An Optimization-Based Approach to Determine System Requirements under Multiple Domain-Specific Uncertainties

Parithi Govindaraju, Navindran Davendralingam
and William Crossley
School of Aeronautics and Astronautics, Purdue University

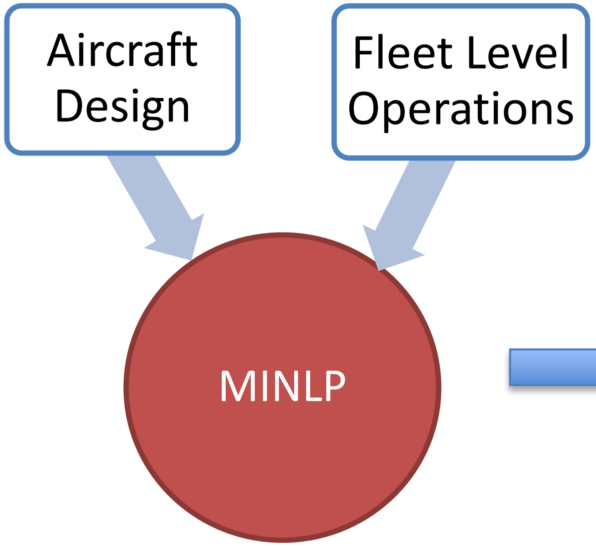
13th Annual Acquisition Research Symposium
May 5, 2016

Research Question

- Improve Requirements Definition
- Can we identify a quantitative approach to determine the “right requirements” for a new system?
  - New system must work in a “fleet” with existing systems
  - Adding new system to improve “fleet-level” objectives
  - Make use of methods from operations research, operations analysis
- Can this approach address uncertainties?
  - New system design
  - Fleet-level operations
- Application here is military air cargo
  - Introduce new aircraft
  - Minimize fuel consumption, maximize productivity
  - Display tradeoffs

What are the right requirements for a new strategic cargo aircraft?

Approach: Decomposition Strategy

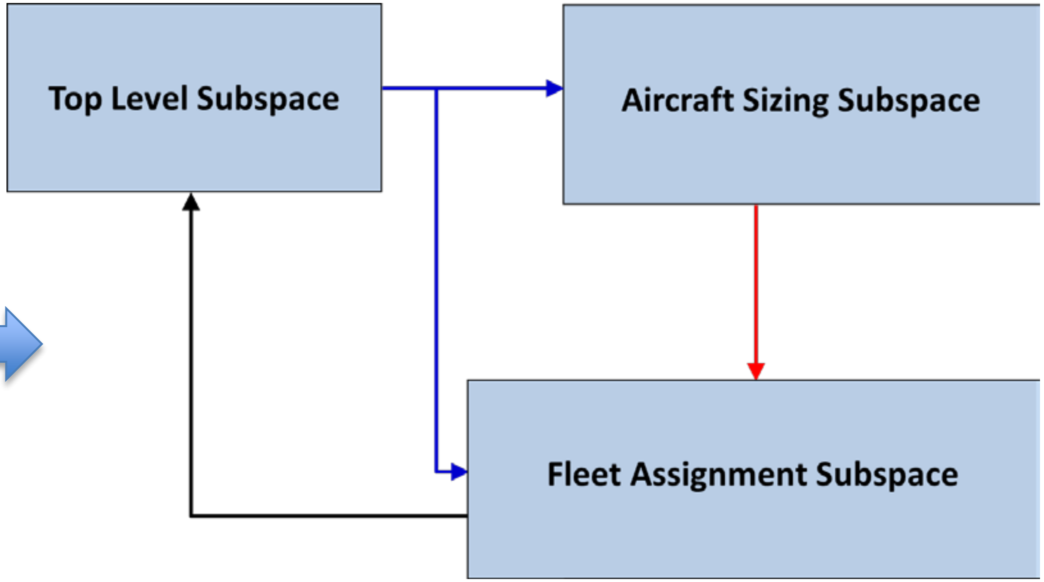


Aircraft Design

Fleet Level Operations

MINLP

Monolithic Formulation



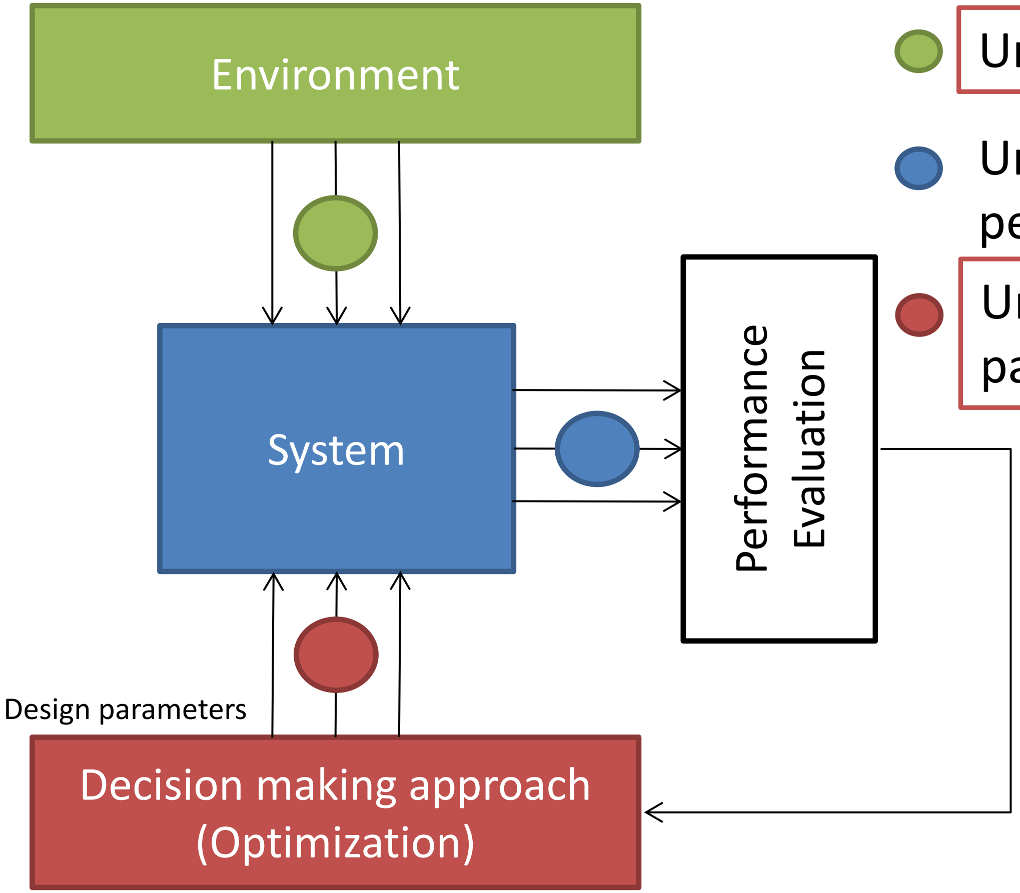
Top Level Subspace

Aircraft Sizing Subspace

Fleet Assignment Subspace

Strategy: Subspace Decomposition

Uncertainty



Environment

System

Performance Evaluation

Design parameters

Decision making approach (Optimization)

