azengineering

# Blockmodeling and the Estimation of Evolutionary Architectural Growth in Major Defense Acquisition Programs

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# Agenda

- Purpose, Question, and Contribution
- Challenge and Opportunity in Pre-MS A Cost Analysis
- Mapping DoDAF to COSYSMO
- Leveraging SE and Exploiting the SV-3
- Estimating Unforeseen Architectural Growth in MDAPs
  - Microstructure
  - Macrostructure
- Simulating Growth and Estimating Cost
- "Blockmodeling" Beyond Architectural Communities
- Future work
- Questions

**Overarching Purpose**: To transform Model-Based System Engineering (MBSE) artifacts into computational knowledge that can be leveraged early in the system lifecycle when uncertainty is high and confidence is low

↓

**Focused Question**: Can parametric cost estimation, in conjunction with DoD Architecture Framework (DoDAF) models, capture the monetary impact of architectural changes early in the system lifecycle?

↓

**Principal Contribution**: A network science-based algorithm for estimating the cost of unforeseen architectural growth

# Challenge and Opportunity in Pre-MS A Cost Analysis

**We find ourselves in challenging times . . .**

- Sequestration in 2013 + CRs =
  Reduced production +
  Hard modernization decisions +
  ⋯ + **Difficult cost planning**

**. . . and times were already tough . . .**

- 1997-2009: 47 MDAPs had cost overruns of at least 15%/30% over their current/baseline estimates

**. . . especially early in the life cycle . . .**

- ~ 28% of a system's baseline requirements will change

**. . . as late adds carry substantial costs.**

- 2014: 6 of 14 largest cost overruns due to new capabilities

**But there is an appetite for change . . .**

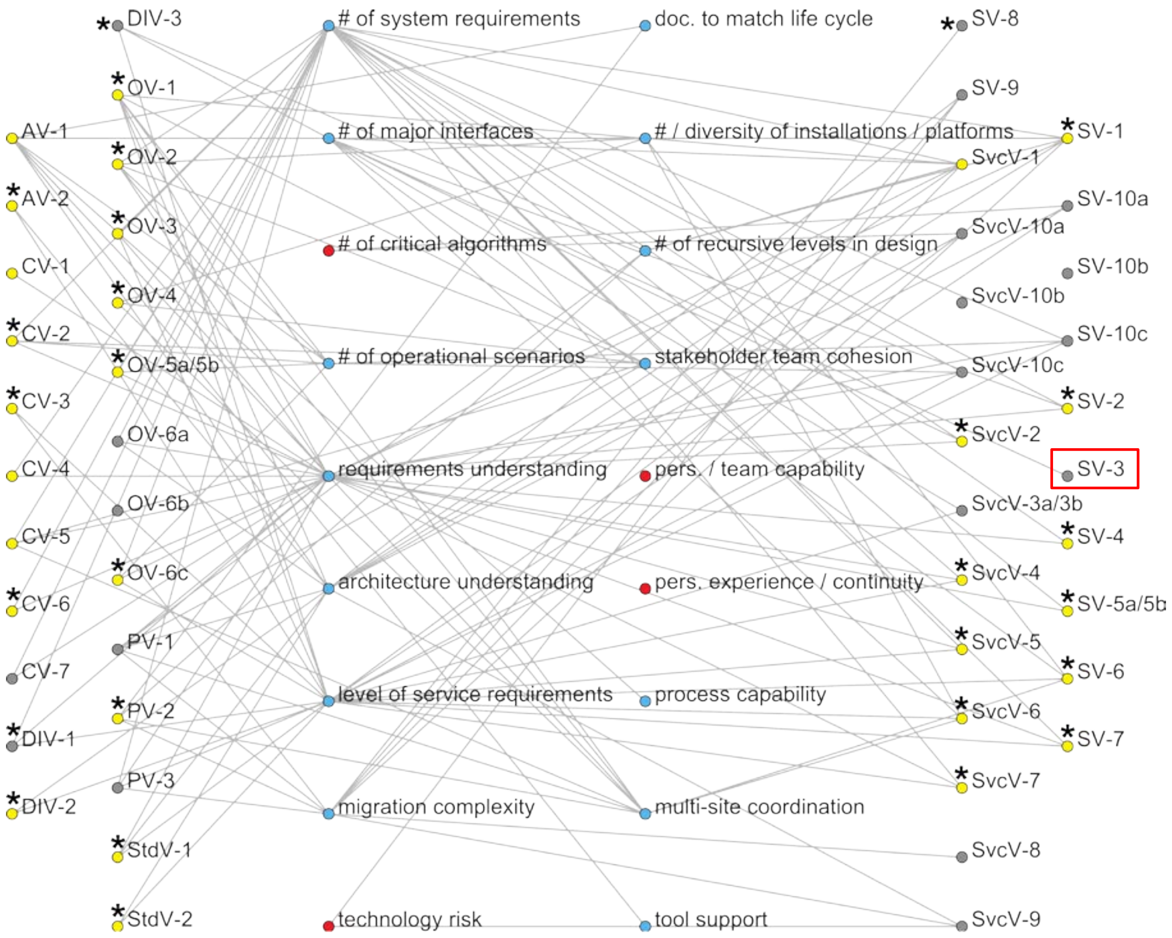
- *WSARA* (2009): Increased the rigor of Pre-MS A cost analysis (baseline shifted from MS B to MS A)
- *DoDI 5000.02* (2013): Mandated a draft CDD, with required DoDAF models, be submitted Pre-MS A

**. . . and this presents an opportunity.**

- DoDAF includes factors that influence system engineering (SE) effort (e.g., interfaces)
- COSYSMO estimates SE effort

***DoDAF's models appear to map to COSYSMO's parameters***

# DoDAF models required Pre-MS A nearly span COSYSMO's drivers*



***78% of COSYSMO's drivers map to DoDAF models submitted early in the life cycle***

Legend

▬ - DoDAF model X is relevant for rating COSYSMO driver Y

🟡 - Model required Pre-MS A (2012-2015)

* - Model required Pre-MS A (2015-)

🔵 - Pre-MS A model(s) maps to driver

🔴 - No Pre-MS A model maps to driver

| | |
|---|---|
| AV | - All (2 models) |
| CV | - Capability (7 models) |
| DIV | - Data and information (3 models) |
| OV | - Operational (9 models) |
| PV | - Project (3 models) |
| StdV | - Standards (2 models) |
| SvcV | - Services (13 models) |
| SV | - Systems (13 models) |

* Valerdi, R., Dabkowski, M., & Dixit, I. (2015). Reliability Improvement of Major Defense Acquisition Program Cost Estimates – Mapping DoDAF to COSYSMO. *Systems Engineering*, 18(5), 530-547. doi:10.1002/sys.21327

