URI-LM-13-100

ACQUISITION RESEARCH PROGRAM
SPONSORED REPORT SERIES

Acquisition Research for Design and Service Enterprises

02 February 2014

Dr. Manbir Sodhi, Professor

Department of Mechanical, Industrial, and Systems Engineering

University of Rhode Island

Prepared for the Naval Postgraduate School, Monterey, CA 93943.

The research presented in this report was supported by the Acquisition Research Program of the Graduate School of Business & Public Policy at the Naval Postgraduate School.

Executive Summary

In organizations responsible for the design, service, and management and distribution of products, acquisition decisions and product upgrades must be synchronized with service tasks and fleet operations. In partnership with the U.S. Navy's torpedo enterprise, this research investigates operations and acquisition concepts for such organizations using mathematical and simulation models.
The tasks proposed in this project are as follows:

- **Modeling of functions representing deterioration patterns governing the efficiency of real systems and development of solution methods for optimization using these functions.** This was to include the development of system efficiency metrics, factoring in the impact of obsolescence and appropriate risk mitigations strategies to be designed and evaluated.
- **Development of algorithms for solving obsolescence problems with realistic dimensions.**
- **Models for determining and evaluating acquisition decisions and product upgrades which must be synchronized with service tasks and fleet operations while maintaining an effective inventory were to be constructed.**

As stated in the original proposal, several research theses were completed on related topics. The following are some of the theses which have already been completed:

- Schulze, M. (2010). *Optimization using common part strategies in closed-loop supply chains*, MS thesis, Department of Manufacturing, Industrial and Manufacturing Systems, The University of Rhode Island, Kingston, RI. Major professor: Dr. Manbir Sodhi.
- Hanisch, C. (2010). *Maximizing component reuse for sustainable operations*, MS thesis, Department of Manufacturing, Industrial and Manufacturing Systems, The University of Rhode Island, Kingston, RI. Major professor: Dr. Manbir Sodhi.
- Vollenberg, F. (2010). *Optimization of system efficiency with consideration of deterioration processes*, MS thesis, Department of Manufacturing, Industrial and Manufacturing Systems, The University of Rhode Island, Kingston, RI. Major professor: Dr. Manbir Sodhi.

- Husen, T. (2011). *Development of solution approaches for optimally solving the stochastic parallel machine replacement problem*, MS thesis, Department of Manufacturing, Industrial and Manufacturing Systems, The University of Rhode Island, Kingston, RI. Major professor: Dr. Manbir Sodhi.
- McKeon, B. (2011). *Utilizing modular upgrades to maintain an effective inventory of complex machines*, PhD dissertation, Department of Manufacturing, Industrial and Manufacturing Systems, The University of Rhode Island, Kingston, RI. Major professor: Dr. Manbir Sodhi.

Additional work on concepts, models, and algorithms has been and is currently being investigated by the principal investigator and by graduate students at the University of Rhode Island, including the following:

1. Vaziri, M., & Sodhi, M. (2013). *Spare parts management and game theory*, Working paper, Department of Mechanical, Industrial and Systems Engineering, The University of Rhode Island, Kingston, RI.
2. Vaziri, M., & Sodhi, M. (2013). *Applications of game theory to inventory level decision making*, Working paper, Department of Mechanical, Industrial and Systems Engineering, The University of Rhode Island, Kingston, RI.

The narrative that follows summarizes some of the research results and is direclty related to the thesis reports.

Keywords: optimization model, system efficiency, torpedo enterprise, component replacement, lot sizing, inventory

About the Author

Manbir Sodhi is a professor of systems and industrial engineering, Department of Mechanical, Industrial, and Systems Engineering, University of Rhode Island. Professor Sodhi obtained his graduate degrees from the University of Arizona and has taught courses in systems design, systems simulation, and deterministic and stochastic optimization, among others. In addition to consulting for several companies, he has worked as a visiting scientist at the Naval Undersea Warfare Center Division, Newport, RI, and at the NATO Undersea Research Center in La Spezia, Italy. His recent work has appeared in professional journals that include the *Journal of Scheduling*, *International Journal of Production Research*, and *IIE Transactions*. He is currently exploring decision models supporting supply chain planning in defense operations and is developing tools and concepts of operations for the use of unmanned undersea vehicles (UUVs) for a variety of search operations.

Dr. Manbir Sodhi
Department of Mechanical, Industrial, and Systems Engineering
University of Rhode Island
Kingston, RI 02881
Tel: 401-874-5189
Fax: 401-874-2355
Email: sodhi@egr.uri.edu

THIS PAGE INTENTIONALLY LEFT BLANK

URI-LM-13-100

ACQUISITION RESEARCH PROGRAM
SPONSORED REPORT SERIES

Acquisition Research for Design and Service Enterprises

02 February 2014

Dr. Manbir Sodhi, Professor

Department of Mechanical, Industrial, and Systems Engineering

University of Rhode Island

THIS PAGE INTENTIONALLY LEFT BLANK

Table of Contents

THIS PAGE INTENTIONALLY LEFT BLANK

List of Figures

THIS PAGE INTENTIONALLY LEFT BLANK

List of Tables

THIS PAGE INTENTIONALLY LEFT BLANK

Acquisition Research for Design and Service Enterprises

Modeling Replacement Decisions for System's Efficiency

As products become increasingly more complex and have shorter life cycles, common part strategies allow companies to use components across families of products and improve the reliability of machines and products in service. The use of common parts for different product families makes it possible to manage complexity by re-using recovered components multiple times. This is certainly the case for a large number of defense-related products including torpedoes, aircraft, ships, and other durable products. Thus, common parts allow an increase of the product variety whilst keeping complexity controllable and manageable.

In order to keep systems at acceptable levels of efficiency, a decision-maker must know the optimal points of time when components within a system can be replaced.

Figure 1 helps clarify the concept of components reuse and replacement.

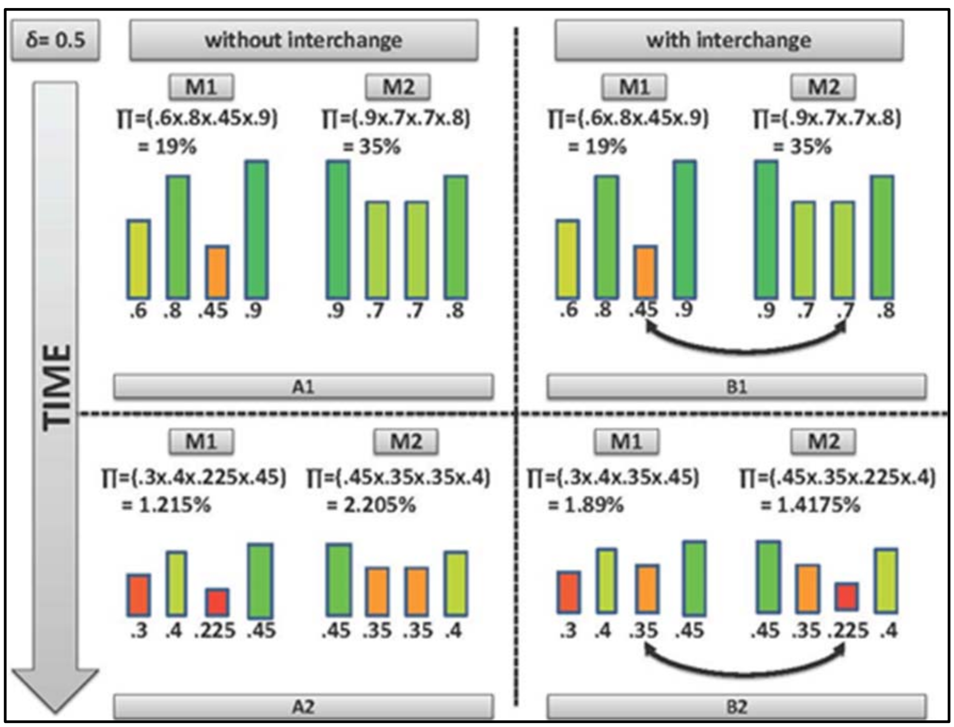

Figure 1. Components Reuse and Replacement Illustration

Two machines (M1 and M2) are each equipped with four components of the same type. All components are in a different condition. Without a component interchange, the components deteriorate with the deterioration factor $\delta = 0.5$, as shown on the left side (A1 to A2) in Figure 1. Since the performance of the entire machine is equal to the product of its components deterioration, M1 deteriorates from 19% to 1.215% and M2 deteriorates from 35% to 2.205%.

In order to increase the reliability and to avoid a possible malfunction, a component could be simply interchanged, as shown on the right side in Figure 1. Thus, the machine with the worst performance has a higher level of efficiency after its deterioration (B2) than in the case of no interchange (M2 = 1.4175 in A2 > M1 = 1.215 in B2).

In other words, the performance level of the machine with the worst performance in the case of an interchange (M2 in B2) is better than the performance level of the worst machine in the case of no interchange (M1 in A2). This phenomenon—due to a component interchange—increases the system performance significantly.

In order to find an optimal common part strategy, a decision-maker must know both the optimal replacement type and the optimal replacement time. However, according to the definition above, it does not make sense to interchange components in the case of a single machine in order to increase the overall reliability and efficiency. Hence, in the single-machine case, the challenge is to figure out the optimal points in time for replacing components by new ones. In the multiple-machines case, a decision-maker must know both, the replacement times of used components by other used ones as well as new ones.

A comprehensive literature review of obsolescence, replacements, and closed loop supply chains can be found in Shulze (2010), Christian (2010), Vollenberg (2010), McKeon (2012), and Husen (2012). It is clear that although the existing literature has dealt with several variants of the part replacement problem, a systems efficiency approach for optimizing repair, replacement and refurbishments has not been developed. Before presenting some of the models developed in this research, some basic definitions need to be established (see Christian, 2010; Husen, 2012; Shulze, 2010; Vollenberg, 2010).

After used products and components are returned to the original equipment manufacturer (OEM), several opportunities are available to reutilize the products or components. For instance, remanufacturing is one possible recovery option. All the options considered for this research, with the exception of reuse, improve the condition of the components. The following are the options considered:

- **Reuse:** The reuse of a component implies that the component will be reintroduced into the market as-is. For instance, a component can be dismantled from a machine and installed into a different one. For this choice, it is assumed that the component will operate without reduction of performance. In the case of a component's reuse, the condition remains constant.
- **Repair:** The repair of a component may be necessary if the component cannot perform its original function. After repairing a component, its original state is assumed to be reached again.
- **Refurbishment:** The refurbishment of a component improves its condition to a certain level of efficiency, but does not restore it completely. Assuming that a new component has a performance rating (or an efficiency) of 100%, then the level of efficiency after refurbishment is between the condition level before refurbishment and at most 100%, but never equal to 100%.
- **Remanufacturing:** A remanufactured component improves the condition to a level between the former condition level and at most 100%, but never exceeds 100%. In contrast to refurbishment, the performance upgrades the condition of a new machine. Thus, the level of performance after remanufacturing can be better than refurbishment.
- **Replacement:** Replacing a component means to swap in a new component. Consequently, the efficiency after replacement is equal to 100%.
- **Redesign:** The above options are commonly considered in the literature. This research defines an additional option—redesign. A redesigned component achieves an efficiency in excess of 100%. This may be because of improved materials used, improved manufacturing processes, changes to the structure, and so forth, giving a redesigned component the possibility of reaching a level of efficiency greater than 100%.

