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Determining New System Design Requirements to Optimize Fleet Level Metrics under Uncertainty

3 July 2017

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Prof. Crossley has interacted with industry in many roles, including several consulting arrangements, summer visits and research collaborations. He was a Boeing Welliver Faculty Summer Fellow in 2002. Before joining Purdue, he participated in the McDonnell Douglas Helicopter Systems / Arizona State University Industrial Fellows program, in which he was a member of the Advanced Concept Development Group responsible for conceptual design studies of advanced helicopters and rotorcraft, working part-time (20 hours) during the academic year and full time (40 hours) during summers while earning his MS and Ph.D. degrees.

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Executive Summary

Traditional approaches to design and optimize a new system often do not consider how the operator will use this new system alongside the other existing systems. This "hand-off" between the designs of the new system and how this new system operates with the group of systems, leads to the sub-optimal performance of the new system when measured with respect to system-level objective. In the case of aircraft design, choices made to meet a set of requirements dictate the performance of the aircraft, and this aircraft performance in turn influences how the operator might use the aircraft. Further, the presence of uncertainties in predictions of the new aircraft performance and costs and uncertainties in the amount of payload / passenger to transport further exacerbate the problem of determining these requirements. Recent efforts have posed approaches to address this problem, but generally with a deterministic perspective.

This research improves upon prior work by extending a prior developed subspace decomposition framework to enable capability that addresses multi-domain uncertainties. The framework addresses uncertainties arising in one domain and its propagation to the next connected domain. The framework employs a Reliability-Based Design Optimization (RBDO) approach to address the uncertainties arising from the aircraft design optimization subspace and employs an Interval Robust Counterpart (IRC) formulation to address the uncertainty propagation from the design subspace to the allocation subspace.

The research adopts a previously developed subspace decomposition approach and integrates features from robust / reliability based optimization to address the uncertainties and solves two application problems – a military and a commercial airline application. The military application involves an Air Mobility Command (AMC) fleet problem, and, the commercial airline applications reflects typical operations of a US based carrier. The framework demonstrates its ability to acceptably handle uncertainties arising from various domains. Results of application also demonstrates the ability of the framework to identify the design requirements for the new aircraft, and a posterior analysis indicates that the framework acceptably handles the uncertainties.

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Technical Report

Background and Introduction

The ‘Better Buying Power 3.0’ (Kendall, F., 2014) document states: “Defining requirements well is a challenging but essential prerequisite in achieving desired service acquisition outcomes”. Traditional acquisition processes focus on development of requirements at the system-level. Current acquisition analyses of design alternatives are disjointed from considering operations (the way an end user operates these new systems alongside existing ones), resulting in inefficiencies at the higher aggregate level (Taylor, C., & de Weck, O., 2007, Mane et al., 2007). Typical design practice for new systems assumes a “handoff” between the design of the new, yet-to-be introduced system, and the operations on how the system impacts top-level performance.

The authors proposed an approach that would include top-level requirements for a new system as decision variables in an optimization problem. With the objective to maximize (or minimize) a fleet-level performance metric, then an optimization algorithm should determine the “right requirements” as part of finding the optimal set of decision variable values. Using aviation examples, one can pose the optimization problem that included top-level requirements as decision variables along with new system design variables and operational decision variables. The resulting formulation is a mixed-integer nonlinear programming problem that is very difficult if not impossible to solve in reasonable time. The authors and their colleagues have developed a decomposition approach that allows solution of this problem, with a few minor modifications from the original problem.

The initial efforts concentrated on demonstrating that solving the decomposition approach was practical and that the results were useful; however, those initial efforts could not address data uncertainties in the problem. The recent work has identified and demonstrated how to include consideration for various types of data driven uncertainties as well. With the focus on aviation examples, the work first considered an application of the decomposition approach under uncertainty to military air cargo transportation using actual data from the US Air Force Air Mobility Command (AMC) as the basis for a set of example problems. Then, to explore the flexibility of the decomposition approach under uncertainty, data from the Bureau of Transportation Statistics provided the basis for another set of example problems representative of commercial airlines.