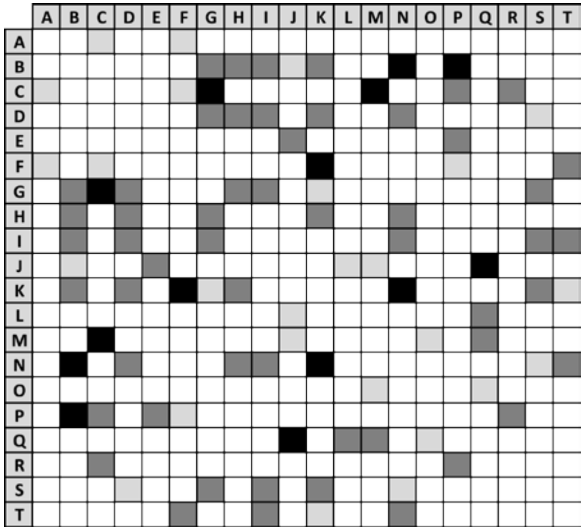
# Leveraging SE and Exploiting the SV-3

- From the 2008 National Research Council report “Pre-Milestone A and Early-Phase Systems Engineering” . . .
  - The “application of SE to decisions made in the pre-Milestone A period is critical to avoiding (or at least minimizing) cost and schedule overruns” (p. 3)
  - 3 of the 6 primary drivers of cost growth addressable by SE are:
    1. Incomplete requirements at MS B,
    2. System complexity (via internal, architectural design), and
    3. External interface complexity (via network-centric operations or “systems of systems” constructs) (pp. 82-85)
- The SV-3 (or Systems-Systems Matrix) provides an abstraction of all 3, as requirements (however incomplete) drive the selection of subsystems (nodes) which are connected by interfaces (edges), both internal and external

***Formally evaluating the SV-3 Pre-MS A and estimating its potential growth holds promise for minimizing cost overruns***

# Hypothetical SV-3

- 20 subsystems with 47 interfaces of varying complexity
- Without loss of generality, assume there are . . .
  - 200 easy, 200 nominal, and 50 difficult requirements
  - 5 difficult critical algorithms
- Using additional $w_{ik}$ and $EM_j$ data,* apply CER to obtain an initial estimate of $PM_{NS}$



| | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | | | □ | | | □ | | | | | | | | | | | | | | |
| B | | | | | | | ▨ | ▨ | ▨ | □ | ▨ | | | ■ | | ■ | | | | |
| C | □ | | | | | □ | ■ | | | | | | ■ | | | ▨ | | ▨ | | |
| D | | | | | | | ▨ | ▨ | ▨ | | ▨ | | | ▨ | | | | | □ | |
| E | | | | | | | | | | ▨ | | | | | | ▨ | | | | |
| F | □ | | □ | | | | | | | | ■ | | | | | □ | | | | ▨ |
| G | | ▨ | ■ | ▨ | | | | ▨ | ▨ | | □ | | | | | | | | ▨ | |
| H | | ▨ | | ▨ | | | ▨ | | | | ▨ | | | ▨ | | | | | | |
| I | | ▨ | | ▨ | | | ▨ | | | | | | | ▨ | | | | | ▨ | ▨ |
| J | | □ | | | ▨ | | | | | | | □ | □ | | | | ■ | | | |
| K | | ▨ | | ▨ | | ■ | □ | ▨ | | | | | | ■ | | | | | ▨ | □ |
| L | | | | | | | | | | □ | | | | | | | ▨ | | | |
| M | | | ■ | | | | | | | □ | | | | | □ | | ▨ | | | |
| N | | ■ | | ▨ | | | | ▨ | ▨ | | ■ | | | | | | | | □ | ▨ |
| O | | | | | | | | | | | | | □ | | | | □ | | | |
| P | | ■ | ▨ | | ▨ | □ | | | | | | | | | | | | ▨ | | |
| Q | | | | | | | | | | ■ | | ▨ | ▨ | | □ | | | | | |
| R | | | ▨ | | | | | | | | | | | | | ▨ | | | | |
| S | | | | □ | | | ▨ | | ▨ | | ▨ | | | □ | | | | | | |
| T | | | | | | ▨ | | | ▨ | | □ | | | ▨ | | | | | | |