Table 1 summarizes the different condition-improvement options. The system efficiency specifies the current condition of the component after applying the specific option.

Table 1. Recovery Options

Option	System Efficiency (SE) After Accomplishing Option
Reuse	System Efficiency (SE) = former condition < 1
Repair	SE = former condition < 1
Refurbish	former condition < SE < 1
Remanufacture	former condition < SE < 1
Replace	SE = 1
Redesign	SE > 1

Optimization Models for Maximizing the System Efficiency Over a Finite Time Horizon

In this section, some optimization models for selecting from the various choices of decisions listed above are developed. These are recounted in greater detail in Shulze (2010), Christian (2010), Vollenberg (2010), McKeon (2012), and Husen (2012), and in working papers referred to in the beginning of this paper.

The traditional way to improve the condition or efficiency of a machine is the replacement of degraded components by new ones. The trouble in this context is that the regarded framework is drawn too narrowly in many practical applications. In the case of a multiple-machines consideration, a scheduler considers every machine separately in many cases. Even though a single machine should be improved, the consideration refers in many cases to just two periods.

The replacement models are developed according to a categorization shown in Figure 2.

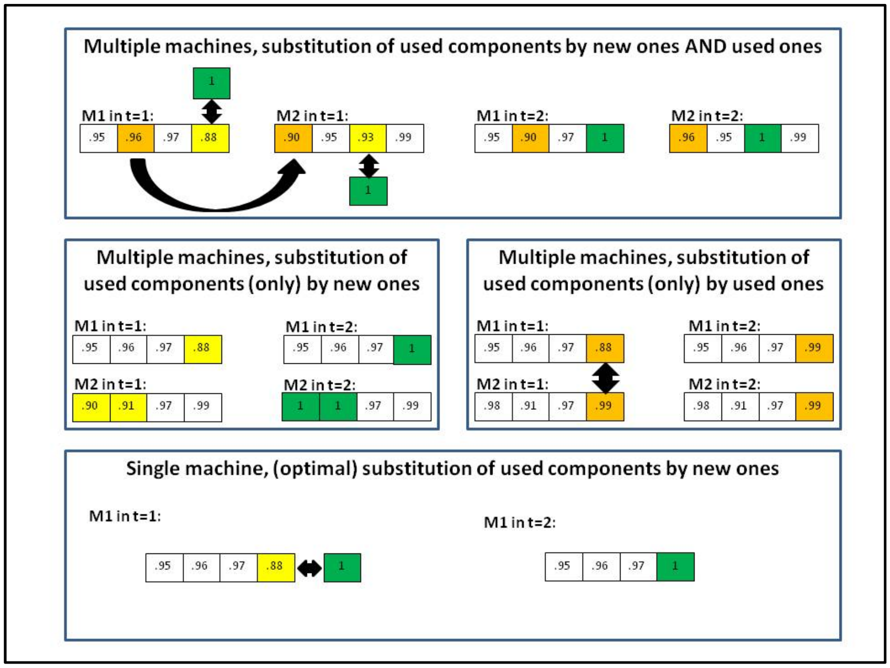

Figure 2. Replacement Scenarios

The initial models consider replacements within a single machine. It is constrained by a given budget for each period as well as a deterioration function for each component in every single period. Subsequently, the model is extended to a multiple-machines model. This model determines the time for optimal replacement of used components by new ones. The goal is to achieve an optimal level of aggregated system efficiency, in consideration of given constraints. While the previous multiple-machines model just allows the replacement of used components by new ones, the next step is the development of a model that simulates the interchange of different used components. In other words, the third model deals with the interchange of modular, different used components within different machines. The last model combines both, the optimal replacement time of used components by new ones and the replacement by different used components.

Single-Machine Component-Replacement Problem

The first analysis deals with the optimal components replacement strategy for a single machine. The is called the Single-Machine Component-Replacement Problem (SMCRP), and it computes the optimal points in time for the replacement of common parts and modular components to maximize the machine's overall efficiency over a defined period of time, under given budget constraints. The problem is illustrated in Figure 3. The goal is to find the optimal components for replacement at an optimal time.

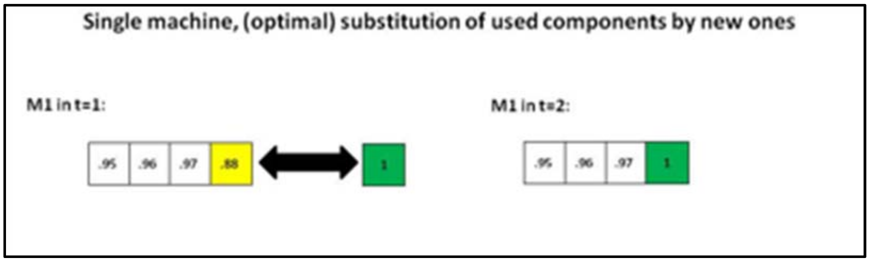

Figure 3. Single-Machine Component-Replacement Problem

Mathematical Model

Assuming that a machine is made of changeable or modular components, the following notation is used:

The efficiency of a component as well as of the entire machine is defined as a percentage. A new component has a defined maximum efficiency of 100% or 1.0. All used components offer an efficiency greater than 0% but less than 100%.

Decision Variables:

$x_{c,t}$ = 1, if component c is selected for upgrade in period t
$x_{c,t}$ = 0, otherwise

Variables:

$e_{c,t}$ = efficiency of component c in period t
SE_t = overall system efficiency in period t = machine efficiency in period t
Z = least system efficiency

Parameters:

ζ_c = cost per component change
δ_c = deterioration
β_c = budget

Sets:

M = set of machines m_1 to m_M
C = set of components c_1 to c_C
N = set of copies of C n_1 to n_N
T = set of times in quarters t_1 to t_T

Objective:

Based on the definition, that the overall efficiency is equal to the worst machine's efficiency over all periods, the objective is the maximization of the least efficient system, Z, for each period t in order to maximize the overall system efficiency S_t. In the case of a components series connection, the efficiency of the machine can be assumed as the product of the different components' efficiencies. Thus, the objective can be formulated as the following:

Maximize Minimize System Effectiveness = Z

Constraints:

Every component is subject to deterioration. This deterioration can mathematically be formulated as the multiplication of the components efficiency by a defined deterioration factor δ_c. The starting condition for each component can be arbitrary, and is assumed to be 100% here. The deterioration of the next period is defined as the product of the component's efficiency in the prior period. Hence, the efficiency or condition of a component *c* in each period *t* is either—in the case of no replacement—the age efficiency expressed by the multiplication as described above or 1 in the event of a replacement. This replacement can be mathematically formulated as

$$e_{c,t} = \max \{x_{c,t},\ e_{c,t} \times e_{c,(t-1)}\} \qquad \forall\, c,\, t\,. \tag{1}$$

The last step is the consideration of the replacement costs. By definition, ζ_c is the fixed cost component of the replacement of component *c*. The expenses from all component replacements must be less than a given budget β_t in every period ***t***. This can be formulated as the following:

$$\beta_t \geq \sum_{c=1}^{C} (X_{c,t} \times \zeta_c). \tag{2}$$

The budget constraint assumes that any money that is not spent will not be transferred into the next period, which is typical of defense budgets. This assumption is valid for all following considerations.

Multiple-Machine Component-Purchase Problem

The model above is now extended by including additional machines. Several cases have been considered, as described below.

The first model for multiple machines, called Multiple-Machine Component-Purchase Problem (MMCPP), deals with the optimal time of a component's exchange with a new component. The problem is demonstrated in Figure 4.

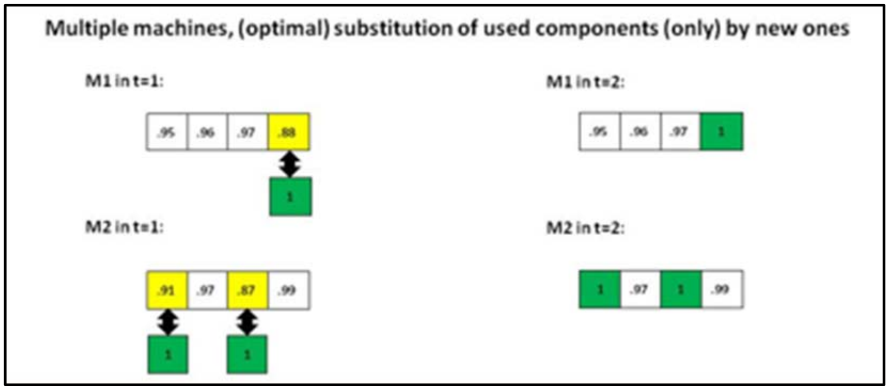

Figure 4. Multiple-Machine Component-Purchase Problem

A discussion on the formulation and solution of this model is in Schulze (2010) and Christian (2010).

Multiple-Machine Component-Interchange Problem

In the Multiple-Machine Component-Interchange Problem (MMCIP), components can be moved between machines. This is shown in Figure 5.

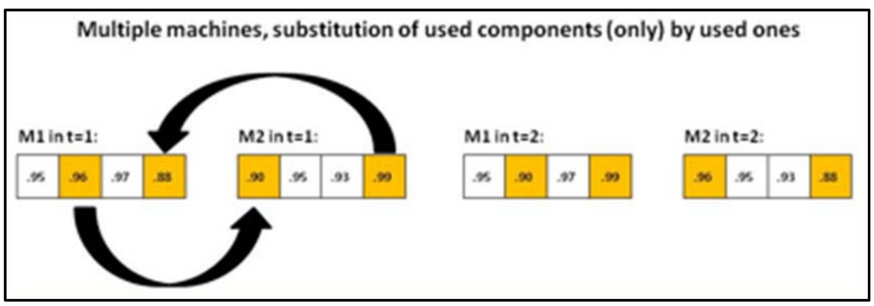

Figure 5. Multiple-Machine Component-Interchange Problem

In order to distinguish between replacement of used components, every component has to be labeled more specifically. Because each component type exists more than once in the whole system, it is necessary to introduce an additional index. The introduction of a set of copies N permits a distinct labeling. The nomenclature developed is shown in Figure 6, with the component type c and the copy of this type n defining the specific component. To complete the exact identification of each component, the identification is expanded by the machine and the time period. Hence, it is now possible to locate every single component in the overall system by referring to it using the nomenclature shown in Figure 6.