🟢 Uncertainty in operations

🔵 Uncertainty in observed system performance

🔴 Uncertainty in design parameters

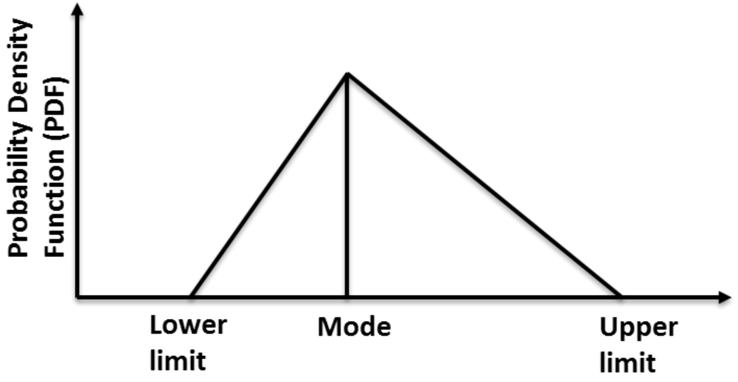
Optimization-based Approach

- **Objectives**
  - Minimize Fleet fuel consumption
  - Maximize Fleet productivity (speed of payload delivered)
- **Variables**
  - New aircraft requirements (pallet capacity, range, speed)
  - New aircraft design variables (NLP: Nonlinear Programming)
    - Wing loading, aspect ratio, thrust-to-weight ratio, etc.
  - Assignment variables (MIP: Mixed integer programming)
    - Flights, payload on a particular route
- **Constraints**
  - Cargo demand
  - Aircraft performance (takeoff distance, landing distance etc.)
  - Fleet operations (maximum operational hours, number of each aircraft types etc.)

Aircraft Design (Sizing) Uncertainty

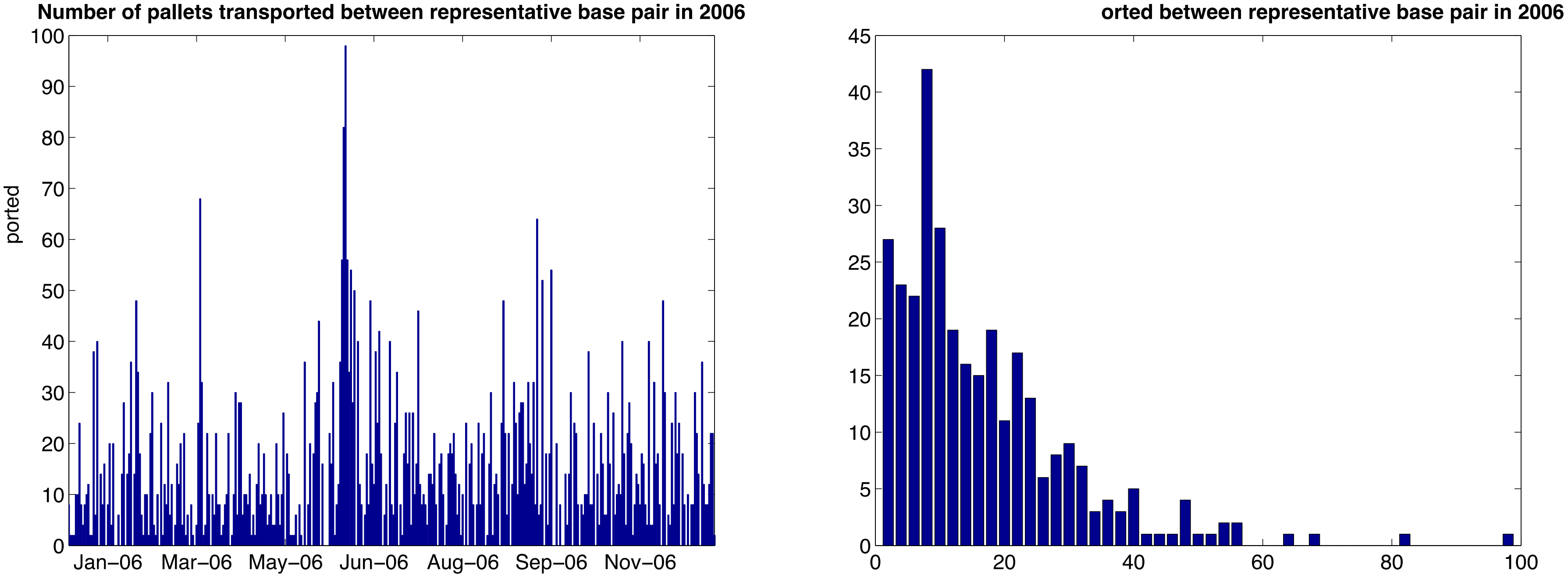
- Uncertain parameters characterized via scaling factors with triangular distributions
- Aircraft performance predictions follow distributions

$$C_{D_0} = k_{C_D} \times (C_{D_0\ predicted})$$



Uncertain Parameters (ξ)	Lower limit	Mode	Upper Limit
C_{D_0} multiplier, k_{C_D}	0.90	1.0	1.10
SFC	0.45	0.5	0.55
Oswald efficiency multiplier, k_{e_0}	0.95	1.0	1.05