SV-3

Interface Complexity

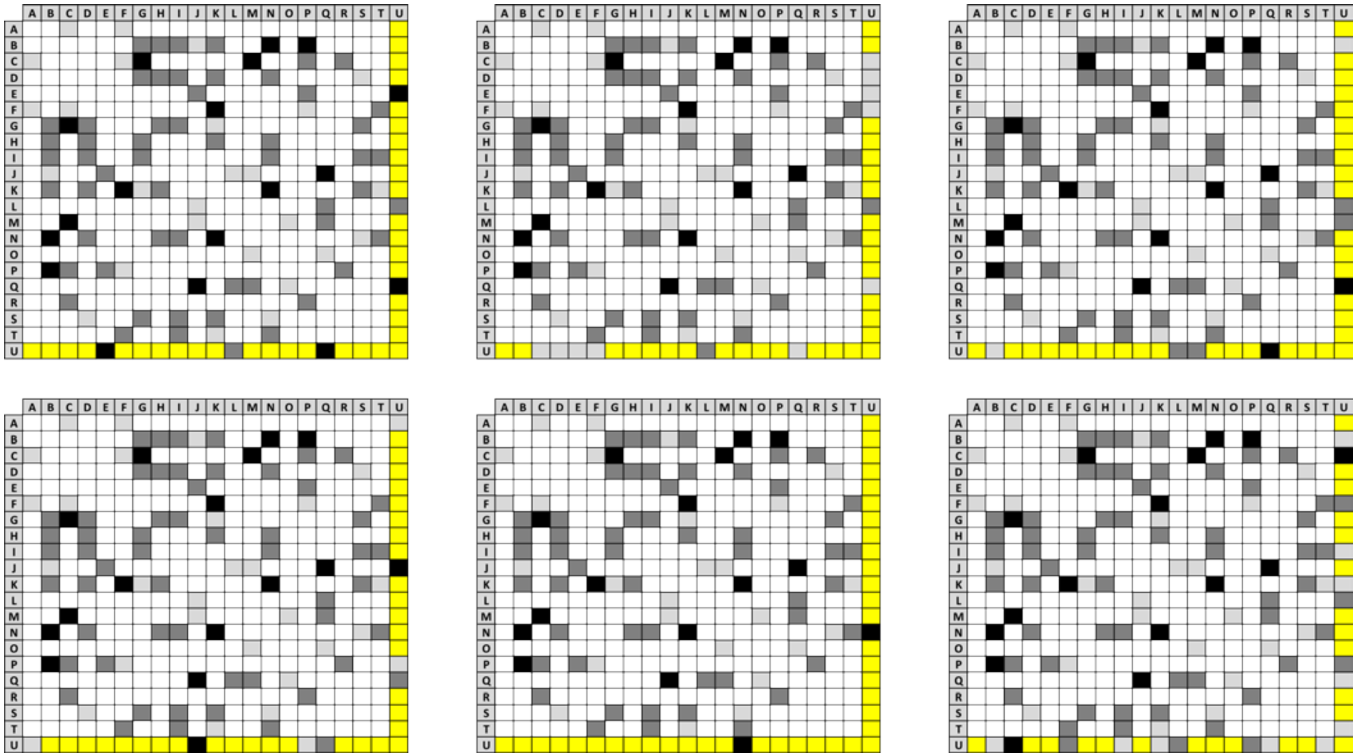
□ = Easy, ▨= Nominal, ■ = Difficult

$$PM_{NS} = 0.25\cdot\left(\underbrace{(0.5\times 200+1.0\times 200+5.0\times 50)}_{\text{requirements}}+\underbrace{(11.5\times 5)}_{\text{algorithms}}+\underbrace{(1.1\times 13+2.8\times 27+6.3\times 7)}_{\text{interfaces}}\right)^{1.06}\cdot 0.89 = 245.27$$

* Valerdi, R. (2008). *The Constructive Systems Engineering Cost Model (COSYSMO): Quantifying the Costs of Systems Engineering Effort in Complex Systems*. Saarbrücken, Germany: VDM Verlag.

# What about inevitable, unforeseen change?

- This is Pre-MS A ⇒ requirements will change



***If we add a new subsystem U to the existing architecture, how will it connect?***

***What will it cost?***

# The analytical task

***Estimate the number of interfaces***
***(by complexity level) U will generate***

(Q1) How many subsystems should **U** connect to (**degree, *m***)?,

(Q2) If **U** connects to *m* subsystems, which *m* subsystems should it connect to (**adjacency**)?, and

(Q3) If **U** connects to a specific set of *m* subsystems, what should the complexity of these interfaces be (**weights**)?

# Network Science – A mechanism for generating unforeseen architectural growth (microstructure)

Fundamental assumption: Current architecture foretells future architecture

*Degree*: Treat degree of **U** ($M_U$) as a random variable with PMF equal to the degree distribution of the existing system ("rich-by-birth") (Dorogovtsev & Mendes, 2003)

| $m$ | 2 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|
| $n_m$ | 5 | 3 | 5 | 3 | 3 | 1 |
| $P(M_U = m)$ | 0.25 | 0.15 | 0.25 | 0.15 | 0.15 | 0.05 |

*Adjacency*: Utilize Barabási–Albert (1999) preferential attachment (PA) model, where highly connected subsystems are more likely to interface with **U** ("rich-get-richer")

| System ($i$) | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| $d_i$ | 2 | 7 | 6 | 6 | 2 | 5 | 7 | 5 | 6 | 5 |
| $p_i$ | 0.021 | 0.074 | 0.064 | 0.064 | 0.021 | 0.053 | 0.074 | 0.053 | 0.064 | 0.053 |
| **System ($i$)** | **K** | **L** | **M** | **N** | **O** | **P** | **Q** | **R** | **S** | **T** |
| $d_i$ | 8 | 2 | 4 | 7 | 2 | 5 | 4 | 2 | 5 | 4 |
| $p_i$ | 0.085 | 0.021 | 0.043 | 0.074 | 0.021 | 0.053 | 0.043 | 0.021 | 0.053 | 0.043 |

$$p_i = \frac{d_i}{\sum_{j=1}^{N} d_j}$$

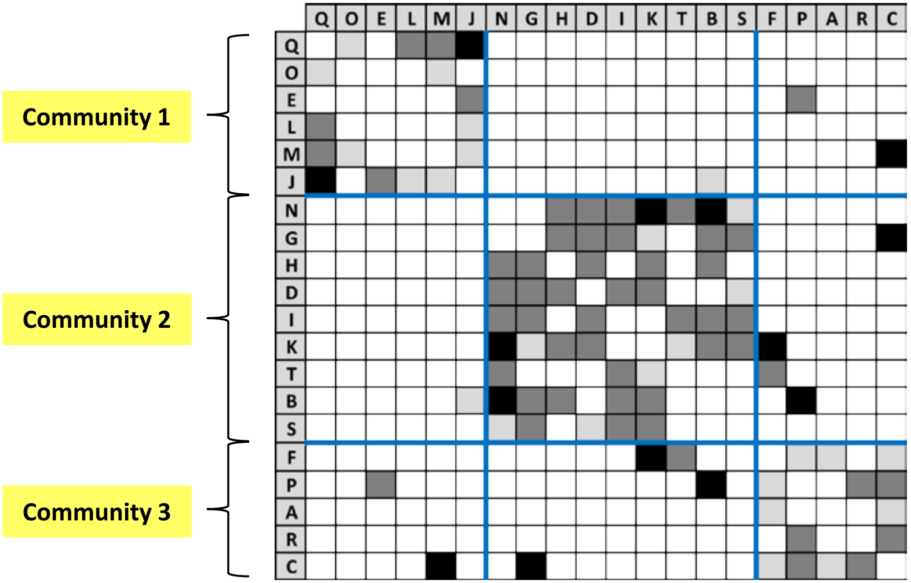
*Weights*: Model complexity of the interface between **U** and subsystem ***i*** ($w_{iU}$) as a random variable, where the pmf for $w_{iU}$ is ***i***'s interface complexity distribution

# Network Science – A mechanism for generating unforeseen architectural growth (macrostructure)

Fundamental assumption: Current architecture foretells future architecture

From *The Art of Systems Architecting*: "The most important aggregation and partitioning heuristics are to **minimize external coupling** and **maximize internal cohesion**"*

*Architectural communities*: Utilize Girvan-Newman (2002) to identify groups of subsystems such that the number of interfaces is sparse between and dense within groups


Community 1
Community 2
Community 3
Q O E L M J N G H D I K T B S F P A R C
Q O E L M J N G H D I K T B S F P A R C

| Intracommunity<br>Community | Δ | Intercommunity<br>Communities | Δ |
|---|---|---|---|
| 1: {Q, O, E, L, M, J} | 0.5333 | 1 and 2 | 0.0095 |
| 2: {N, G, H, D, I, K, T, B, S} | 0.6944 | 1 and 3 | 0.0364 |
| 3: {F, P, A, R, C} | 0.7000 | 2 and 3 | 0.0440 |

* Maier, M., & Rechtin, E. (2000). *The Art of Systems Architecting*. (2nd ed.). New York, NY: CRC Press.