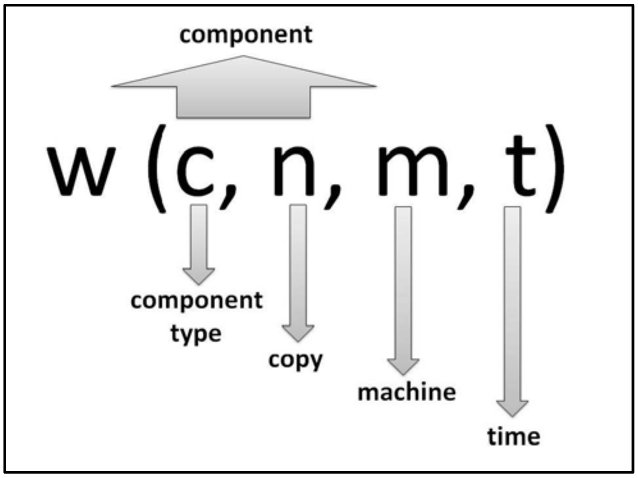

Figure 6. Component Nomenclature

Figure 7 illustrates the use of the nomenclature. In this example, two machines are shown as a closed system. Each machine is composed of four subsystems/components. There are two component types, c_1 and c_2. Each machine is made up of two components of each type; hence, the closed system—both machines include four components of each of the two types, or each component type *c* has four copies *n*. In this way, it becomes possible to address each component clearly. This is essential for developing an optimization program.

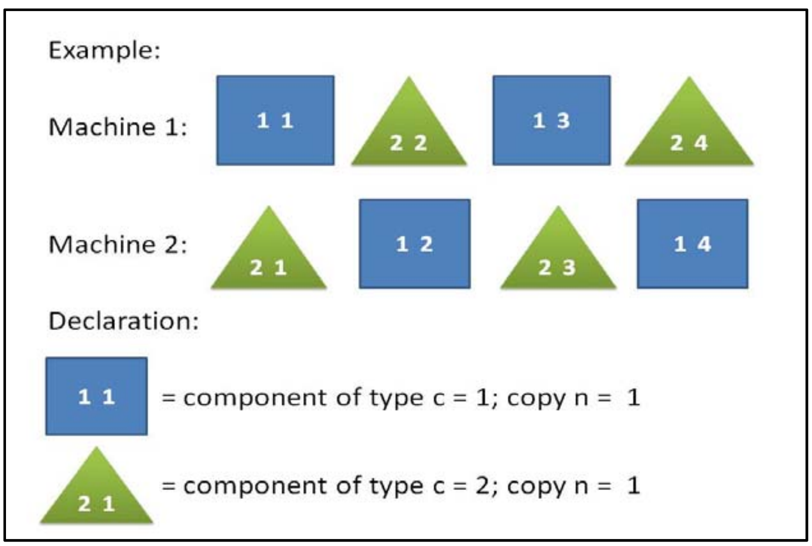

Figure 7. Illustration for Two Machines Each With Four Components of Two Different Types

Implementing a component exchange in optimization software is nontrivial. For putting such a component exchange into practice, a black box approach, as illustrated in Figure 8, is used. After each period, all installed components of all machines within the overall system are, in theory, disassembled and packed in a black box. Subsequently, every machine is composed newly in an optimal manner in every single period with the aid of different assignment constraints. Thus, an optimal re-formation is determined for every single machine in every single period. Thereby, the exchange of different components within the system can be described in an elegant way that is much easier to implement within an optimization software than a pairwise interchange.

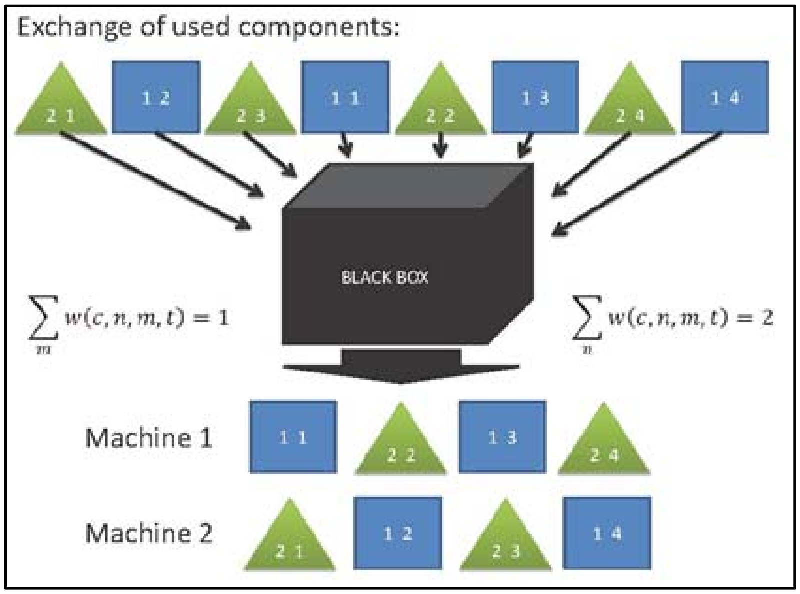

Figure 8. Exchange of Used Components

Multiple-Machine Optimal-Replacement Problem

A final model for planning replacements with deterioration considers all possible replacement/upgrade options, and it is illustrated in Figure 9.

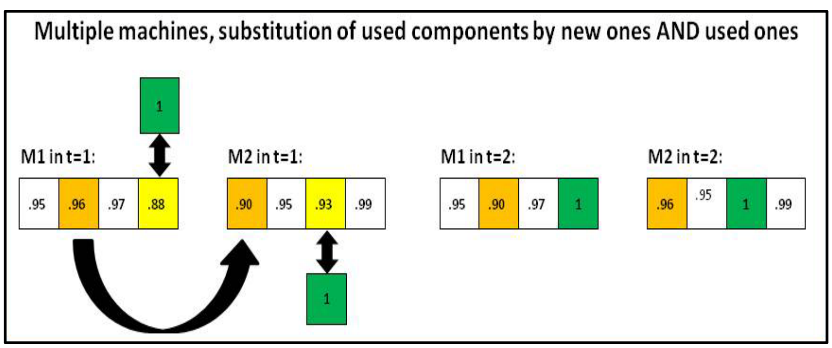

Figure 9. Multiple-Machine Optimal-Replacement Problem

In practice, a common problem is the optimal replacement strategy in respect to a batch of machines M. An OEM wants to know the optimal times to replace different components by new ones within its machinery in order to maximize the level of efficiency over a regarded time line. To help a company answer this question, an algebraic model has been formulated and an optimization model has been developed (Christian, 2010; Schulze, 2010).

Heuristics for Maximizing the System Efficiency Over a Finite Time Horizon

This subsection deals with heuristic approaches concerning the optimal component exchange and composition. Although heuristic approaches fall short of optimal solutions in many cases, they can be used to find acceptable solutions in reasonable computation time and with commonly available calculating resources. For large and complex problems, heuristic approaches are the only viable option for finding solutions.

In Schulze (2010) and Christian (2010), heuristics were developed that focus on single-machine and multiple-machine component replacements. In McKeon (2012), more advanced algorithms were designed and implemented using C++ and these can solve larger problems with more complex structures. The simpler single-machine heuristics have been coded in Excel, and the multiple machines have been programmed using commercial solvers accessed through the GAMS interface. This has been used for solving a real-world problem.

A Practical Application

The models described above have been used to find an optimal replacement strategy for a self-propelled torpedo. A self-propelled torpedo is an explosive projectile weapon that is propelled underwater toward a target where it detonates.

In a torpedo, because of limited space for redundancy, optimal performance of every single component is necessary. If a single component breaks down, the whole torpedo can fail. In addition to the high cost of the torpedo leading to a loss during exercises, this can be of vital importance in the case of a malfunction during a military operation.

Torpedo Composition and Problem Description

Since military data is generally classified, the notation is kept abstract. Figure 10 shows the main composition of this torpedo, which consists of 57 different main components which can be further divided in the following four main subassemblies:

- Sensor system: The sensor system consists of 11 different components. Most of these components are electronic devices like transmitters, sensors, and microchips.
- Warhead: The warhead carries the explosive payload of the torpedo. The subassembly contains additional electronic components like the

initiator, microchips, and cables. In this instance, the warhead consists of 14 different components.

- Fuel: In addition to the fuel tank, the subassembly fuel comprises devices for the fuel injection and other electronic devices. A total of eight different components are in the subassembly.
- Motor: The subassembly motor contains all components that make up the engine of the torpedo. Different kinds of fuel pumps and engine components such as the stator and rotor components are some examples of the 24 different components of this assembly.

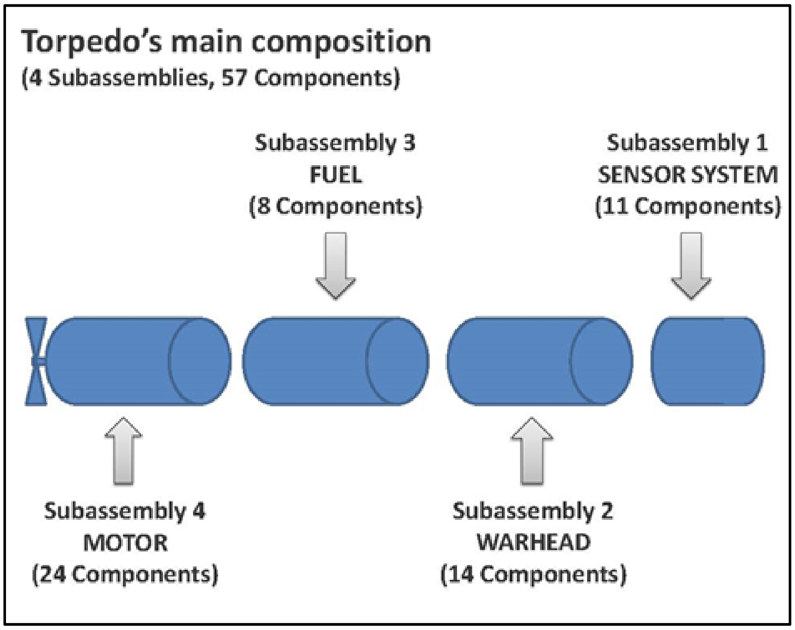

Figure 10. Main Components of the Torpedo

All components are assumed to be essential. In other words, the entire torpedo will not be able to perform in the case of a single component's malfunction. Thus, the overall efficiency can be assumed as the product of all component efficiencies. The regarded time horizon is one year, which is divided into twelve months, or periods.

All components are subject to a monthly degeneration between 0.09% and 0.1%. Degenerations in this range are quite realistic. Although the degeneration depends significantly on the component type, the degenerations are assumed to be randomly distributed. Hence, the degeneration factors for all different components are in the range of 0.999 to 0.9991.