Operational Uncertainty in Pallet Demand

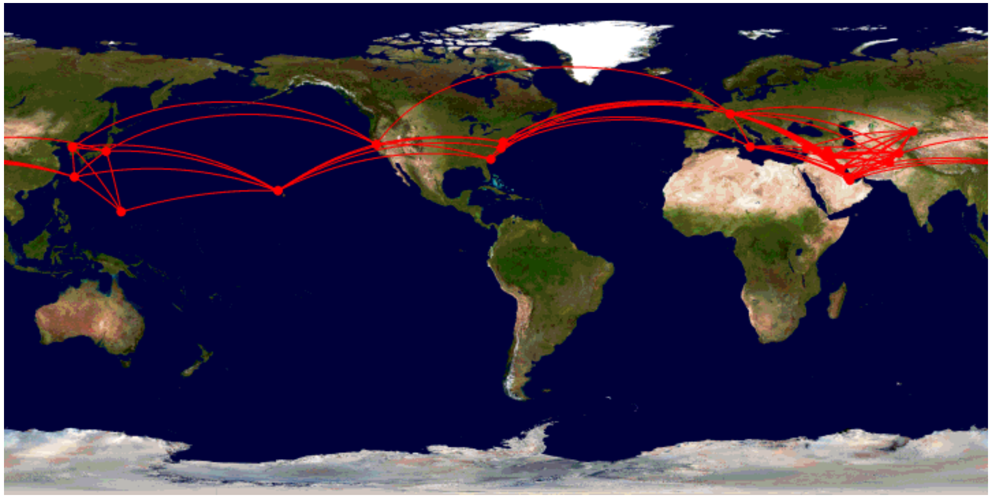


- GATES dataset shows large variation in daily cargo transported, asymmetric demand between base pairs
- From this, treat future daily pallet demand as uncertain

Approach: Handling Uncertainty

- Reliability-based design optimization (RBDO) formulation to handle *uncertainty in new system design*
- Descriptive sampling approach to handle *uncertainty in pallet demand*
- Propagation of uncertainty from aircraft sizing subspace
  - Performance of new aircraft is uncertain
  - Coefficients in assignment problem are distributions
- Used a 'Robust Optimization' approach
  - Interval Robust Counterpart (IRC) formulation: Optimize the worst-case values of parameters within an uncertainty set
  - Insensitive to data uncertainty in the problem

Case Study: 25-base Network



- Determine the requirements for a new aircraft (type X) that would improve fleet-level objectives
- 25-base problem consisting of 219 directional routes
  - Extracted from the GATES dataset, so reflects actual levels of demand
- Existing fleet for AMC
  - 28 C-5, 44 C-17, and 21 B747-F operated on 25 base subset

The fleet can add five new aircraft (all of type X)

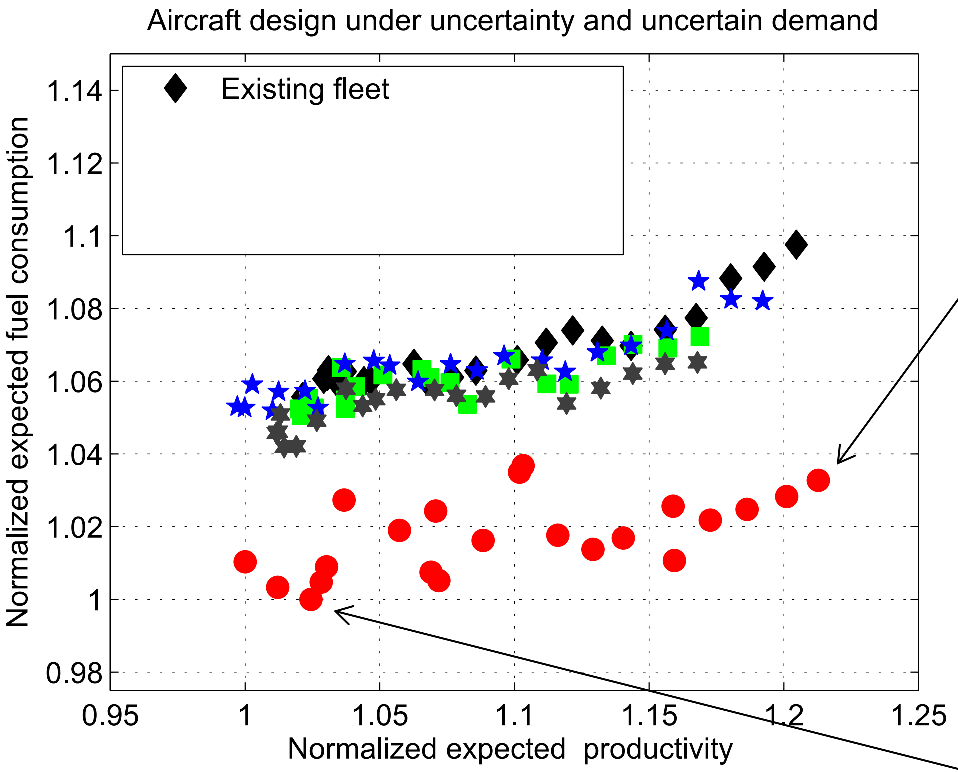
C-5

C-17

B747-F chartered from Civil Reserve Air Fleet

Source: *www.amc.af.mil*

Combined Results



New Aircraft X:
Pallet capacity = 24
Design range = 2992 nmi
Cruise speed = 550 knots
AR = 9.20 *T/W* = 0.24 *W/S* = 161 lb/ft^2
Engine BPR = 13.13
Wing Sweep = 10 deg
Taper Ratio = 0.25

New Aircraft X:
Pallet capacity = 16
Design range = 3800 nmi
Cruise speed = 549.37 knots
AR = 9.06 *T/W* = 0.24 *W/S* = 161 lb/ft^2
Engine BPR = 12.11
Wing Sweep = 10 deg
Taper Ratio = 0.30

- “Optimal” requirements and design of new aircraft to improve fleet-level capabilities
- Tradeoff of fuel consumption and productivity
- Formulation addresses uncertainty

Concluding Statements

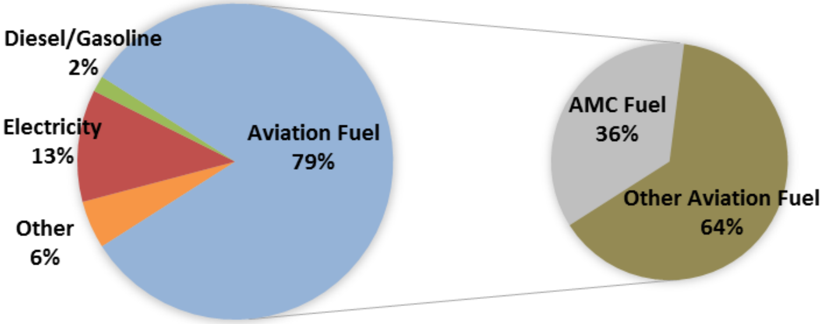
- Decision support framework to assist decision-maker or acquisition practitioner
  - Assess tradeoffs of different fleet-level metrics
  - Each tradeoff solution describes the design requirements for the new system
  - Addressed multi-domain uncertainty and uncertainty propagation
- Tradespace evaluation based on quantitative metrics
  - Shows impact of system requirements on fleet-level capabilities
  - Results here are limited by the accuracy of the aircraft sizing methodology