# Simulating Growth and Estimating Cost – Dabkowski et al. (2014)

For a specified number of iterations . . .

*Preprocessing*

1. Initialize the system as the current system

2. Use Girvan-Newman (2002) to identify architectural communities

3. Randomly assign **U** to community *k*

*Intracommunity Growth*

4. Generate a realization for $M_{\mathbf{U},intra}$ given **U** is assigned to community *k* ($m_{intra}$)

5. Connect **U** to $m_{intra}$ subsystems inside community *k* using the BA model

6. For each interface established in (5), assign complexity ($w_{i\mathbf{U},intra}$)

*Intercommunity Growth*

7. Generate a realization for $M_{\mathbf{U},inter}$ given **U** is assigned to community *k* ($m_{inter}$)

8. Connect **U** to $m_{inter}$ communities using the BA model

9. For each interface established in (8), assign complexity ($w_{i\mathbf{U},inter}$)

*Cost Estimation*

10. Estimate cost for augmented system using COSYSMO ($PM_{NS}$*)

11. Calculate additional cost of adding subsystem **U** ($PM_{NS}$* − $PM_{NS}$)

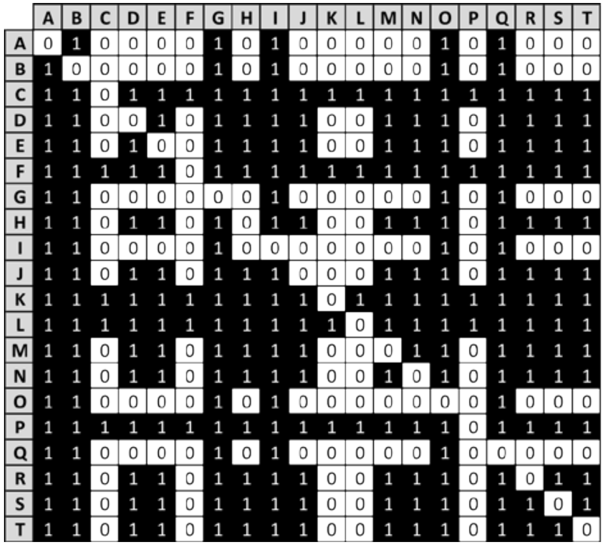
12. Store results and return to (3)

*"Did I build the right model?* ***Is it general enough?"***

# Community detection may miss key macrostructure . . .

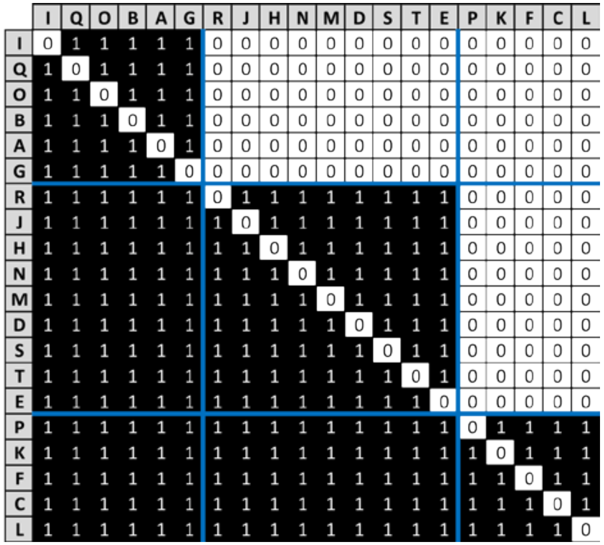
Hypothetical SV-3

Original representation



| | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| B | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| C | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| D | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| E | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| F | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| G | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| H | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| I | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| J | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| K | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| L | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| M | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| N | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| O | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| P | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| Q | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| R | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 |
| S | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| T | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 |

Representation following permutation



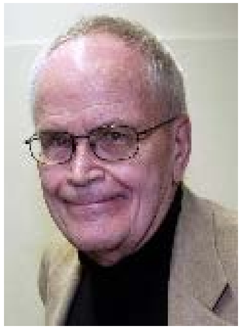
| | I | Q | O | B | A | G | R | J | H | N | M | D | S | T | E | P | K | F | C | L |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| I | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Q | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| O | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| B | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| A | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| G | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| R | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| J | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| H | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| N | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| M | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| D | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| S | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| T | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| E | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| P | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| K | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 |
| F | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |
| C | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 |
| L | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

- $N = 20$ subsystems and $E = 251$ *directed* interfaces; relatively dense ($\Delta = 0.661$)
- Girvan-Newman (2002) identifies 6 architectural communities with a modularity of just 0.017

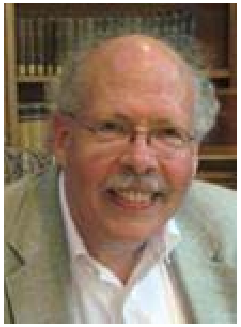
***Girvan-Newman misses the indisputable, hierarchical clustering of subsystems!***

# . . . but blockmodeling does not.

- Blockmodeling
  - Partitions a network consisting of $i = 1, \cdots, N$ objects (i.e., the SV-3) into $k = 1, \cdots, P$ non-overlapping positions, where the positions generally abide the structure represented in a $(P \times P)$ image matrix such that $P \ll N$
  - Developed by computational sociologists at Harvard in the mid-1970's