Because the exact costs are classified, random costs between $300 and $1300 are assumed. Even though outliers in prices are not considered, these numbers should be realistic for many of the components.

Under the assumption that a decision-maker has to manage a specific budget for one year, she has to find the best possible budget allocation. In other words, the goal is to find a best possible replacement strategy for the torpedo's components under a given annual budget.

Several strategies are considered for evaluation of the solution methods in Schulze (2010) and Christian (2010)[2]. These examples use an annual budget of $118,500 for all cases. We start with a "fresh" torpedo—that is, a torpedo that has all new components. This costs $50,000 up front, leaving a balance of $68,500 for the 11 following months.

Optimal Allocation

The optimal allocation is computed based on the mathematical programming formulations described earlier. The solution (Figure 11) shows the replacement strategy recommended for the 57 components (listed in the columns), and the system efficiency computed as a product function in this case. Figure 12 shows the results obtained using the heuristic, and it can be seen that the performance of the heuristic is within 5% of the optimal solution.

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3	0.998	1.000	0.998	0.998	0.998	0.998	0.998	0.998	1.000	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	1.000
4	0.997	0.999	0.997	0.997	0.997	0.997	0.997	0.997	0.999	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.999
5	0.996	0.998	0.996	1.000	0.996	1.000	1.000	1.000	0.998	0.996	1.000	1.000	0.996	0.996	0.996	0.996	0.996	0.996	0.996	0.998
6	0.995	0.997	1.000	0.999	0.995	0.999	0.999	0.999	1.000	0.995	0.999	0.999	0.995	1.000	1.000	0.995	0.995	0.995	0.995	1.000
7	0.995	0.996	0.999	0.998	1.000	0.998	0.998	0.998	0.999	1.000	0.998	0.998	1.000	0.999	0.999	0.994	0.994	1.000	0.994	0.999
8	1.000	0.995	0.998	0.997	0.999	0.997	0.997	1.000	0.998	0.999	0.997	0.997	0.999	0.998	0.998	0.993	0.993	0.999	0.993	0.998
9	0.999	0.994	0.997	0.996	0.998	0.996	0.996	0.999	1.000	0.998	1.000	0.996	0.998	0.997	0.997	1.000	0.992	0.998	0.992	1.000
10	0.998	0.994	0.996	0.995	0.997	0.995	0.995	0.998	0.999	0.997	0.999	0.995	0.997	0.996	0.996	0.999	0.991	0.997	1.000	0.999
11	0.997	0.993	0.995	0.994	0.996	0.994	1.000	0.997	1.000	1.000	0.998	0.994	0.996	0.995	0.995	0.998	0.990	0.996	0.999	0.998
12	0.996	0.992	0.994	0.993	1.000	1.000	0.999	0.996	0.999	0.999	0.997	0.993	0.995	1.000	1.000	0.997	1.000	0.995	0.998	0.997

C	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999
3	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	1.000	0.998	0.998	0.998	0.998	1.000	1.000	0.998	0.998	0.998
4	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.999	0.997	0.997	0.997	0.997	0.999	0.999	1.000	0.997	0.997
5	0.996	0.996	1.000	1.000	1.000	1.000	1.000	0.996	0.996	0.996	0.998	1.000	0.996	0.996	0.996	0.998	0.998	0.999	0.996	0.996
6	1.000	0.995	0.999	0.999	0.999	0.999	0.999	0.995	1.000	1.000	0.997	0.999	0.995	0.995	0.995	0.997	0.997	0.998	0.995	1.000
7	0.999	0.994	0.998	0.998	0.998	0.998	0.998	0.994	0.999	0.999	1.000	0.998	1.000	0.995	1.000	1.000	0.996	0.997	0.995	0.999
8	0.998	1.000	0.997	0.997	0.997	0.997	0.997	0.993	0.998	0.998	0.999	0.997	0.999	0.994	0.999	0.999	0.995	0.996	0.994	0.998
9	0.997	0.999	0.996	0.996	0.996	0.996	0.996	0.992	0.997	0.997	0.998	0.996	0.998	0.993	0.998	0.998	1.000	0.995	1.000	1.000
10	0.996	0.998	0.995	0.995	0.995	1.000	1.000	1.000	0.996	0.996	0.997	1.000	0.997	1.000	0.997	0.997	0.999	1.000	0.999	0.999
11	1.000	0.997	0.994	1.000	1.000	0.999	0.999	0.999	0.995	0.995	1.000	0.999	0.996	0.999	0.996	0.996	0.998	0.999	0.998	0.998
12	0.999	0.996	1.000	0.999	0.999	0.998	0.998	0.998	1.000	1.000	0.999	0.998	0.995	0.998	0.995	0.995	0.997	0.998	0.997	0.997

C	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57		Π
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000		1.000
2	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999		0.947
3	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	0.998	1.000	0.998		0.910
4	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.999	0.997		0.864
5	0.996	0.996	0.996	1.000	0.996	0.996	0.996	0.996	0.996	1.000	0.996	0.996	0.996	0.996	0.996	0.998	0.996		0.863
6	0.995	1.000	0.995	0.999	0.995	0.995	1.000	1.000	0.995	0.999	0.995	0.995	0.995	0.995	0.995	0.997	0.995		0.863
7	0.994	0.999	0.994	0.998	0.994	0.994	0.999	0.999	0.994	0.998	0.995	0.994	1.000	0.994	0.994	0.996	1.000		0.862
8	1.000	0.998	0.993	0.997	1.000	1.000	0.998	0.998	1.000	0.997	1.000	0.993	0.999	1.000	1.000	1.000	0.999		0.873
9	0.999	1.000	0.992	0.996	0.999	0.999	0.997	0.997	0.999	0.996	0.999	1.000	0.998	0.999	0.999	0.999	0.998		0.864
10	0.998	0.999	1.000	0.995	0.998	0.998	0.996	0.996	0.998	0.995	0.998	0.999	0.997	0.998	0.998	0.998	0.997		0.864
11	0.997	0.998	0.999	1.000	0.997	0.997	1.000	0.995	0.997	1.000	0.997	0.998	1.000	0.997	0.997	1.000	0.996		0.864
12	0.996	0.997	0.998	0.999	0.996	0.996	0.999	0.994	0.996	0.999	0.996	0.997	0.999	0.996	0.996	0.999	0.995		0.862

Figure 11. Optimal Annual Expenditure

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	0.999	0.999	0.999	0.999	1.000	1.000	0.999	0.999	1.000	0.999	0.999	0.999	0.999	1.000	1.000	0.999	0.999	0.999	0.999	1.000
3	0.998	0.998	0.998	0.998	0.999	0.999	0.998	0.998	0.999	0.998	0.998	0.998	0.998	0.999	0.999	0.998	0.998	0.998	0.998	0.999
4	0.997	0.997	0.997	0.997	1.000	1.000	0.997	0.997	0.998	0.997	0.997	0.997	0.997	0.998	0.998	0.997	0.997	0.997	0.997	0.998
5	0.996	0.996	0.996	0.996	0.999	0.999	0.996	0.996	1.000	0.996	0.996	0.996	1.000	1.000	1.000	0.996	0.996	1.000	0.996	1.000
6	0.995	0.995	0.995	0.995	1.000	1.000	0.995	1.000	0.999	1.000	1.000	1.000	0.999	0.999	0.999	0.995	0.995	0.999	0.995	0.999
7	0.995	0.994	1.000	0.994	0.999	0.999	0.995	0.999	0.998	0.999	0.999	0.999	0.998	0.998	0.998	0.994	1.000	0.998	0.994	0.998
8	0.994	1.000	0.999	0.993	0.998	0.998	0.994	0.998	0.997	0.998	0.998	0.998	0.997	0.997	0.997	0.993	0.999	0.997	0.993	1.000
9	0.993	0.999	0.998	1.000	1.000	1.000	0.993	0.997	0.996	0.997	0.997	0.997	0.996	1.000	1.000	1.000	0.998	0.996	0.992	0.999
10	0.992	0.998	0.997	0.999	0.999	0.999	1.000	0.996	1.000	0.996	0.996	0.996	0.995	0.999	0.999	0.999	0.997	0.995	0.991	0.998
11	0.991	0.997	0.996	0.998	0.998	0.998	0.999	0.995	0.999	0.995	0.995	0.995	0.994	0.998	0.998	0.998	0.996	0.994	0.990	0.997
12	0.990	0.996	0.995	0.997	1.000	0.997	0.998	0.994	0.998	0.994	0.994	0.994	1.000	0.997	0.997	0.997	0.995	1.000	1.000	1.000

C	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	1.000	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	1.000	1.000	0.999	0.999	0.999	1.000	0.999	0.999	0.999	1.000
3	0.999	0.998	0.998	0.998	0.998	0.998	1.000	0.998	0.998	0.998	0.999	0.999	1.000	0.998	0.998	0.999	1.000	0.998	0.998	0.999
4	1.000	0.997	0.997	0.997	0.997	0.997	0.999	0.997	0.997	0.997	1.000	0.998	0.999	0.997	1.000	0.998	0.999	0.997	0.997	1.000
5	0.999	0.996	1.000	0.996	0.996	1.000	0.998	0.996	1.000	0.996	0.999	1.000	0.998	0.996	0.999	1.000	0.998	0.996	1.000	0.999
6	0.998	0.995	0.999	0.995	1.000	0.999	0.997	1.000	0.999	0.995	0.998	0.999	0.997	0.995	0.998	0.999	0.997	0.995	0.999	0.998
7	0.997	0.994	0.998	0.995	0.999	0.998	1.000	0.999	0.998	0.995	0.997	0.998	1.000	0.995	0.997	0.998	1.000	0.995	0.998	1.000
8	1.000	0.993	0.997	1.000	0.998	0.997	0.999	0.998	0.997	0.994	1.000	0.997	0.999	1.000	0.996	0.997	0.999	1.000	0.997	0.999
9	0.999	0.993	0.996	0.999	0.997	0.996	0.998	0.997	0.996	1.000	0.999	0.996	0.998	0.999	1.000	1.000	0.998	0.999	0.996	0.998
10	0.998	1.000	0.995	0.998	0.996	0.996	0.997	0.996	0.995	0.999	0.998	1.000	0.997	0.998	0.999	0.999	0.997	0.998	0.995	1.000
11	0.997	0.999	0.994	0.997	0.995	0.995	0.996	0.995	0.994	0.998	0.997	0.999	1.000	0.997	0.998	0.998	0.996	0.997	0.995	0.999
12	0.996	0.998	0.993	0.996	0.994	1.000	0.995	0.994	0.993	0.997	1.000	0.998	0.999	0.996	0.997	0.997	1.000	0.996	0.994	0.998