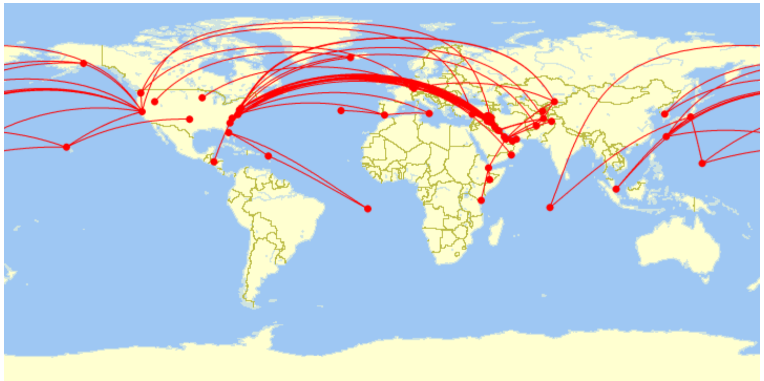
Thank You

BACKUP SLIDES

Application: Air Mobility Command (AMC)

- AMC: One of the major command centers of the U.S. Air Force
- AMC is the DoD's single largest aviation fuel consumer*
- Non-deterministic nature of AMC operations
  - Demand is highly asymmetric
  - Demand fluctuation on a day to day basis
  - Routes flown vary based on demand
- AMC's mission profile includes
  - Worldwide cargo and passenger transport**
- Used Global Air Transportation Execution System (GATES) dataset



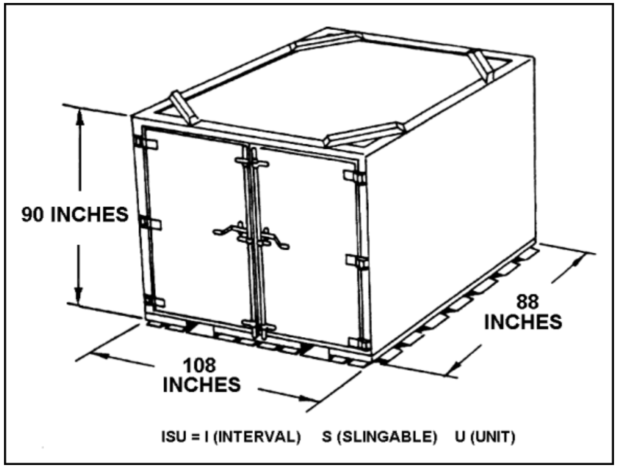


Sample route network from GATES

**Aviation fuel savings: AMC leading the charge. Air Mobility Command*
***This work only addresses cargo transport*

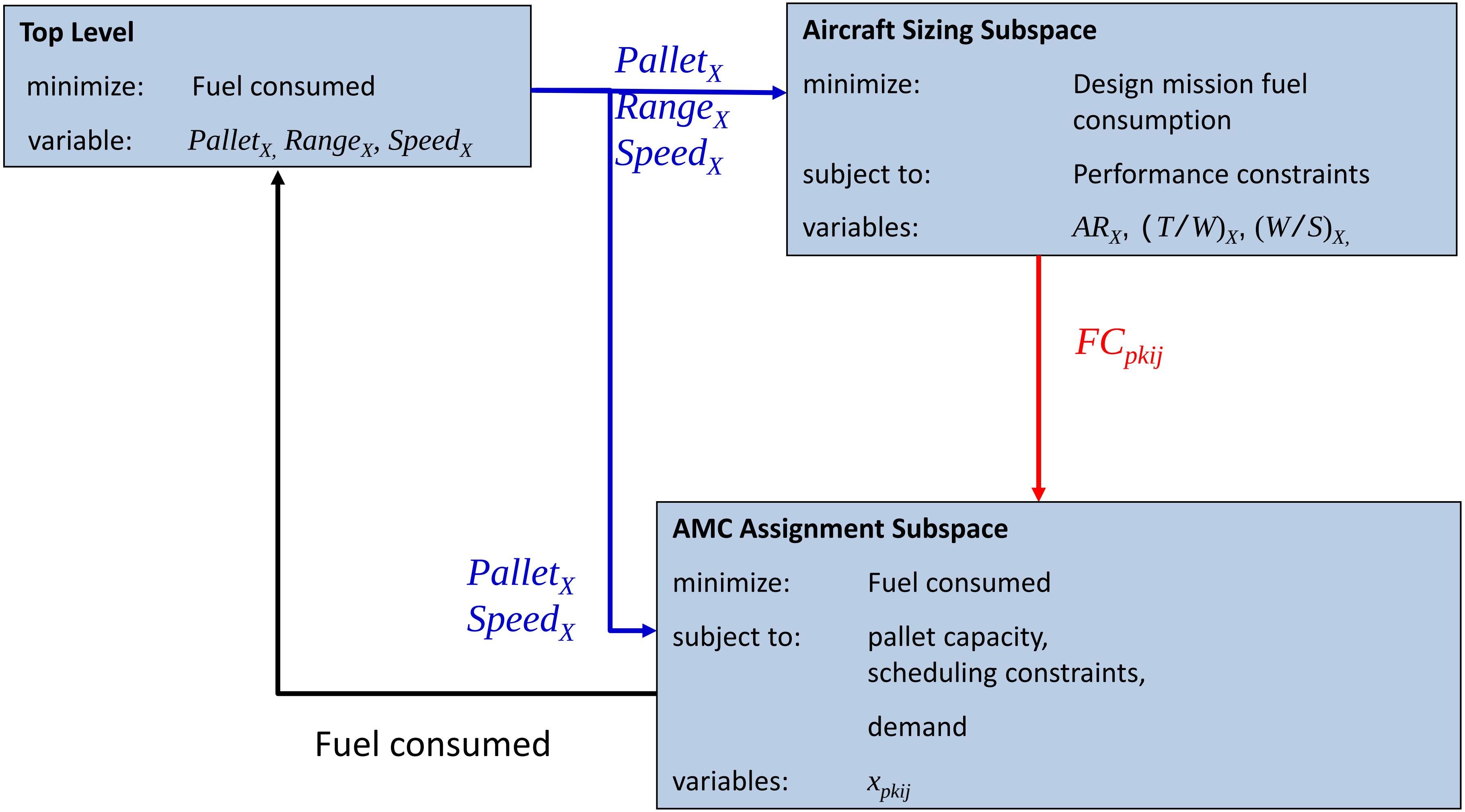
Air Mobility Command

- Used Global Air Transportation Execution System (GATES) dataset
- Filtered route network from GATES dataset
  - Demand for subset served by C-5, C-17 and 747-F (~75% of total demand)
  - Fixed density and dimension of pallet (463 L)
- Our aircraft fleet consists of only the C-5, C-17 and 747-F.