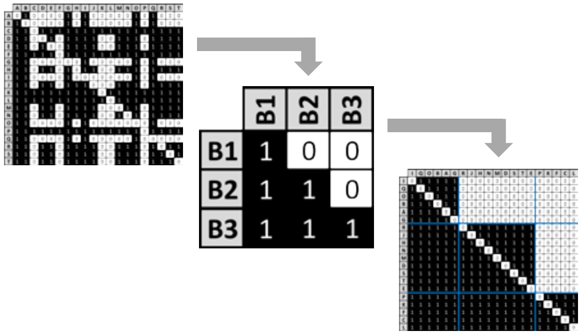
**Harrison White**



**Scott Boorman**

**Ronald Breiger**

**Social structure from multiple networks. I. Blockmodels of roles and positions. *AJS*, 81(4), 730-780.**

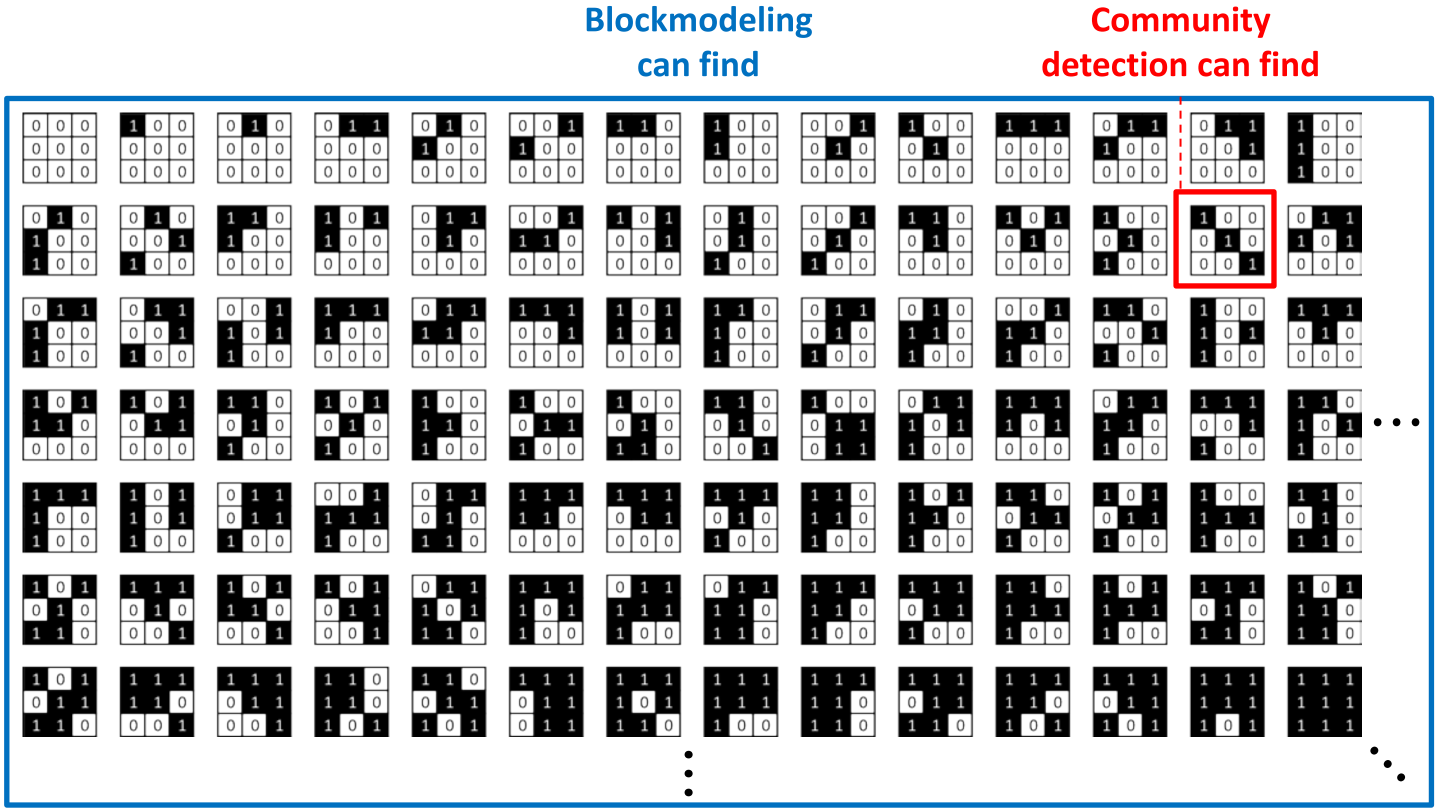


| | B1 | B2 | B3 |
|---|---|---|---|
| B1 | 1 | 0 | 0 |
| B2 | 1 | 1 | 0 |
| B3 | 1 | 1 | 1 |

***Pajek recovers the SV-3's hierarchical structure exactly!***

  - Integrated into popular network analysis software (i.e., Pajek via Doreian, Batagelj, and Ferligoj's (2005) direct approach)

# Blockmodeling is the natural generalization of community detection . . .



***Of the 512 possible $(3 \times 3)$ binary image matrices, community detection can find a partition for 1 – the identity; blockmodeling can accommodate all 512!***

# . . . but blockmodeling is not a panacea.

- Issue #1: Blockmodeling (BM) problems are NP-hard ⇒ time to find globally optimal solutions can *explode* as the # of subsystems/positions ↑

  Consequence #1: BM normally applies heuristics versus exact methods ⇒ better fitting image matrices and partitions may exist

- Issue #2: Exact methods largely confined to *confirmatory* fitting (image matrix is pre-specified) ⇒ exact *exploratory* fitting procedures are lacking

  Consequence #2: An SV-3's macrostructure is not "known in advance" ⇒ available exact BM methods are ill-suited for the task at hand

- Issue #3: Majority of BM heuristics and all exact methods focus on single one-/two-mode networks ⇒ BM multiple relations is an open problem

  Consequence #3: SV-3s are often mixed-mode networks ⇒ new methods are required to accommodate all SV-3s

# WANTED . . . An Efficient Exact Method for Blockmodeling Mixed-Mode SV-3s

Given:

A mixed-mode SV-3

1-mode portion 2-mode portion

| | | Internal Subsystems | | | | | | | | | | External Subsystems | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | I1 | I2 | I3 | I4 | I5 | I6 | I7 | I8 | I9 | I10 | E1 | E2 | E3 | E4 | E5 | E6 | E7 |
| Internal Subsystems | I1 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | I2 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | I3 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | I4 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| | I5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| | I6 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| | I7 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| | I8 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| | I9 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 |
| | I10 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |

Find:

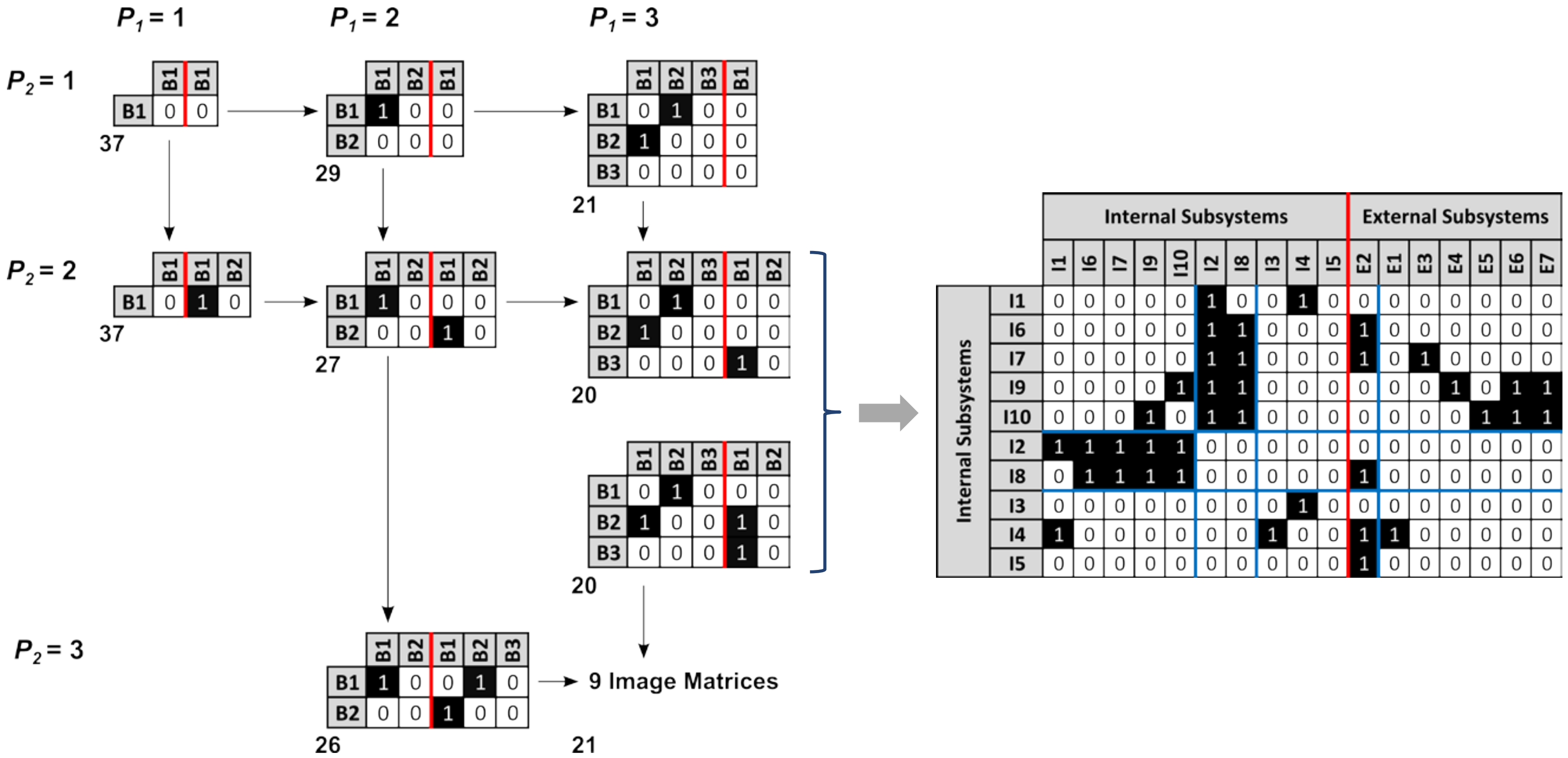
The globally optimal mixed-mode image matrix and corresponding partition with three or fewer internal and external subsystem positions

| | | Internal Subsystems | | | External Subsystems | | |
|---|---|---|---|---|---|---|---|
| | | B1 | B2 | B3 | B1 | B2 | B3 |
| Internal Subsystems | B1 | 0 or 1 | 0 or 1 | 0 or 1 | 0 or 1 | 0 or 1 | 0 or 1 |
| | B2 | 0 or 1 | 0 or 1 | 0 or 1 | 0 or 1 | 0 or 1 | 0 or 1 |
| | B3 | 0 or 1 | 0 or 1 | 0 or 1 | 0 or 1 | 0 or 1 | 0 or 1 |

Idea: Leverage the results in Brusco and Steinley's (2009) paper "Integer programs for one- and two-mode blockmodeling based on prespecified image matrices for structural and regular equivalence"

# Globally Optimal IMs and Partition

- Formulated a series of IPs using C++; solved in IBM's ILOG CPLEX



$P_1 = 1$, $P_2 = 1$ (37)

|  | B1 | B1 |
|---|---|---|
| B1 | 0 | 0 |

$P_1 = 2$, $P_2 = 1$ (29)

|  | B1 | B2 | B1 |
|---|---|---|---|
| B1 | 1 | 0 | 0 |
| B2 | 0 | 0 | 0 |

$P_1 = 3$, $P_2 = 1$ (21)

|  | B1 | B2 | B3 | B1 |
|---|---|---|---|---|
| B1 | 0 | 1 | 0 | 0 |
| B2 | 1 | 0 | 0 | 0 |
| B3 | 0 | 0 | 0 | 0 |

$P_1 = 1$, $P_2 = 2$ (37)

|  | B1 | B1 | B2 |
|---|---|---|---|
| B1 | 0 | 1 | 0 |

$P_1 = 2$, $P_2 = 2$ (27)

|  | B1 | B2 | B1 | B2 |
|---|---|---|---|---|
| B1 | 1 | 0 | 0 | 0 |
| B2 | 0 | 0 | 1 | 0 |

$P_1 = 3$, $P_2 = 2$ (20)

|  | B1 | B2 | B3 | B1 | B2 |
|---|---|---|---|---|---|
| B1 | 0 | 1 | 0 | 0 | 0 |
| B2 | 1 | 0 | 0 | 0 | 0 |
| B3 | 0 | 0 | 0 | 1 | 0 |

(20)

|  | B1 | B2 | B3 | B1 | B2 |
|---|---|---|---|---|---|
| B1 | 0 | 1 | 0 | 0 | 0 |
| B2 | 1 | 0 | 0 | 1 | 0 |
| B3 | 0 | 0 | 0 | 1 | 0 |

$P_1 = 2$, $P_2 = 3$ (26)

|  | B1 | B2 | B1 | B2 | B3 |
|---|---|---|---|---|---|
| B1 | 1 | 0 | 0 | 1 | 0 |
| B2 | 0 | 0 | 1 | 0 | 0 |

→ 9 Image Matrices (21)

|  |  | Internal Subsystems |  |  |  |  |  |  |  |  |  | External Subsystems |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
|  |  | I1 | I6 | I7 | I9 | I10 | I2 | I8 | I3 | I4 | I5 | E2 | E1 | E3 | E4 | E5 | E6 | E7 |
| Internal Subsystems | I1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|  | I6 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
|  | I7 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
|  | I9 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 |
|  | I10 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
|  | I2 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|  | I8 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
|  | I3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
|  | I4 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
|  | I5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |

***Partition of internal subsystems driven by "outside" interfaces***

# Generalizing Dabkowski et al. (2014) via Blockmodeling

For a specified number of iterations . . .