C	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57		Π
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000		1.000
2	0.999	0.999	0.999	0.999	0.999	0.999	1.000	0.999	0.999	0.999	0.999	0.999	1.000	0.999	0.999	0.999	0.999		0.959
3	0.998	1.000	0.998	1.000	0.998	0.998	0.999	0.998	0.998	0.998	0.998	0.998	1.000	0.998	0.998	0.998	0.998		0.918
4	0.997	0.999	0.997	0.999	1.000	0.997	0.998	0.997	0.997	0.997	0.997	0.997	0.999	0.997	0.997	1.000	0.997		0.886
5	1.000	0.998	0.996	0.998	0.999	0.996	1.000	0.996	0.996	1.000	0.996	0.996	1.000	0.996	0.996	0.999	0.996		0.884
6	0.999	0.997	0.995	0.997	0.998	1.000	0.999	0.995	0.995	0.999	0.995	1.000	0.999	0.995	0.995	0.998	0.995		0.874
7	0.998	0.996	0.994	1.000	0.997	0.999	0.998	0.994	0.994	0.998	0.995	0.999	0.998	0.994	0.994	0.997	0.994		0.853
8	0.997	1.000	1.000	0.999	1.000	0.998	0.997	1.000	1.000	0.997	0.994	0.998	1.000	0.994	1.000	0.996	0.994		0.871
9	0.996	0.999	0.999	0.998	0.999	0.997	1.000	0.999	0.999	0.996	0.993	0.997	0.999	0.993	0.999	1.000	0.993		0.870
10	0.995	0.998	0.998	0.997	0.998	0.996	0.999	0.998	0.998	0.995	0.992	0.996	0.998	0.992	0.998	0.999	0.992		0.849
11	0.994	0.997	0.997	0.996	0.997	0.995	0.998	0.997	0.997	0.995	1.000	0.995	1.000	0.991	0.997	0.998	0.991		0.817
12	1.000	0.996	0.996	0.995	0.996	0.994	0.997	0.996	0.996	1.000	0.999	0.994	0.999	1.000	0.996	0.997	0.990		0.829

Figure 12. Heuristic Solution for Torpedo Maintenance Strategy

In addition to these analyses, models using a variety of deterioration functions have been evaluated in Vollenberg (2010). These include age-dependent deterioration and exponentially varying deterioration functions. Incremental updates have also been investigated in McKeon (2012), and an exact solution method using branch and bound for solving the deterministic problem as well as solutions for stochastic cases was discussed in Husen (2012).

Determining and Evaluating Acquisition Decisions and Product Upgrades Which Must Be Synchronized With Service Tasks and Fleet Operations While Maintaining an Effective Inventory

This section defines the problem of maintaining an effective inventory in such a manner so that some service level is achieved. Managing complex products that have long lifetimes is not an easy task. However, most defense and many industrial organizations deal with such products on a daily basis. Whereas non-durable goods (i.e., goods with lifetimes of less than three years) can be sold in large volumes with very little post-sales support, durable goods such as commercial grade printing and photo-copying systems, enterprise-wide computing systems, weapons, and weapon systems are designed to accommodate evolutionary updates of the design of key components or technology refreshes and insertions that either fix existing bugs and/or introduce new features by upgrades to modules. The complicating factor here is that the upgrades/insertions have to be done to a large inventory of in-service products while meeting promised deliveries. In the context of some defense organizations such as the torpedo enterprise, there are mandates on reserve quantities for different types of weapons, scheduled rotations between training and warshot inventory, mandatory maintenance schedules, and so forth. Furthermore, issues such as obsolescence and part failures must also be taken into consideration, and contracts for acquiring new and replacement parts must also be matched with the budgets and promised deliveries to the fleet.

Following Keynes (2006), it is generally accepted that the main motives for holding money are precautionary, speculative, and transaction. As explained in Arrown, Karlin, and Scarf (1958), precautionary motives protect against uncertainty; speculative objectives are fueled by anticipation of future gains; and transaction encapsulates the reluctance to change currencies/investments because of the fixed or variable fees incurred in flipping from one type of investment to another. Reasons for holding an inventory of goods are generally the same as those for holding currency. It can be argued that the exception is when goods are held in reserve to meet uncertain demands, with the objective of exceeding some level of customer satisfaction. The accounting of costs and benefits in defense organizations is somewhat different, and this paper seeks to develop the argument that the goal of holding inventory in this sector is to respond sufficiently to future threats. In an environment of rapidly changing threats (Hilsenrath, 2011), the utility of an inventory of weapons is not just in its ability to meet current needs, but also in its ability to meet future requirements with minimal transformation effort.

Costs Involved in Defense Logistics

The costs considered when modeling inventory decisions in commercial enterprises are typically holding, ordering, shortage, and backorder costs. Holding costs include the cost of money (opportunity loss because of the money tied up in inventory or the cost of capital borrowed to purchase inventory). Shortage costs include the cost of lost sales implying profit. Backorder costs are the costs incurred when orders not delivered in a timely manner must be rushed to the customer using more expensive logistics channels. Other costs considered when analyzing inventory decisions are lateral transfer cost (Lee, 1987), multiple-channel supply costs, and others; and additional issues include buyer–vendor coordination, including price discounts (Goyal & Gupta, 1989), and so forth. In terms of maximizing inventory effectiveness, in the commercial world, companies maximize profit, and demand serves as the primary constraint. In other words, profit is king, and demand is the main constraint to maximizing profit. As we will see (and as would be expected), this is not the case when supporting weapon systems.

The nature of costs in the defense sector is considerably different. Defense logistics agencies are issued annual budgets for maintaining supply chains with the goal of stocking adequate levels of weapons and supplies to meet contingency demands. Stated slightly differently, the fleet requirements drive inventory need and the main constraint is the allowable budget; other constraints include Intermediate Maintenance Activity (IMA) capacity in terms of personnel and test equipment. To use the language from the previous paragraph, demand is king, and the budget (a type of profit) is the primary constraint when maximizing demand fulfillment. This brings out the point that in the Department of Defense (DoD), cash flow is controlled by a higher authority and cannot be increased based on “selling” more inventory. The budget is set (at some point in time), and support of the weapon system must be optimized based on that amount. This type of inventory effectiveness optimization does not lend itself to commercial enterprise, because in the retail world, profits will change based on company performance.

Logistics Costs in the Torpedo Enterprise

Another level of complexity is added to the torpedo enterprise’s inventory system in that its inventory is stored at three IMAs, each with differing cost models. The IMA in Pearl Harbor, HI, is contractor run and was awarded based on a competitive services contract. The IMA in Yorktown, VA, is run by the U.S. Navy; the labor at this IMA is “free” as it is supplied by sailors. The third IMA is located at Naval Undersea Warfare Center (NUWC), Division Keyport, and is staffed with government civil service labor. These differing structures (commercial, military, and federal) sometimes cause issues in regards to standardization of processes and organizational cohesiveness. Further, the torpedo enterprise, because it supports a weapon for war, is also governed by legal statutes related to safety, hazardous material, RFID, and UID, to name a few; these are all cost drivers.

There are also inventory considerations below the torpedo All Up Round (AUR) level. Torpedo-unique parts are inventoried by the Naval Inventory Control Point (NAVICP), and items common between torpedoes and other DoD systems are inventoried by the Defense Logistics Agency (DLA). Demands for these parts are tracked through the use of in-house databases. Problems with inventory re-order are sent to the NUWC for technical recommendations (e.g., suitable replacements when obsolescence is encountered).

The torpedo enterprise inventory, for the purposes of this paper, is the warshot and exercise inventory maintained at the AUR configuration in bunkers at or near the IMAs. These torpedo inventories are stored for both the Atlantic Fleet and the Pacific Fleet, and the torpedoes are available for the fleet to requisition. The quantity goal for the torpedo enterprise inventory is Non-Nuclear Ordnance Requirements with a wartime surge capability referred to as WAR RESERVE. At one time, the planning to support the Atlantic Fleet and Pacific Fleet requirements was handled separately, but several years ago, the enterprise moved to centralized inventory planning and handling (i.e., one Planning Cell). The Planning Cell meets with the fleet representatives quarterly, at a minimum, to discuss warshot and exercise requirements; exercise torpedoes are units capable of being fired and recovered for the fleet to maintain proficiency. These warshot and exercise requirements are translated to IMA capacity, and torpedo build requirements are determined to workload the IMAs. So, the flexibility of the inventory at the AUR level is the IMA's capacity to build exercise and warshot torpedoes and to turn one into the other and vice versa. Fleet/ship requirements can also be met through a mix of torpedo configurations (i.e., MK48 Mod 6 vs. MK48 Mod 7) that are tailored to the target operating theatre. Additionally, there is flexibility of inventory at the AUR torpedo level through the upgrade of operational software via download capability. Versions of operational software can be downloaded at IMAs during weapon maintenance and preparation or even on board ships. Operational software brings flexibility to AUR torpedoes with improved and varying performance.

Since our enterprise is not in production of AUR torpedoes at this time and has not been for many years, foreign military sales can both limit and enhance our flexibility. To sell AUR torpedoes to other nations at this time has a negative impact on the United States' inventory quantity, but provides valuable resources to reconstitute production capability or performance enhancements in both hardware and software, which are helpful in the long run of the program (i.e., financing torpedo upgrades in the future).

Use of older torpedo configuration hardware that has been “mothballed” (e.g., MK48 Mod 4) brings with it the flexibility of “quantity versus quality.” Older torpedo hardware that has been slated for demilitarization can be revitalized to add quantity to the inventory with calculated performance degradation. Unrelated to the purpose of this research, performance-versus-quantity models exist to evaluate overall torpedo enterprise inventory effectiveness.

Modeling Inventory Effectiveness

In the discussion that follows, details of some preliminary models investigating the impact of flexibility on inventory operations are presented. The first approach utilizes an established two-level service model with conversion options between different part types to estimate the benefit that may be garnered by pooling inventory. The second approach presents a mathematical programming approach for determining optimal inventory decisions with transfers and conversions between different part types and common subassemblies. A brief literature review is first presented.

A two-class inventory system for modeling consumable items in a defense setting was presented in Deshpande, Cohen, and Donohue (2003). The authors constructed a model approximating the management of consumables by the DLA and proposed a threshold for determining backorders for different classes of items. This model is useful when considering the allocation of pooled inventory items, but requires the setting of priorities for different classes externally. Clearly, this is difficult to do. However, this paper explains many of the issues particular to inventory management in defense settings.

Multi-echelon models for inventory management of spares in the defense industry have been considered by Simon (1971) and Yanmei, Jiangsheng, Sujian, and Weimin (2008), among others. However, most multi-echelon models consider single-item types and the location of inventory pools at different levels to meet demand changes at different end points by cross-shipping when necessary. A fundamental analysis of the two-level case for repairable items is in Simon (1971), Muckstadt (1973), and Graves (1985). Although substitution of items, examined in Karaesmen and Van Ryzin (2004), can result in significant savings, it has not generally been considered in these multi-echelon models. Begnaud, Benjaafar, and Miller (2009) considered multi-echelon inventory planning with flexible substitution opportunities, but the decision for interchanging items with an associated transaction cost was not developed.