Source: *www.amc.af.mil*

Subspace Decomposition Approach (Deterministic Formulation)



Top Level

minimize: Fuel consumed

variable: $Pallet_{X}, Range_{X}, Speed_{X}$

$Pallet_{X}$
$Range_{X}$
$Speed_{X}$

Aircraft Sizing Subspace

minimize: Design mission fuel consumption

subject to: Performance constraints

variables: $AR_{X},\ (T/W)_{X},\ (W/S)_{X},$

FC_{pkij}

$Pallet_{X}$
$Speed_{X}$

AMC Assignment Subspace

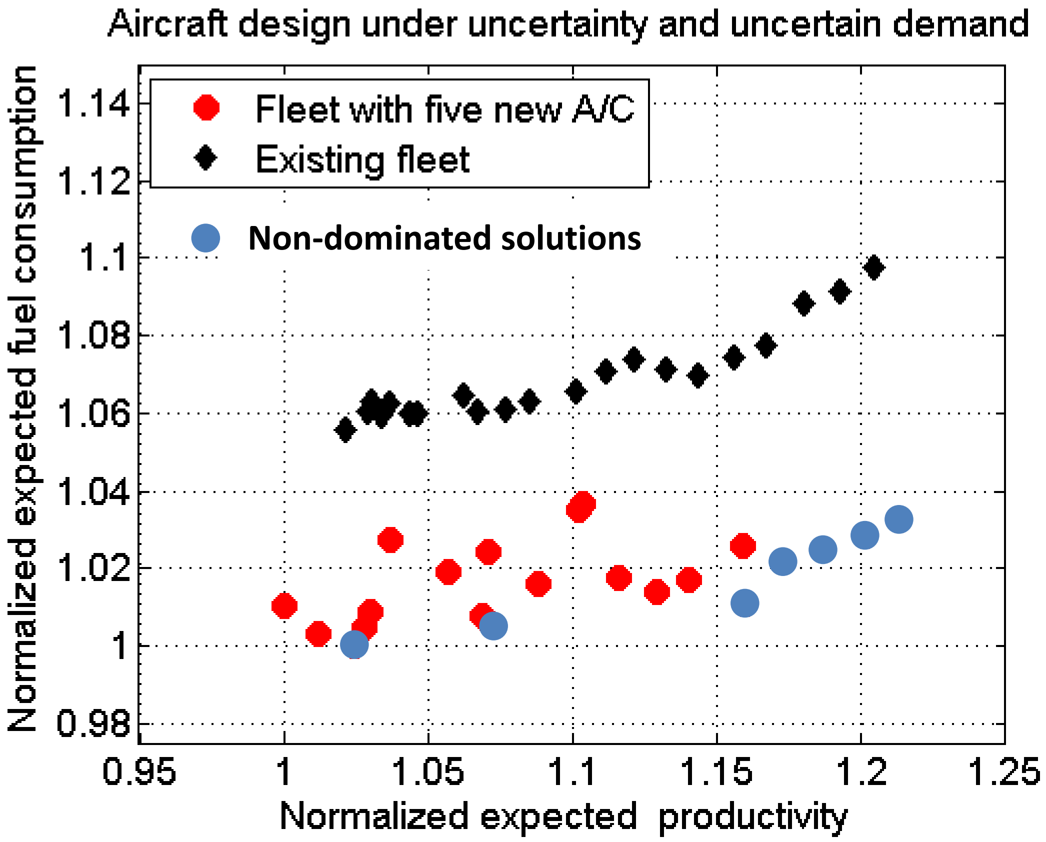
minimize: Fuel consumed

subject to: pallet capacity, scheduling constraints, demand

variables: x_{pkij}

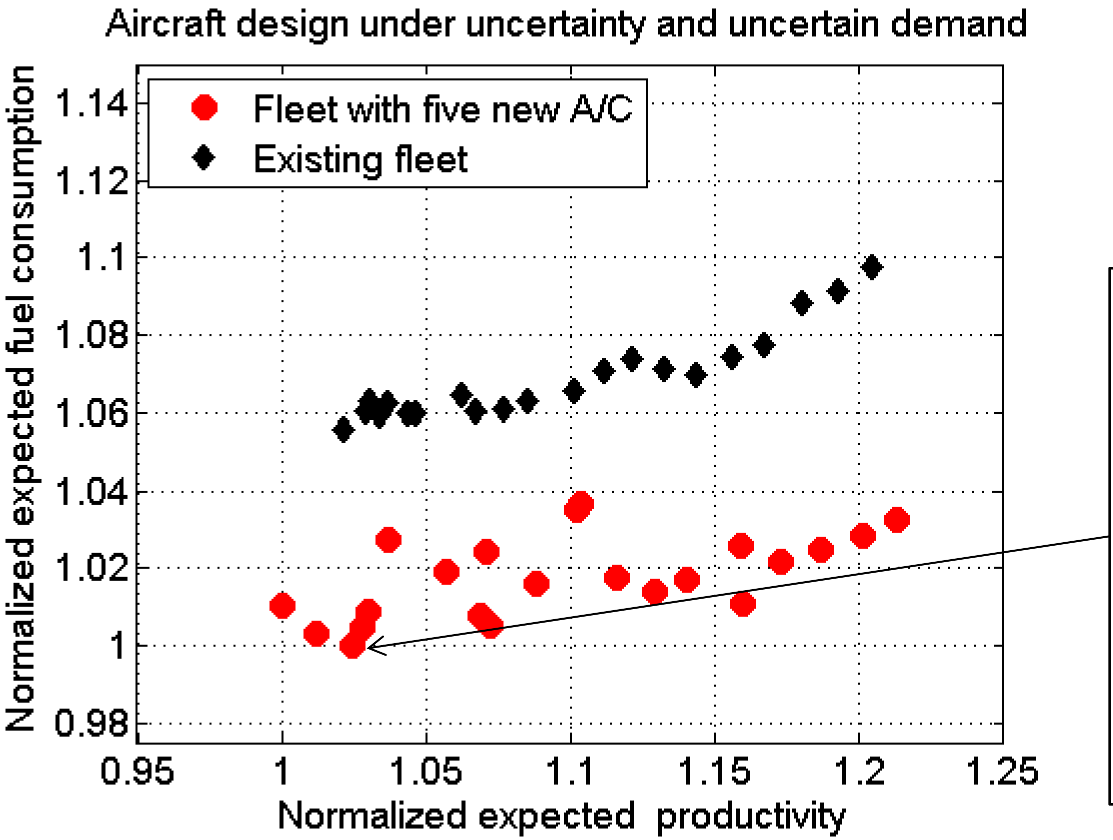
Fuel consumed