*Preprocessing*

1. Initialize the system as the current system

2. Build an optimal set of $\{P(M=m),\beta\}$ pairs

3. Use Dabkowski-Fan-Breiger (2015; 2016) to identify an optimal $P$-position image matrix and partition of subsystems

*Growth*

4. Randomly select a member from the optimal set of $\{P(M=m),\beta\}$ pairs

5. Generate a realization for the incoming subsystem's (**X**'s) number of interfaces using $P(M=m)$; **if the IM and partition suggest a compelling, underlying architectural structure**, use *Connection Option A*; otherwise, use *Connection Option B*

*Connection Option A (use blockmodel)*

6a. Randomly assign **X** to position *k*,

6b. Model assignment of **X**'s $m$ interfaces to positions as a random $(1 \times P)$ vector $\boldsymbol{C}$, where $\boldsymbol{C}$ follows a Multinomial$(m, \boldsymbol{p})$ distribution and $\boldsymbol{p}$ is the $(1 \times P)$ vector of multinomial probabilities given by

$$\frac{\text{\# interfaces in block } (k,l) \text{ of the partitioned, permuted SV−3}}{\text{\# interfaces in blocks } (k,\bullet) \text{ of the partitioned, permuted SV−3}};$$

generate a feasible realization for $\boldsymbol{C}$

6c. For $l = 1, \cdots, P$, attach **X** to $\boldsymbol{c}_l$ subsystems inside position $l$ using attachment probabilities $p_i = {d_i}^{\beta} / \sum_{j=1}^{N} {d_j}^{\beta}$

6d. For each interface established in (6c), assign complexity ($w_{i\mathbf{X}}$)

*Connection Option B (do not use blockmodel)*

6a. Attach **X** to $m$ subsystems using attachment probabilities $p_i = {d_i}^{\beta} / \sum_{j=1}^{N} {d_j}^{\beta}$

6b. For each interface established in (6a), assign complexity ($w_{i\mathbf{X}}$)

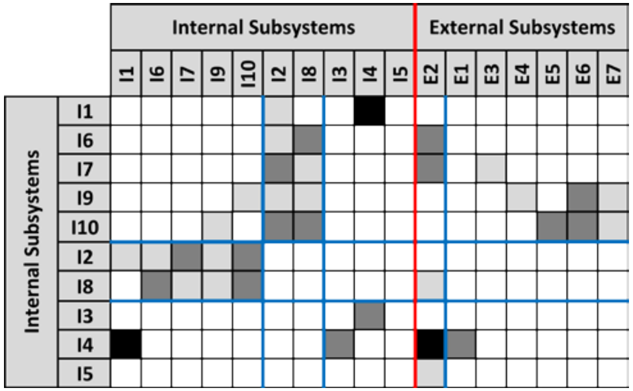
*Cost Estimation (same as Dabkowski et. al (2014), goto Step 4)*

# Estimating the Cost of Architectural Growth

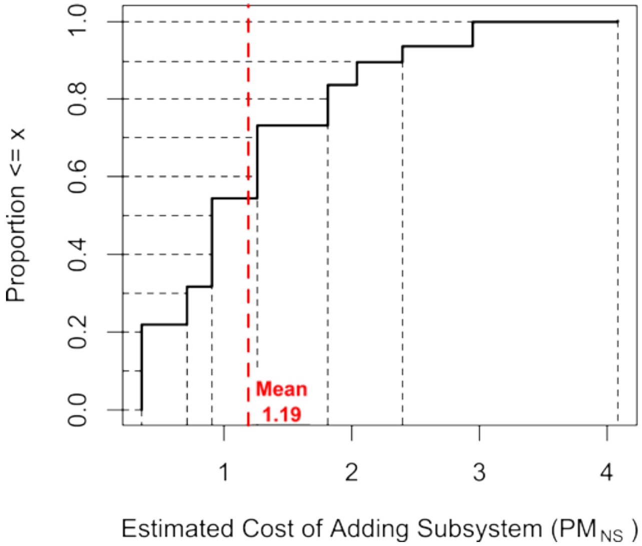
- Assumed the following:
  - $A = 0.25; E = 1.06; \prod_{j=1}^{14} EM_j = 0.89$
  - Requirements: 75 easy, 50 nominal, 10 difficult
  - Internal interfaces: 6 easy, 5 nominal, 1 difficult
  - External interfaces: 6 easy, 6 nominal, 1 difficult

  $\Rightarrow$ 59.24 $PM_{NS}$ of SE effort required
- Coded algorithm in R
- Ran 10,000 iterations

***Expected cost to connect an additional subsystem to the SV-3's internal subsystems is (1.177, 1.206) PM$_{NS}$***



| | | Internal Subsystems | | | | | | | | | | External Subsystems | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | I1 | I6 | I7 | I9 | I10 | I2 | I8 | I3 | I4 | I5 | E2 | E1 | E3 | E4 | E5 | E6 | E7 |
| Internal Subsystems | I1 | | | | | | ░ | | | █ | | | | | | | | |
| | I6 | | | | | | ░ | ▓ | | | | ▓ | | | | | | |
| | I7 | | | | | | ▓ | ░ | | | | ▓ | | ░ | | | | |
| | I9 | | | | | ░ | ░ | ░ | | | | | | | ░ | | ▓ | ░ |
| | I10 | | | | ░ | | ▓ | ▓ | | | | | | | | ▓ | ▓ | ░ |
| | I2 | ░ | ░ | ▓ | ░ | ▓ | | | | | | | | | | | | |
| | I8 | | ▓ | ░ | ░ | ▓ | | | | | | ░ | | | | | | |
| | I3 | | | | | | | | | ▓ | | | | | | | | |
| | I4 | █ | | | | | | | ▓ | | | █ | ▓ | | | | | |
| | I5 | | | | | | | | | | | ░ | | | | | | |



# Future Work

- Gather additional data for further validation and refinement
  - Secure sponsored research to weight SV-3s by interface complexity
  - Work with PMs to obtain multiple snapshots of SV-3s over time
- Explore additional connection options (e.g., model the probability that subsystem **X** is assigned to position *k* as a function of position *k*'s size)
- Modify algorithm to address external architectural growth
- Investigate the evolution of non-DoD architectures (e.g., open-source software architectures, non-militarized space systems, etc.)

# Questions

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# IP formulation

**For all possible mixed-mode image matrices solve . . .**

**Data**

$\boldsymbol{S}^{(1)}$: $(N_1 \times N_1)$ 1-mode portion of the SV-3

$\boldsymbol{S}^{(2)}$: $(N_1 \times N_2)$ 2-mode portion of the SV-3

$\left(\boldsymbol{B}_1^{(r)} | \boldsymbol{B}_2^{(q)}\right)$: Binary $(P_1 \times P_1 | P_1 \times P_2)$ mixed-mode image matrix (IM), where $\boldsymbol{B}_1^{(r)}$ and $\boldsymbol{B}_2^{(q)}$ are the 1- and 2-mode portions

**Indices**

$i, j \in \mathcal{N}_1 \quad m \in \mathcal{N}_2$

$p, l \in \mathcal{P}_1 \quad k \in \mathcal{P}_2$

**Decision variables**

When using image matrix $\left(\boldsymbol{B}_1^{(r)} | \boldsymbol{B}_2^{(q)}\right)$:

$x_{i,p}^{(r,q)} = 1 \Rightarrow$ internal subsystem $i$ assigned to internal position $p$