There is a vast body of literature related to mathematical programming models for lot sizing. Starting with Wagner and Whitin (1958), Crowston and Wagner (1973), and so forth, the solution approaches for such problems have involved either dynamic programming approaches, specialized algorithms, or integer programming formulations and solutions (Belvaux & Wolsey, 2000; Wolsey, 2002). As noted in Wolsey (2002), many real-world lot sizing problems can now be adequately solved using commercial-off-the-shelf mathematical programming software. Wolsey further classified lot sizing problems using three fields: [x,y,z]. The first field, x, indicates the problem version, and its choices are *LS* (lot sizing), *WW* (Wagner–Whitin), *DLSI* (discrete lot sizing with initial stock), and *DLS* (discrete lot sizing without initial stock). The second field describes the production capabilities: *C* for capacitated, *CC* for constant production, and *U* for uncapacitated. When multiple items share production capacities, the additional qualifier *BB* is prepended to *DLSI*. The third field describes extensions/variants and includes *B* (backlogging), *SC* (startup costs), *ST* (startup times), *LB* (minimum production levels), *SL* (sales constraints), and *SS* (safety stock considerations). The first two fields of problems considered here could then be described as *DLSI*-*CC*. Since the nomenclature proposed does not capture transformations, we suggest an extension to the nomenclature—*T* for transformation, whereby items can be transformed from one product type to another. Although there are a large number of additional combinations that can be proposed, for now the nomenclature used to describe the multi-item lot sizing problem with transformations can be *BB*/*DLSI*-*CC*-*T*.

Based on the previous discussion, we propose the thesis that for a defense logistics operation, a fundamental measure of inventory effectiveness is the flexibility to meet a variety of potential needs for future operations. Based on this assumption, two preliminary models are developed to show how the increase in flexibility can indeed result in improvements to service levels. The first approach is based on an established two-level service operation, first explored in Sherbrooke (1968) and further developed in Simon (1971), Muckstadt (1973), and others. The second model presented is a multi-product lot sizing model with transformations between different product types.

A Preliminary Investigation of the Impact of Flexibility in Two-Level (Base-Depot) Operations

Following Sherbrooke (1968), a two-level operation for recoverable parts is described as follows: Several distributed maintenance facilities (j = 1, …, N) restore incoming recoverable parts. While most parts can be repaired locally, some fraction of incoming parts have to be sent to the central depot for repair. The base and depot each maintain their own levels of inventory independently, and this inventory of parts is used for immediate replacement of incoming parts that undergo repair. When this inventory is depleted, the turnaround of outgoing parts is delayed until some refurbished units are available. The organization of this system is shown in Figure 13. As indicated in the figure, the parts are assumed to arrive at base j with exponential inter-arrival times, at rates λ_j, respectively. The service time at each base is μ_j. The depot is designated by the index 0. The total transfer time between the base and the depot is denoted as τ_j, and the stock levels maintained at the depot and bases are (S_0, S_0, …, S_n).

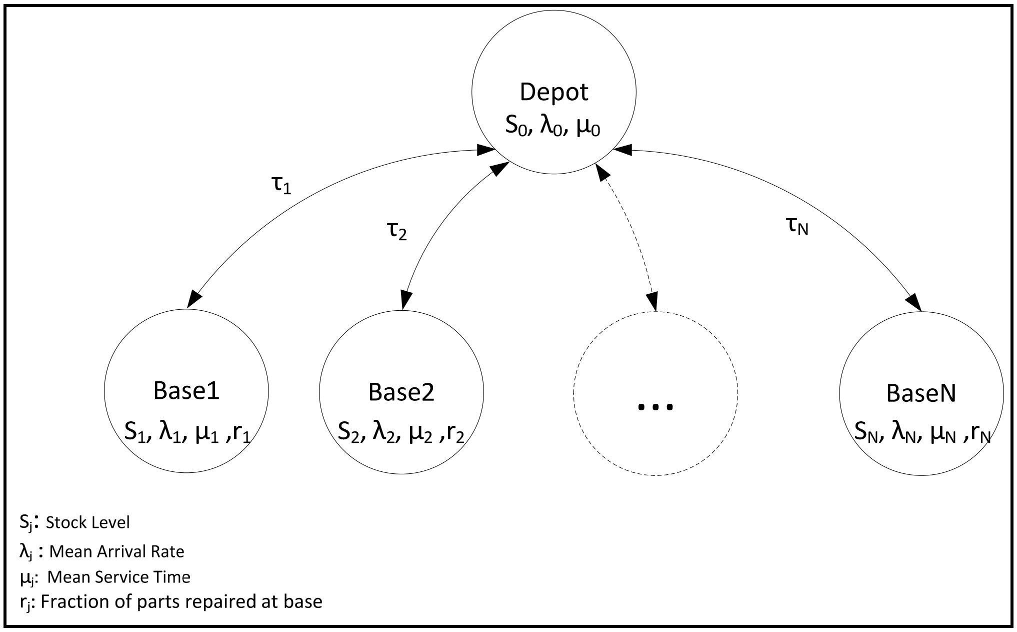

Figure 13. Two-Level Structure for Repairable Items

For such a scenario, given an allocation of spares (S_0, …, S_N) amongst the bases and depot, the average number of parts waiting in the system at the base and the depot (L_0 , L_1, …, L_N) are computed as

$$L_0 = \sum_{k=s_0}^{\infty} (k - S_0) e^{-\lambda_0 \mu_0} \frac{(\lambda_0 \mu_0)^k}{k!} \tag{3}$$

$$L_j \approx \sum_{k=s_j}^{\infty} (k - S_j) e^{-\lambda_j \beta_j} \frac{(\lambda_j \beta_0)^k}{k!}, \ j = 1, \dots, N \tag{4}$$

where,

$$\lambda_0 = \sum_{j=1}^{N} (1 - r_j) \lambda_j \text{ , and} \tag{5}$$

$$\beta_j = r_j \mu_j + (\tau_j + \frac{L_0}{\lambda_0})(1 - r_j) \tag{6}$$

A detailed discussion can be found in Tijms (2003).

Now, let us assume that the system handles two part types, k = 1,2. The repair protocol is the same—that is, base j repairs incoming parts with probabilities r_{j1} and r_{j2}, respectively. The stock levels at the depot and the bases are (S_{01}, S_{02}, S_{11}, S_{02}, …, S_{N1}, S_{N2}) respectively. A simulation experiment was conducted in which arrival and service rates were randomly selected (with a service ratio of ½ for the bases and the depot). The transportation time between the base and the depot was set to $2^*\mu_j$. The total inventory level was varied, as shown in the following graph. This was done for each product type, and an optimal distribution of inventory was determined. The expected number of items in the system for each product type was recorded as $\mathcal{L}_1$ and $\mathcal{L}_2$. Finally, an optimal allocation of inventory for the combined system was determined using an evolutionary algorithm, and the total number of items in the system was noted as $\mathcal{L}_3$. A graph comparing $\mathcal{L}_1 + \mathcal{L}_2$ and $\mathcal{L}_3$ is shown in Figure 14. As expected, the performance of the pooled system is significantly superior to that of the separate systems. For the parameters used here, the number of parts in the system required to maintain an equivalent service level is smaller by a factor of 4, on average.

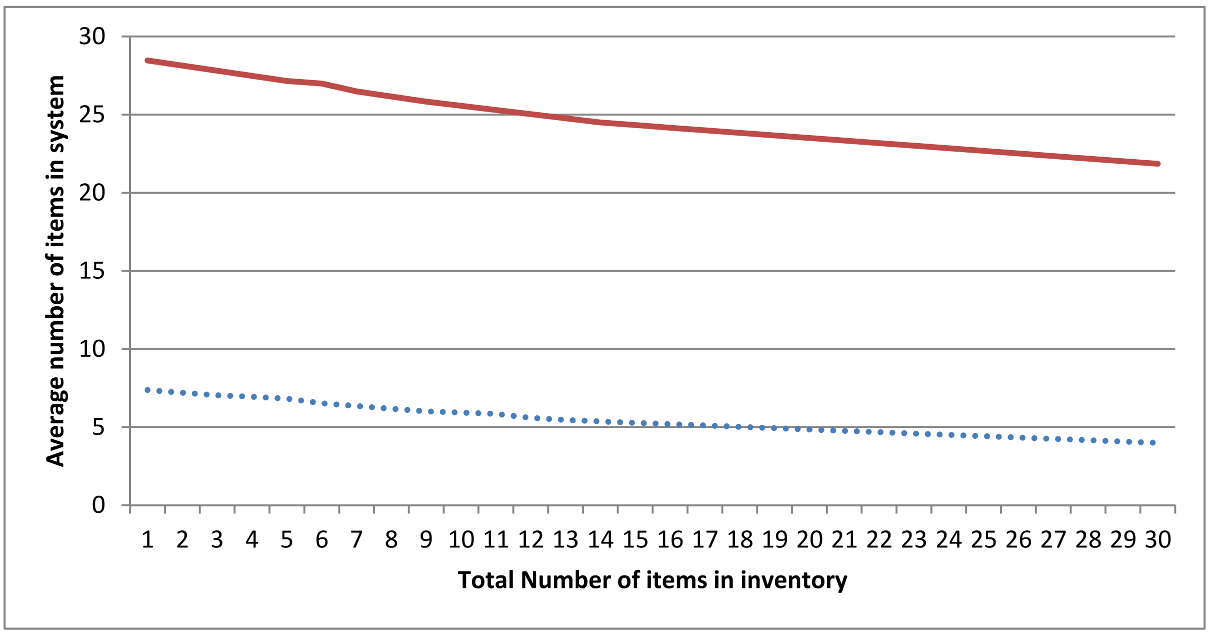

Figure 14. Comparison of Pooled vs. Segregated Inventory Performance

The example presented here emphasizes the advantages of a pooled inventory and transformations between two product types. This analysis is a part of ongoing work focused on developing metrics for effective inventory with transformations in the context of defense organizations.

Basic Lot Sizing Model

The model being expanded in this paper to mimic the torpedo enterprise's inventory is a lot sizing problem. The assumptions of this model are unlimited and instantaneous production, unlimited inventory storage, no incoming or outgoing inventory, and deterministic demand. However, these assumptions can easily be altered by adding the proper constraints. The constraining costs in the model are inventory carry-over ($/period/unit), set-up costs ($/set-up), and production costs ($/production unit). The objective of this model is to meet demand for each period while minimizing cost over the periods being studied, and allowing transformations between products/subassemblies during the planning horizon.