Results: 25-base Network



Non convex Pareto front
Some non-dominated solutions

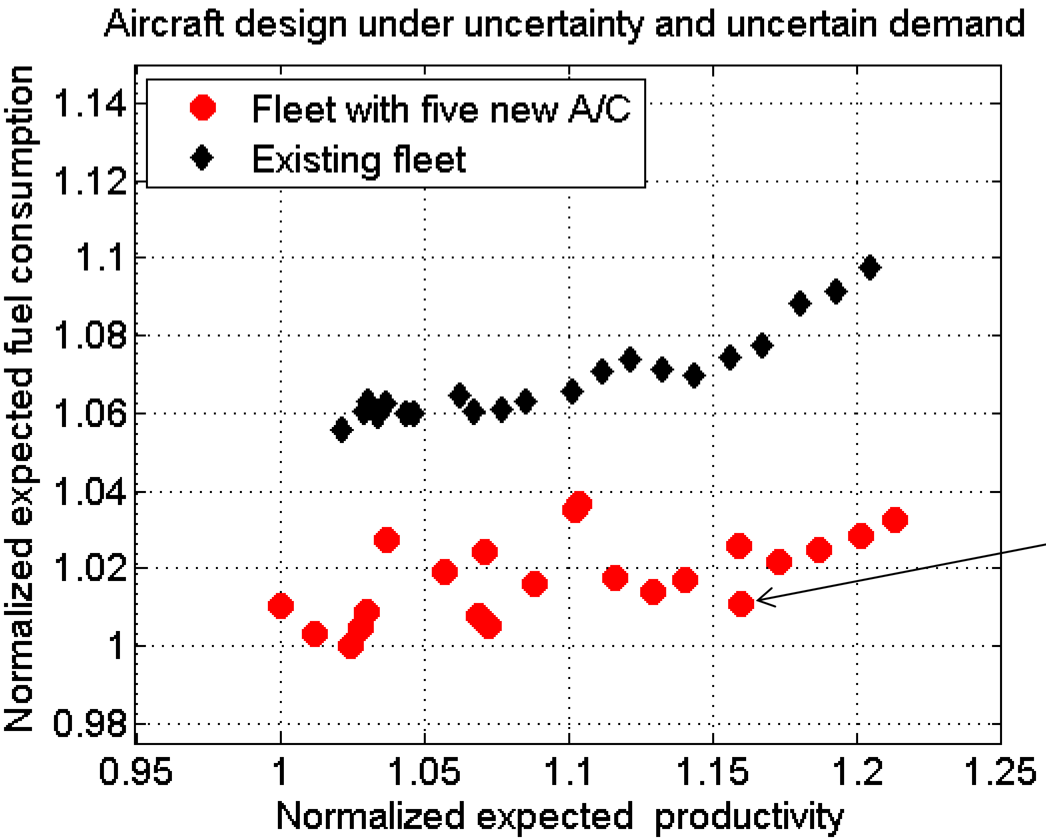
Results: 25-base Network



New Aircraft X:

Pallet capacity = 16
Design range = 3800 nmi
Cruise speed = 549.37 knots
AR = 9.06
T/W = 0.24
W/S = 161 lb/ft^2
Engine BPR = 12.11
Wing Sweep = 10 deg
Taper Ratio = 0.30

Results: 25-base Network



New Aircraft X:

Pallet capacity = 17

Design range = 3800 nmi

Cruise speed = 525.28 knots

AR = 9.37

T/W = 0.24

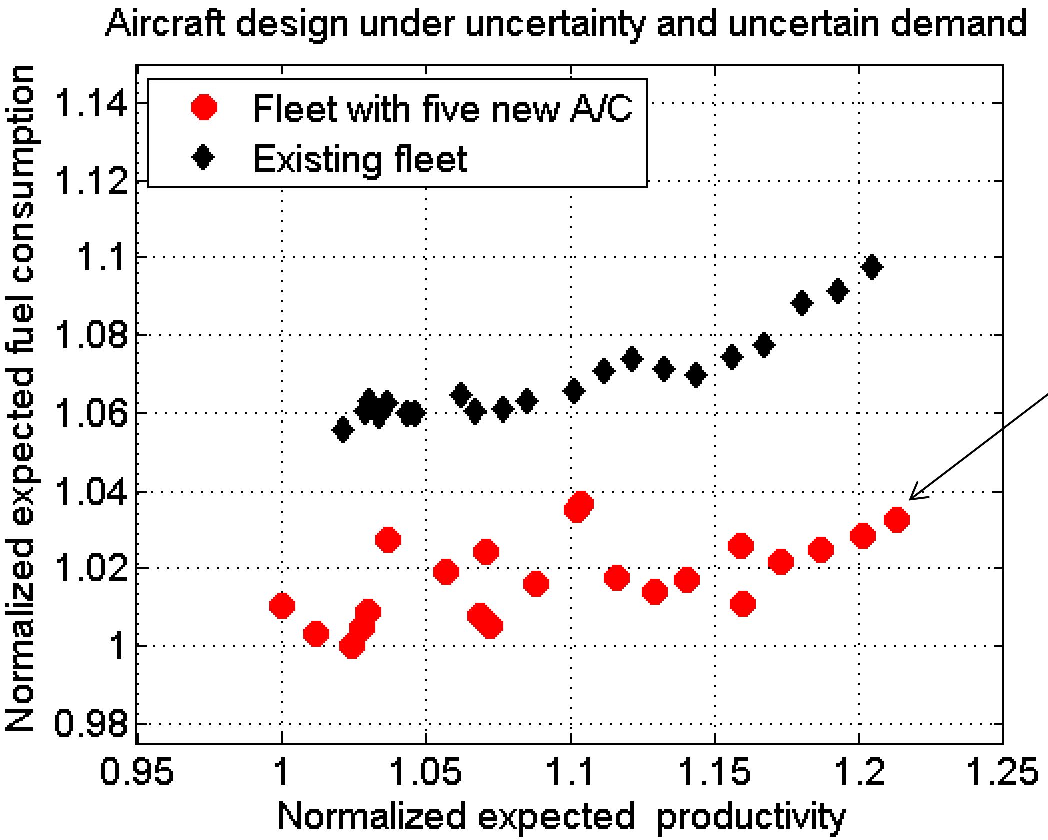
W/S = 161 lb/ft^2

Engine BPR = 12.92

Wing Sweep = 10 deg

Taper Ratio = 0.26

Results: 25-base Network



New Aircraft X:

Pallet capacity = 24

Design range = 2991.7 nmi

Cruise speed = 550 knots

AR = 9.2

T/W = 0.24

W/S = 161 lb/ft^2

Engine BPR = 13.13

Wing Sweep = 10 deg

Taper Ratio = 0.25

Subspace Decomposition Approach (Deterministic Formulation)

Top level subspace	Minimize	Fleet fuel consumption
	Subject to	Bounds on $Pallet_X$, $Range_X$, $Speed_X$
Aircraft sizing subspace	Minimize	Fuel consumption of Aircraft X for design mission
	Subject to	Performance constraints Bounds on AR, W/S, T/W
Fleet assignment subspace	Minimize	Fleet fuel consumption
	Subject to	Demand constraints Node balance constraints Starting location of aircraft constraints Daily utilization limits Trip limits $x_{pkij} \in \{0,1\}$

25-base, 219-route Network

- Top level
  - Three decision variables
  - Bounds on decision variables
- Aircraft sizing
  - Six continuous decision variables
  - Four nonlinear constraints
  - Five uncertain parameters
  - Bounds on decision variables
- Fleet assignment
  - 183,750 binary decision variables
  - 134,203 constraints
  - Uncertainty in pallet demand on each route along with uncertainty propagation from aircraft sizing

INTERVAL ROBUST COUNTERPART MODEL

Deterministic Formulation

$$
\begin{aligned}
&Minimize: && c'x \\
&Subject\ to: && Ax \le b \\
& && x_j \in \{0,1\}
\end{aligned}
$$

$c: n-vector,\ b: m-vector,\ A: m \times n\ matrix$

IRC Model

- (ε, δ)-Interval Robust Counterpart (IRC) formulation* for bounded uncertainty
  - δ: infeasibility tolerance, ε – data uncertainty

$$\left|\widehat{a_{ij}} - a_{ij}\right| \leq \varepsilon\left|a_{ij}\right|, \left|\widehat{b_i} - b_i\right| \leq \varepsilon|b_i|$$

  - Uncertainty in objective function: Transform objective function as constraint
  - ε and δ can change for each constraint

- A solution $\boldsymbol{x}$ is **robust** if
  - x is feasible for the nominal values
  - Whatever are the true values of the coefficients and RHS parameters within the corresponding intervals, must satisfy the i-th inequality constraint with an error at most $\delta \times \max(1, b_i)$

Lin et al., A new robust optimization approach for scheduling under uncertainty: I. Bounded uncertainty

IRC $[\varepsilon, \delta]$ Formulation

$$Minimize: \quad c'x$$

$$Subject\ to: \quad Ax \le b$$

$$\sum_{j \notin J_i} a_{ij} x_j + \sum_{j \in J_i} \left(a_{ij} + \varepsilon \left| a_{ij} \right| \right) x_j \le b_i - \varepsilon \left| b_i \right| + \delta_i \times \max\left(1,\ \left| b_i \right| \right) \quad \forall i$$

$$x_j \in \{0,1\}$$

$c : n-vector,\ b : m-vector,\ A : m \times n\ matrix$

J_i : set of indices of the x variables with uncertain coefficients in the i-th inequality constraint

- The additional constraints consider the worst-case values of the uncertain parameters
  - With tolerable violations of the constraint
  - Enforced using user-defined factors, δ_i

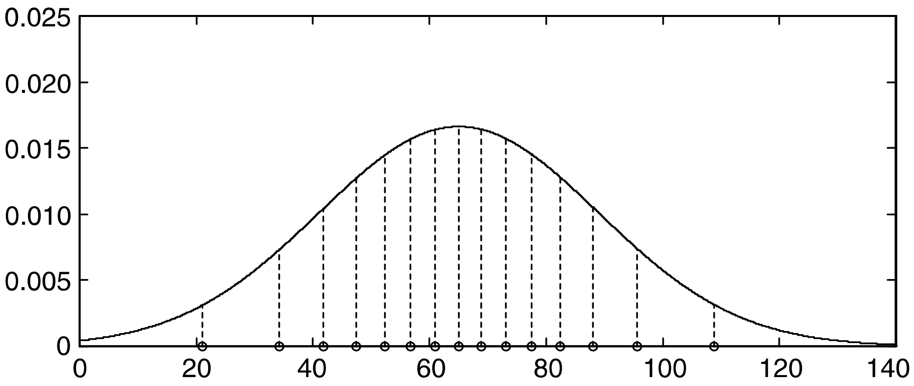
Demand Uncertainty

- Applying IRC model to the demand constraint
  - ‘Immunized’ against the worst-case scenario (maximum value) of demand
  - Leads to a ‘conservative’ solution
- Instead, handled through a stratified sampling technique to reduce computational expense
  - On-demand nature of fleet operations
  - Large fluctuations in pallet demand

How can our approach help AMC?

- Our methodology
  - Helps determine the requirements for – and describe the design of – a new aircraft for use in the AMC fleet
  - Optimize fleet-level metrics that address performance and fuel use
  - Account for uncertainties in fleet operations and new aircraft performance
- Describe how design requirements of the new aircraft would change for different tradeoff opportunities between productivity and fuel consumption

Descriptive Sampling



$$dem_{i,j}[a] = F_{i,j}^{-1}\left(\frac{a-0.5}{B}\right) \qquad a = 1,2,3\ldots,B.$$

Random sampling = random set × random sequence
Descriptive sampling = deterministic set × random sequence

- Discretize the distribution to generate B demand scenarios
  - Sample more from high-density and less from low-density regions
- Random permutation of the demand values for each route

Saliby, E., *"Descriptive sampling: A better approach to Monte Carlo simulation"*

Listes, O. and Dekker, R., "A scenario aggregation–based approach for determining a robust airline fleet composition for dynamic capacity allocation"

