium*. Monterey, CA: Naval Postgraduate School.

THIS PAGE INENTIONALLY LEFT BLANK

2003 - 2006 Sponsored Acquisition Research Topics

Acquisition Management

- Software Requirements for OA
- Managing Services Supply Chain
- Acquiring Combat Capability via Public-Private Partnerships (PPPs)
- Knowledge Value Added (KVA) + Real Options (RO) Applied to Shipyard Planning Processes
- Portfolio Optimization via KVA + RO
- MOSA Contracting Implications
- Strategy for Defense Acquisition Research
- Spiral Development
- BCA: Contractor vs. Organic Growth

Contract Management

- USAF IT Commodity Council
- Contractors in 21st Century Combat Zone
- Joint Contingency Contracting
- Navy Contract Writing Guide
- Commodity Sourcing Strategies
- Past Performance in Source Selection
- USMC Contingency Contracting
- Transforming DoD Contract Closeout
- Model for Optimizing Contingency Contracting Planning and Execution

Financial Management

- PPPs and Government Financing
- Energy Saving Contracts/DoD Mobile Assets
- Capital Budgeting for DoD
- Financing DoD Budget via PPPs
- ROI of Information Warfare Systems
- Acquisitions via leasing: MPS case
- Special Termination Liability in MDAPs

Logistics Management

- R-TOC Aegis Microwave Power Tubes
- Privatization-NOSL/NAWCI
- Army LOG MOD

- PBL (4)
- Contractors Supporting Military Operations
- RFID (4)
- Strategic Sourcing
- ASDS Product Support Analysis
- Analysis of LAV Depot Maintenance
- Diffusion/Variability on Vendor Performance Evaluation
- Optimizing CIWS Lifecycle Support (LCS)

Program Management

- Building Collaborative Capacity
- Knowledge, Responsibilities and Decision Rights in MDAPs
- KVA Applied to Aegis and SSDS
- Business Process Reengineering (BPR) for LCS Mission Module Acquisition
- Terminating Your Own Program
- Collaborative IT Tools Leveraging Competence

A complete listing and electronic copies of published research within the Acquisition Research Program are available on our website: www.acquisitionresearch.org

Acquisition research Program
Graduate school of business & public policy
Naval postgraduate school
555 DYER ROAD, INGERSOLL HALL
MONTEREY, CALIFORNIA 93943

www.acquisitionresearch.org