Mathematically, this model can be written as follows:

$P_{it} = production\ of\ product\ i\ in\ time\ t$

$I_{it} = inventory\ carry - over\ of\ product\ i\ in\ time\ t$

$S_{it} = setup\ of\ production\ for\ product\ i\ in\ period\ t$

$\chi_{ij} = cost\ of\ producing\ (i = j)\ products$

$\sigma_i = setup\ cost\ of\ product\ i$

$\phi_i = cost\ of\ holding\ product\ i$

$\delta_{it} = demand\ of\ product\ i\ in\ time\ t$

$$Min\ Z = \left\{\sum_i \sum_j \sum_t \left(\chi_{ij} P_{ijt} \middle| i = j\right) + \sum_i \sum_t (\phi_i I_{it}) + \sum_i \sum_t (\sigma_i S_{it}) \,\middle|\forall\ i, t\right\} \tag{7}$$

$S.t.$

$$P_{it} + I_{i(t-1)} = \delta_{it} + I_{it} \tag{8}$$

$$P_{it} \leq S_{it} * M \tag{9}$$

$$P_{it}, I_{it} \geq 0 \tag{10}$$

$$P_{it},, I_{it}, = Integer \tag{11}$$

$$S_{it} = Binary \tag{12}$$

Equation 7 is the objective function that minimizes the production inventory and setup costs of the system. Equation 8 ensures the conservation of material within the model flow. Equation 9 uses Big M logic to set the setup decision for product *i* to 1 if production for product *i* is needed. Equations 10–12 incorporate the necessary non-negativity, integer, and binary constraints, respectively. A flowchart of the base model is shown in **Error! Reference source not found.**5.

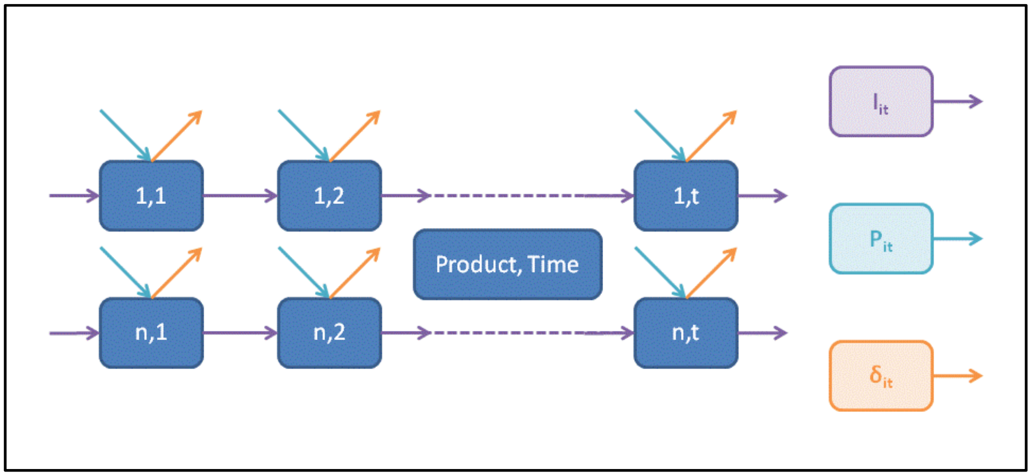

Figure 15. Simple Model for Transformations Among Different Part Types

The first expansion to be integrated into the lot sizing model is that of product transformation. Consider the problem in which two distinct products can, at a price, be converted from one to the other. An example is the production of modern automobiles, where the base model can be upgraded to more "deluxe" or "luxury" models. Another similar example, for which this model was developed, is the transformation of torpedoes from one model to another. The ability to transform products in an inventory creates a more flexible inventory and provides the opportunity for cost savings depending on the transformation and setup costs of a particular system.

In order to expand the model to include transformations, the following variable is added to the model's environment:

$$T_{ijt} = transformation\ of\ product\ i\ into\ j\ in\ time\ t$$

And the following constant is changed to include transformation costs from one product to another.

$$\chi_{ij} = cost\ of\ producing\ (i = j) or\ transforming\ (i \neq j)\ products$$

Furthermore, Equations 7 and 8 are expanded to include the new variable and constant.

$$Min\ Z = \{\sum_i \sum_j \sum_t (\chi_{ij} P_{ijt} | i = j) + \sum_i \sum_t (\phi_i I_{it}) + \sum_i \sum_t (\sigma_i S_{it}) + \sum_i \sum_j \sum_t (\chi_{ij} T_{ijt} | i \neq j) \,|\forall\, i, t\} \quad (13)$$

$$P_{it} + I_{i(t-1)} + \sum_j T_{jit} = \delta_{it} + I_{it} + \sum_j T_{ijt} \quad (14)$$

Note that that in Equation 13, the same cost matrix is used for both production and transformation. For production, *i* = *j*, while for transformation, *i* ≠ *j*.

For the conservation of material constraint, the left-hand side (incoming) of the constraint adds the summation of the transformations from all products *j* into product *i* for the given period, while the right-hand side (outgoing) adds the summation of the transformations from product *i* into all products *j* for the given period. A flowchart of the transformation expanded model can be seen in Figure 16.

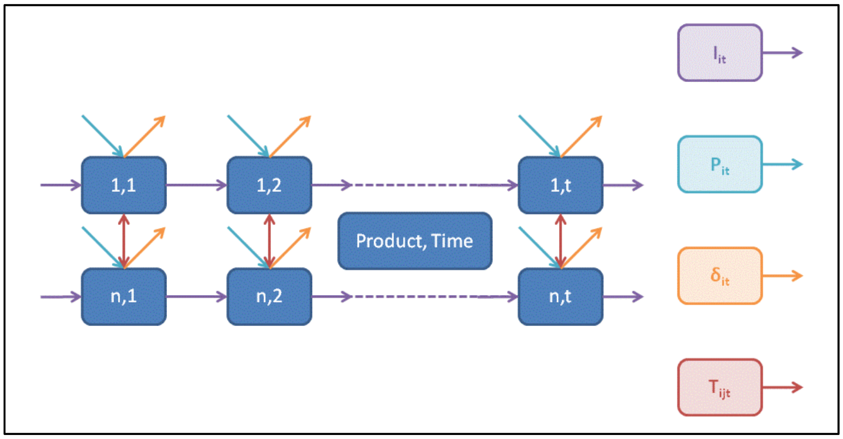

Figure 16. Transformation Expansion

Move Expansion

The next model expansion considers the system in which there is more than one location for producing and storing products. Each distinct location can have its own associated production, storage, inventory, and setup costs. It is assumed that movement of products between locations is instantaneous. This assumption can, however, be dropped by manipulating the time (*t*) values associated with the move variables in the conservation of material constraint.

In order to expand the model to include transformations, the following variable is added to the model's environment:

$M_{itkl} = move\ of\ product\ i\ in\ time\ t\ from\ location\ k\ to\ location\ l$

And the following constant is changed to include movement costs from one location to another:

$\rho_{ikl} = cost\ of\ moving\ product\ i\ from\ location\ k\ to\ location\ l$

Furthermore, all of the other constraints and variables must have a location subscript added to their definitions.

Equations 15 and 16 are expanded to include the new variable, constant, and location subscript.

$$Min\ Z = \left\{ \begin{matrix} \sum_i \sum_j \sum_t \sum_k \left(\chi_{ijk} T_{ijtk} \middle| i \neq j\right) + \sum_i \sum_j \sum_t \sum_k \left(\chi_{ijk} P_{itk} \middle| i = j\right) + \\ \sum_i \sum_t \sum_k (\phi_{ik} I_{itk}) + \sum_i \sum_t \sum_k \sum_l (\rho_{ikl} M_{itkl}) \end{matrix} \middle| \forall\ i, t, k \right\} \quad (15)$$

$$P_{itk} + I_{i(t-1)k} + \sum_{j} T_{jitk} + \sum_{l} M_{itlk} = \delta_{itk} + I_{itk} + \sum_{j} T_{ijtk} + \sum_{l} M_{itkl} \quad (16)$$

The expansion of Equation 15 adds the term for the movement cost and movement variable. Also, the subscript for location is added to all of the costs and variable definitions. In Equation 16 (conservation of material constraint), the left-hand side (incoming) of the constraint adds the summation of the movements from all locations *l* to location *k* for the given period, while the right-hand side (outgoing) adds the summation of the movements from location *k* to all location *l* for the given period. A flowchart incorporating the movement expanded model can be seen in Figure 17.

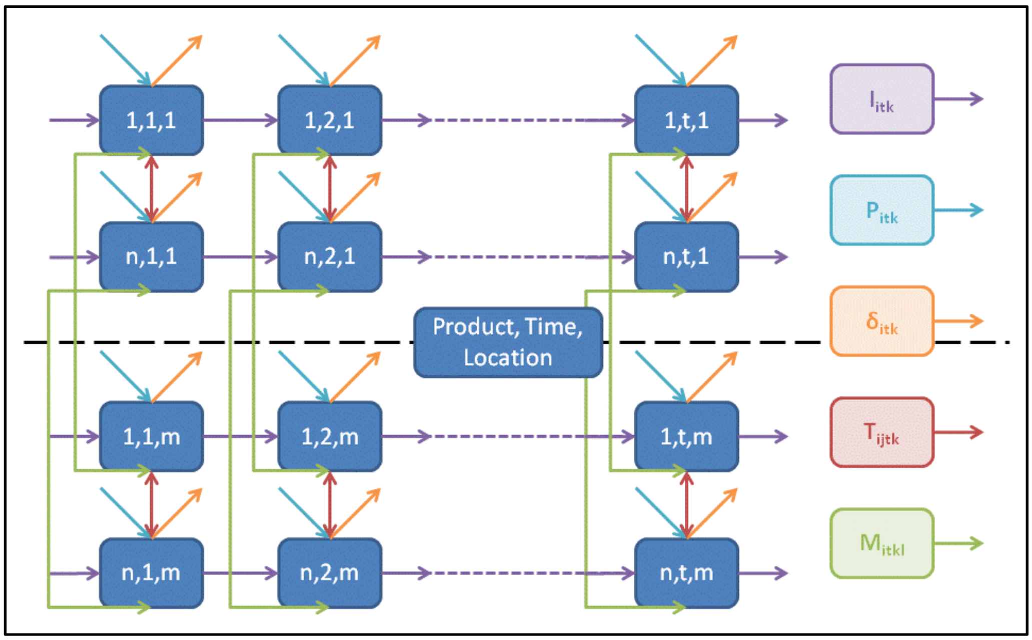

Figure 17. Movement Expansion

Multi-Level Product Expansion

Another possible expansion of this model would be to consider not only the finished products, but also the subassemblies that are used to build them. In order to evaluate such a model, the subassemblies would need their own cost constants for production/purchase, storage, movement, transformation (if applicable), and setup (if applicable). Demand for the subassemblies would be a function of the demand on the finished products. A simple flowchart showing finished products as compositions of subassemblies can be seen in Figure 18.