Aircraft Sizing Problem

Decision variables	Lower Bound	Upper Bound
Wing Aspect Ratio	6.00	9.50
Thrust-to-weight Ratio	0.18	0.35
Wing Loading [lb/ft^2]	65.00	161.00
Engine Bypass Ratio	4.50	14.50
Wing Leading Edge Sweep [deg]	10.00	35.00
Wing Taper Ratio	0.10	0.40
Constraints	**Value**	
Takeoff Distance [ft]	≤ 8500	
Landing Distance [ft]	≤ 5500	
Second segment climb gradient	≥ 0.025	
Top-of-climb rate [ft/min]	≥ 500	

Uncertain Parameters: C_{D_0} multiplier, SFC, Cruise altitude, Pallet mass, Oswald efficiency multiplier

Fleet Assignment Subspace

Minimize

$$\sum_{p=1}^{P}\sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{N} x_{p,k,i,j}\cdot C_{p,k,i,j}$$

Fleet-level DOC

Subject to

$$\sum_{i=1}^{N} x_{p,k,i,j} \geq \sum_{i=1}^{N} x_{p,k+1,i,j} \quad \forall k=1,2,3\ldots K,$$

$$\forall p=1,2,3\ldots P, \quad \forall j=1,2,3\ldots N$$

Node balance constraints

$$\sum_{p=1}^{P}\sum_{k=1}^{K} Cap_{p,k,i,j}\cdot x_{p,k,i,j} \geq dem_{i,j}$$

$$\forall i=1,2,3\ldots N, \forall j=1,2,3\ldots N$$

Demand constraints

$$\sum_{i=1}^{N} x_{p,1,i,j} \leq O_{p,i} \quad \forall p=1,2,3\ldots P, \forall i=1,2,3\ldots N$$

Starting location of aircraft constraints

$$\sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{N} x_{p,k,i,j}\cdot BH_{p,k,i,j} \leq B_P \quad \forall p=1,2,3\ldots P$$

Daily utilization limit

$$\sum_{i=1}^{N}\sum_{j=1}^{N} x_{p,k,i,j} \leq 1 \quad \forall p=1,2,3\ldots P, \forall k=1,2,3\ldots K$$

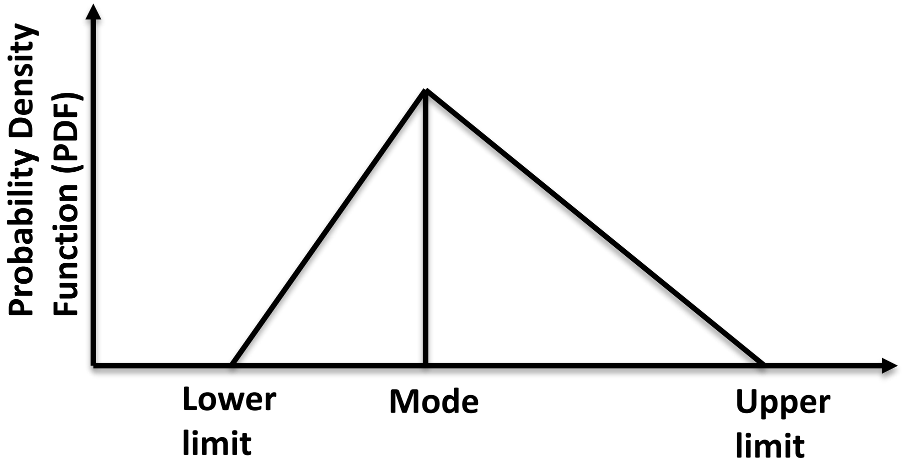
Trip limit

$$x_{p,k,i,j} \in \{0,1\}$$

Boolean Variable

Uncertainty in Aircraft Sizing

- Two major types of uncertainty
  - **Aleatoric uncertainty:** Inherent or natural randomness
  - **Epistemic uncertainty:** Imprecise or absence of complete information
- Some uncertain parameters used as scaling factors
- Represented using assumed triangular distributions

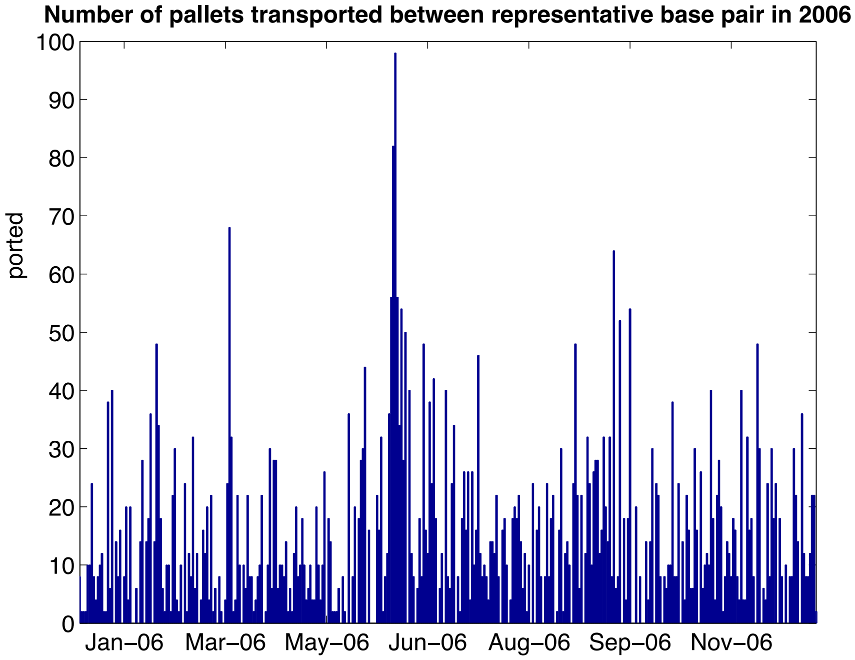


$$C_{D_0} = k_{C_D} \times (C_{D_0 \ predicted})$$

Uncertain Parameters (ξ)	Lower limit	Mode	Upper Limit
C_{D_0} multiplier, k_{C_D}	0.90	1.0	1.10
SFC	0.45	0.5	0.55
Oswald efficiency multiplier, k_{e_0}	0.95	1.0	1.05

Uncertainty in Pallet Demand

- Reported AMC operations show large variations in daily cargo transported and asymmetrical cargo demand between base pairs
  - From this, treat future daily pallet transport demand as uncertain
  - Demand must address direction in route network



Actual Data from GATES

Multi-objective Formulation

- Two objectives
  - Maximize fleet-level productivity
  - Minimize fleet-level fuel consumption
  - Epsilon (Gaming) constraint formulation