This report presents how the approach applies to both military air cargo problems and to commercial airline problems and how the approach handles uncertainties in the aircraft design sub-problem, propagates those uncertainties to the allocation (commercial airline) or assignment (military air cargo) sub-problem, and additionally considers demand uncertainty in the allocation or assignment sub-problem. While the overall decomposition framework can address these two different aviation problems under uncertainty, there are some specific modifications necessary to represent these two different problems.

The approach is able to identify the best requirements for a new aircraft for both the commercial airline and military air cargo problems. *A posteriori* analysis of the resulting design shows the advantages that the approach under uncertainty has over deterministic approaches to the same problems.

Methodology

This section describes in detail the methodology that uses the previously developed subspace-decomposition approach (Crossley et al. 2004, Mane et al. 2007). The approach serves as a 'meta-algorithm' framework within which specific choices in performance metrics and resource constraints can be made for each of the two problem instantiations we have solved (AMC and Commercial Airline) in prior work (Govindaraju et al. May, 2015, Roy et al. Jan, 2017). The description of each subspace and the information flow between subspaces appears in Figure 1 below.

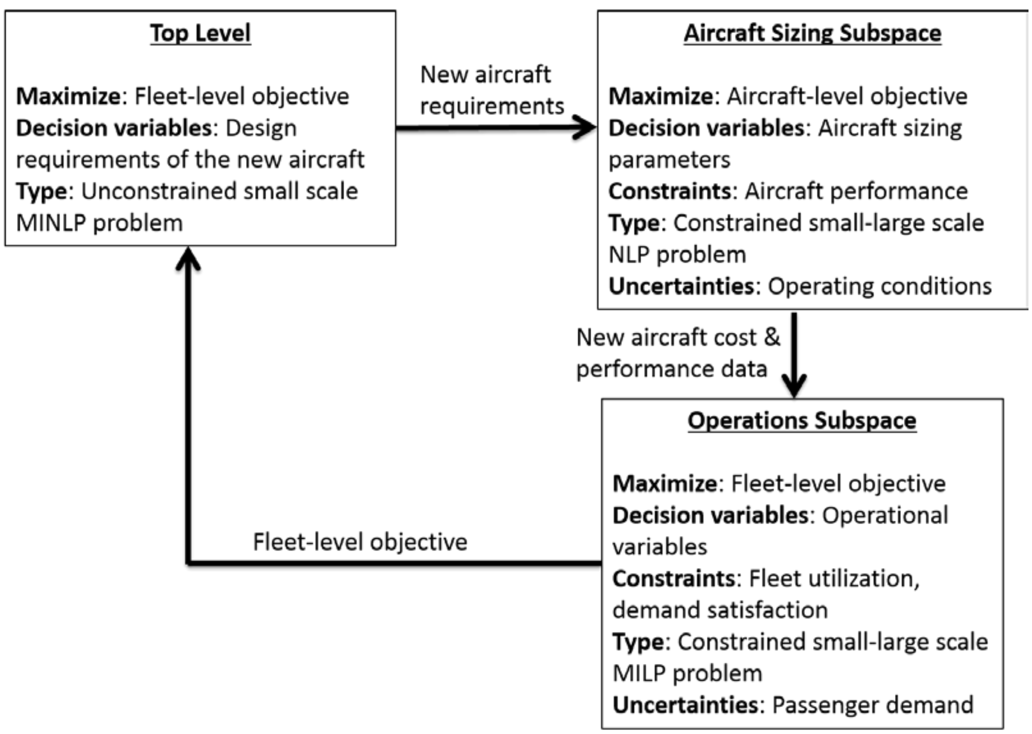


Figure 1. Overview of the sequential decomposition framework

Top-level subspace

The top-level problem seeks to maximize the fleet-level objective of the operator, based upon the choice of the design requirement of the new yet-to-be-designed aircraft. These top-level requirements include design range, payload-carrying capacity etc. of the new yet-to-be-designed aircraft. This level is a small-scale Mixed Integer Non-Linear Programming (MINLP) problem and is solved either using an MINLP solver or by performing a pseudo enumeration.