$y_{m,k}^{(r,q)} = 1 \Rightarrow$ external subsystem $m$ assigned to external position $k$

$$\min \underbrace{\sum_{p,l=1}^{P_1} \sum_{\substack{i,j=1 \\ i \neq j}}^{N_1} w_{i,j,p,l}^{(r,q)} \left(b_{p,l}^{(r)} + s_{i,j}^{(1)} - 2 b_{p,l}^{(r)} s_{i,j}^{(1)}\right)}_{\text{Inconsistencies between one-mode IM and partition of internal subsystems}} + \underbrace{\sum_{i=1}^{N_1} \sum_{m=1}^{N_2} \sum_{p=1}^{P_1} \sum_{k=1}^{P_2} z_{i,m,p,k}^{(r,q)} \left(b_{p,k}^{(q)} + s_{i,m}^{(2)} - 2 b_{p,k}^{(q)} s_{i,m}^{(2)}\right)}_{\text{Inconsistencies between two-mode IM and partitions of internal and external subsystems}}$$

s.t.

$$\sum_{p=1}^{P_1} x_{i,p}^{(r,q)} = 1, \qquad \forall\, i \in \mathcal{N}_1$$

$$\sum_{k=1}^{P_2} y_{m,k}^{(r,q)} = 1, \qquad \forall\, m \in \mathcal{N}_2$$

Each internal / external subsystem assigned to a single internal / external position

$$\sum_{i=1}^{N_1} x_{i,p}^{(r,q)} \geq 1, \qquad \forall\, p \in \mathcal{P}_1$$

$$\sum_{m=1}^{N_2} y_{m,k}^{(r,q)} \geq 1, \qquad \forall\, k \in \mathcal{P}_2$$

No “empty” internal / external positions

$$x_{i,p}^{(r,q)} \in \{0,1\}, \qquad \forall\, i \in \mathcal{N}_1, p \in \mathcal{P}_1$$

$$y_{m,k}^{(r,q)} \in \{0,1\}, \qquad \forall\, m \in \mathcal{N}_2, k \in \mathcal{P}_2$$

Restrict decision variables to be binary

$$\begin{array}{lcll}
w_{i,j,p,l}^{(r,q)} & \leq & x_{i,p}^{(r,q)}, & \forall\, i,j \in \mathcal{N}_1, p,l \in \mathcal{P}_1 \\
w_{i,j,p,l}^{(r,q)} & \leq & x_{j,l}^{(r,q)}, & \forall\, i,j \in \mathcal{N}_1, p,l \in \mathcal{P}_1 \\
w_{i,j,p,l}^{(r,q)} & \geq & x_{i,p}^{(r,q)} + x_{j,l}^{(r,q)} - 1, & \forall\, i,j \in \mathcal{N}_1, p,l \in \mathcal{P}_1 \\
w_{i,j,p,l}^{(r,q)} & \geq & 0, & \forall\, i,j \in \mathcal{N}_1, p,l \in \mathcal{P}_1
\end{array}$$

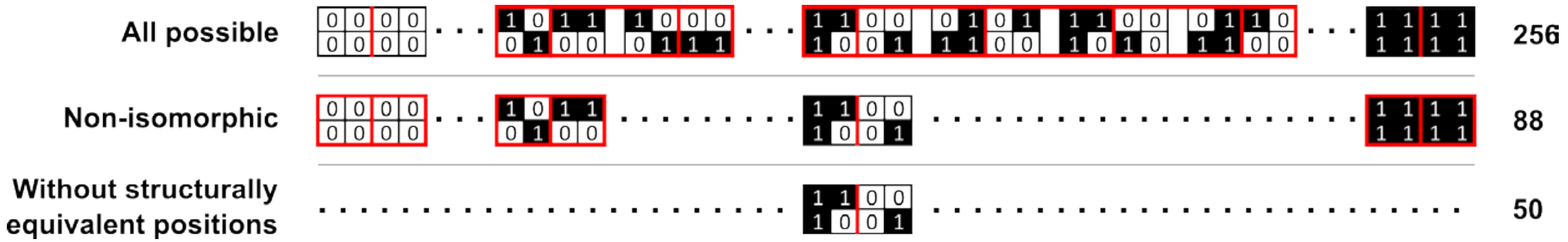
Linearization constraints for $w_{i,j,p,l}^{(r,q)} = x_{i,p}^{(r,q)} x_{j,l}^{(r,q)}$

$$\begin{array}{lcll}
z_{i,m,p,k}^{(r,q)} & \leq & x_{i,p}^{(r,q)}, & \forall\, i \in \mathcal{N}_1, m \in \mathcal{N}_2, p \in \mathcal{P}_1, k \in \mathcal{P}_2 \\
z_{i,m,p,k}^{(r,q)} & \leq & y_{m,k}^{(r,q)}, & \forall\, i \in \mathcal{N}_1, m \in \mathcal{N}_2, p \in \mathcal{P}_1, k \in \mathcal{P}_2 \\
z_{i,m,p,k}^{(r,q)} & \geq & x_{i,p}^{(r,q)} + y_{m,k}^{(r,q)} - 1, & \forall\, i \in \mathcal{N}_1, m \in \mathcal{N}_2, p \in \mathcal{P}_1, k \in \mathcal{P}_2 \\
z_{i,m,p,k}^{(r,q)} & \geq & 0, & \forall\, i \in \mathcal{N}_1, m \in \mathcal{N}_2, p \in \mathcal{P}_1, k \in \mathcal{P}_2.
\end{array}$$

Linearization constraints for $z_{i,m,p,k}^{(r,q)} = x_{i,p}^{(r,q)} y_{m,k}^{(r,q)}$

# Two crucial structural observations

- Observation #1: Some image matrices are created “equal”
- Observation #2: Some positions are created “equal”



| # of positions $(P_1, P_2)$ | All possible IMs | Non-isomorphic IMs | IMs without structurally equivalent positions | Percentage of all possible IMs to fit |
|---|---|---|---|---|
| **(1, 1)** | 4 | 4 | 4 | 100.00% |
| **(1, 2)** | 8 | 6 | 2 | 25.00% |
| **(2, 1)** | 64 | 36 | 32 | 50.00% |
| **(2, 2)** | 256 | 88 | 50 | 19.53% |
| **(2, 3)** | 1024 | 172 | 36 | 3.52% |
| **(3, 1)** | 4096 | 752 | 688 | 16.80% |
| **(3, 2)** | 32768 | 3272 | 2424 | 7.40% |
| **(3, 3)** | 262144 | 10704 | 4912 | 1.87% |
| **Total IMs to fit** | 300364 | 15034 | 8148 | 2.71% |

***Two order of magnitude reduction in the number of IMs to fit***