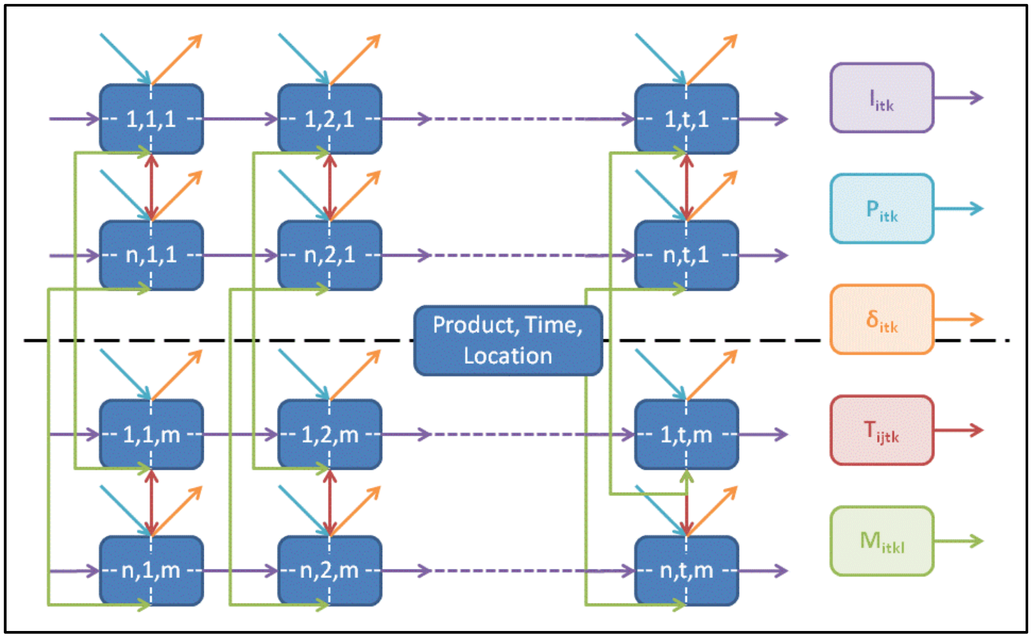

Figure 18. Subassembly Expansion

Expanded Model

The fully expanded model (not including the subassembly expansion) can now be seen as follows:

P_{itk} = *production of product i in time t at location k*

I_{itk} = *inventory carryover of product i in time t at location k*

S_{itk} = *setup of production for product i in period t at location k*

T_{ijtk} = *transformation of product i into j in time t at location k*

M_{itkl} = *move of product i in time t from location k to location l*

χ_{ijk} = *cost of producing* $(i = j)$ *or transforming* $(i \neq j)$ *products at location k*

ϕ_{ik} = *cost of holding product i at location k*

σ_{ik} = *setup cost of product i at location k*

ρ_{ikl} = *cost of moving product i from location k to location l*

δ_{itk} = *demand of product i in time t at location k*

$$Min\ Z = \left\{ \begin{array}{c} \sum_i \sum_j \sum_t \sum_k \left(\chi_{ijk} T_{ijtk} \middle| i \neq j\right) + \sum_i \sum_j \sum_t \sum_k \left(\chi_{ijk} P_{itk} \middle| i = j\right) + \\ \sum_i \sum_t \sum_k (\phi_{ik} I_{itk}) + \sum_i \sum_t \sum_k \sum_l (\rho_{ikl} M_{itkl}) \end{array} \middle| \forall\ i, t, k \right\} \quad (17)$$

$S.t.$

$$P_{itk} + I_{i(t-1)k} + \sum_j T_{jitk} + \sum_l M_{itlk} = \delta_{itk} + I_{itk} + \sum_j T_{ijtk} + \sum_l M_{itkl} \quad (18)$$

$$P_{itk} \leq S_{itk} * M \quad (19)$$

$$P_{itk}, T_{ijtk}, I_{itk}, M_{itkl} \geq 0 \quad (20)$$

$$P_{itk}, T_{ijtk}, I_{itk}, M_{itkl} = Integer \quad (21)$$

$$S_{itk} = Binary \quad (22)$$

As mentioned previously, it is possible to use commercial integer programming solvers, with appropriate reformulations, to attempt to solve this problem; research on this is ongoing

Conclusion

This section examines inventory costs in the context of defense operations. Based on the argument that inventory costs in defense operations are not the same as those in commercial enterprises, it is proposed that inventory effectiveness in this context should be measured in terms of the ability to meet a range of anticipated and sometimes unanticipated threats. This does not necessarily mean that planning can only be for “known knowns” and “known unknowns,” but not for “unknown unknowns.” Initial models have been developed to examine inventory decisions for complex products, that is, those composed of multiple subassemblies in which there are shared subassemblies among different product types. It is possible that the option for storing partially completed assemblies may also help in meeting demand uncertainties. Thus, when faced with uncertain demand for one or more products over a geographically distributed domain, the set of recourses for a manufacturer/planner include excess production (inventory storage), rapid re-location of inventory, production surges, when to upgrade technology or procure new models, the level of assembly at which to store the products and where to store these, as well as in what quantities and ratios of product types. Solutions of mathematical models are illustrated and simulations to assess the utility of the solutions obtained by analytical methods are also presented.

References

Arrow, K. J., Karlin, S., & Scarf, H. (1958). *Studies in the mathematical theory of inventory and production.* Palo Alto, CA: Stanford University Press.

Begnaud, J., Benjaafar, S., & Miller, L. (2009). The multi-level lot sizing problem with flexible production sequences. *IIE Transactions, 41*(8), 702–715.

Belvaux, G., & Wolsey, L. (2000). BC-prod: A specialized branch-and-cut system for lot-sizing problems. *Management Science, 46*(5), 724–738.

Blackburn, J., & Millen, R. (1982). Improved heuristics for multi-stage requirements planning systems. *Management Science, 28*(1), 44–56.

Burke, G., Carrillo, J., & Vakharia, A. (2007). Single versus multiple supplier sourcing strategies. *European Journal of Operational Research, 182*(1), 95–112.

Christian, H. (2010). *Maximizing component reuse for sustainable operations* (Master's thesis). Kingston, RI: University of Rhode Island.

Crowston, W., & Wagner, M. (1973). Dynamic lot size models for multi-stage assembly systems. *Management Science, 20*(1), 14–21.

Deshpande, V., Cohen, M., & Donohue, K. (2003). A threshold inventory rationing policy for service-differentiated demand classes. *Management Science, 49*(6), 683–703.

Fisher, M. (1980). Worst-case analysis of heuristic algorithms. *Management Science, 26*(1), 1–17.

Gerchak, Y., & Mossman, D. (1992). On the effect of demand randomness on inventories and costs. *Operations Research, 40*(4), 804–807.

Goyal, S., & Gupta, Y. (1989). Integrated inventory models: The buyer–vendor coordination. *European Journal of Operational Research, 41*(3), 261–269.

Graves, S. (1985). A multi-echelon inventory model for a repairable item with one-for-one replenishment. *Management Science, 31*(10), 1247–1256.

Hilsenrath, J. (2011, March 28). Gates says Libya not vital national interest. *Wall Street Journal*. Retrieved from http://online.wsj.com/article/SB10001424052748704308904576226704261420430.html?mod=WSJ_hps_sections_world

Husen, T. (2012). *Development of solution approaches for optimally solving the stochastic parallel machine replacement problem* (Master's thesis). Kingston, RI: University of Rhode Island.

Karaesmen, I., & Van Ryzin, G. (2004). Overbooking with substitutable inventory classes. *Operations Research, 52*(1), 83–104.

Keynes, J. (2006). *The general theory of employment, interest and money.* New Delhi, India: Atlantic.

Lee, H. (1987). A multi-echelon inventory model for repairable items with emergency lateral transshipments. *Management Science, 33*(10), 1302–1316.

Love, S. (1972). A facilities in series inventory model with nested schedules. *Management Science, 18*(5), 327–338.

McKeon, B. (2012). *Utilizing modular upgrades to maintain an effective inventory of complex machines* (Doctoral dissertation). Kingston, RI: University of Rhode Island.

Muckstadt, J. (1973). A model for a multi-item, multi-echelon, multi-indenture inventory system. *Management Science, 20*(4), 472–481.

Murphy, M. (1999). *Collocating Air Force weapon systems inventory with the Defense Logistics Agency Premium Service facility.* Maxwell Air Force Base, AL: Department of the Air Force.

Nam, I. (2001). Dynamic scheduling for a flexible processing network. *Operations Research, 49*(2), 305–315.

Perea, F., Puerto, J., & Fernández, F. (2009). Modeling cooperation on a class of distribution problems. *European Journal of Operational Research, 198*(3), 726–733.

Porteus, E. (1986). Optimal lot sizing, process quality improvement and setup cost reduction. *Operations Research, 34*(1), 137–144.

Schulze, M. (2010). *Optimization using common part strategies in closed-loop supply chains* (Master's thesis). Kingston, RI: University of Rhode Island.

Sherbrooke, C. (1968). METRIC: A multi-echelon technique for recoverable item control. *Operations Research, 16*(1), 122–141.

Simon, R. (1971). Stationary properties of a two-echelon inventory model for low demand items. *Operations Research, 19*(3), 761–773.

Suerie, C., & Stadtler, H. (2003). The capacitated lot-sizing problem with linked lot sizes. *Management Science, 49*(8), 1039–1054.

Tempelmeier, H., & Derstroff, M. (1996). A Lagrangean-based heuristic for dynamic multilevel multiitem constrained lotsizing with setup times. *Management Science, 42*(5), 738–757.

Tijms, H. (2003). *A first course in stochastic models*. New York, NY: Wiley.

Vollenberg, F. (2010). *Optimization of system efficiency with consideration of deterioration processes* (Master's thesis). Kingston, RI: University of Rhode Island.

Wagner, H., & Whitin, T. (1958). Dynamic version of the economic lot size model. *Management Science, 50*(12), 1770–1774.

Williams, A. (1963). A stochastic transportation problem. *Operations Research, 11*(5), 759–770.

Wolsey, L. (2002). Solving multi-item lot-sizing problems with an MIP solver using classification and reformulation. *Management Science, 48*(12), 1587–1602.

Yanmei, L., Jiangsheng, S., Sujian, L., & Weimin, L. (2008). Simulation and research on the repairable valuable spare parts inventory model in weapon equipment. In *Proceedings of the IEEE International Conference on Automation and Logistics (ICAL 2008)*. New York City, NY: IEEE.

Zhao, X., & Atkins, D. (2009). Transshipment between competing retailers. *IIE Transactions, 41*(8), 665–676.

THIS PAGE INTENTIONALLY LEFT BLANK

ACQUISITION RESEARCH PROGRAM
GRADUATE SCHOOL OF BUSINESS & PUBLIC POLICY
NAVAL POSTGRADUATE SCHOOL
555 DYER ROAD, INGERSOLL HALL
MONTEREY, CA 93943

www.acquisitionresearch.net