Maximize Fleet level objective

Design variables Design range **Top: MINLP**

Payload carrying capacity

Aircraft sizing subspace

The decision variables from the top level appear in the aircraft sizing sub-space as parameters. Starting from these top-level requirements, this subspace solves an aircraft design optimization problem with the objective that minimizes the design mission direct operating cost. The decision variables for this sub-problem are the variables that defines the wing geometry such as aspect ratio, taper ratio, sweep, etc. and the engine parameters like static thrust, bypass ratio, fan pressure ratio etc. Further, the portion of the aircraft conceptual design phase known as ‘aircraft sizing’, usually uses empirical equation and simplified physical models to predict the cost and performance of the aircraft. The limited knowledge available at this phase of the design process combined with the modeling fidelity results in high uncertainty. For instance, an aircraft is sized for its design mission based on a set of nominal values for operating conditions (e.g., cruise altitude). However, when evaluating the “operating missions” to determine block time and fuel consumed on the flight, there might be a variation in winds aloft, which would alter the block time and fuel consumed. Additionally, predictions of the aircraft performance and characteristics, like parasite drag, that use low-fidelity models will have associated uncertainty. It is therefore necessary to simulate the effect of uncertainties on the design parameters, in the absence of closed form mathematical expressions, for subsequent inclusion in the resulting aircraft sizing optimization problem. We employ a reliability-based design optimization (RBDO) formulation on the new aircraft that is subject to a collection of uncertain parameters. This sub-problem is Non-Linear Programming (NLP) problem that can be solved using a choice of NLP solver such as the *fmincon* function in MATLAB.

Minimize Design mission expected direct operating cost

Design variables Wing design variables

Engine design variables **Size: NLP**

Operations subspace

Operations subspace seeks to solve how the operator uses the new yet-to-be-designed aircraft alongside the existing fleet of aircraft. This is an allocation problem that allocates the new aircraft together with the existing aircraft with the goal to maximize the fleet-level objective. The strategy involves assigning or allocating the fleet on various routes. This sub-problem is posed as a Mixed Integer Linear Programming (MILP) problem with both integer (allocation variables) and the continuous (payload) type variables and is solved using the CPLEX solver available within the GAMS (Brooke et al., 1998) software package. This sub-problem is subjected to operational constraints such as aircraft utilization, demand etc. Further the demand in this subspace is uncertain. The amount of payload to carry across the various routes is an uncertain parameter. Thus, we have two levels of uncertainties that interact and need some strategies to address the propagation of uncertainty from one domain to the other. The new aircraft coming out of the aircraft sizing subspace has uncertain performance and cost coefficients. Our approach employs an Interval Robust Counterpart (IRC) (Lin 2014) formulation to address this uncertainty propagation from the sizing sub-space to the allocation subspace. We size the aircraft at two cases of the uncertain parameters of the aircraft sizing subspace: a nominal case and a worse case and use the IRC formulation to enforce the worse-case performance and cost in the allocation constraints using some tolerance limit. An overview of the operations sub-problem (Alloc: MILP) appears below.

Maximize Expected Fleet-level objective **Design variables** Allocation (integer type) Payload (continuous type)	**Alloc: MILP**

In the following two sections, we detail application of the subspace decomposition approach for the case of setting optimal requirements for military air cargo, and, for commercial airline systems. We mainly highlight key differences in modeling approach for each subsection, to illustrate flexibility of the framework in accommodating unique problem characteristics of each case.

Applications of Subspace Decomposition Approach

Case1 - Military Air Cargo

We use the subspace decomposition approach to determine the optimal requirements of a new, yet-to-be introduced system (here, strategic airlift aircraft), which will operate alongside other strategic military airlift aircraft of the United States Air Force Air Mobility Command (AMC). The problem was motivated by the USAF AMC's emphasis on reducing fleet wide fuel consumption. The objectives are to maximize expected fleet productivity and minimize expected fuel consumption. As these are competing objectives, the problem is posed in a multi-objective sense where fleet wide fuel consumption is minimized and a minimum acceptable fleet productivity level is set as a constraint that is varied to generate a series of non-dominated Pareto solutions. Data on cargo demand is obtained from the Global Air Transportation Execution System (GATES) dataset for the year 2006. Figure 2 illustrates the subspace decomposition of the AMC problem statement.

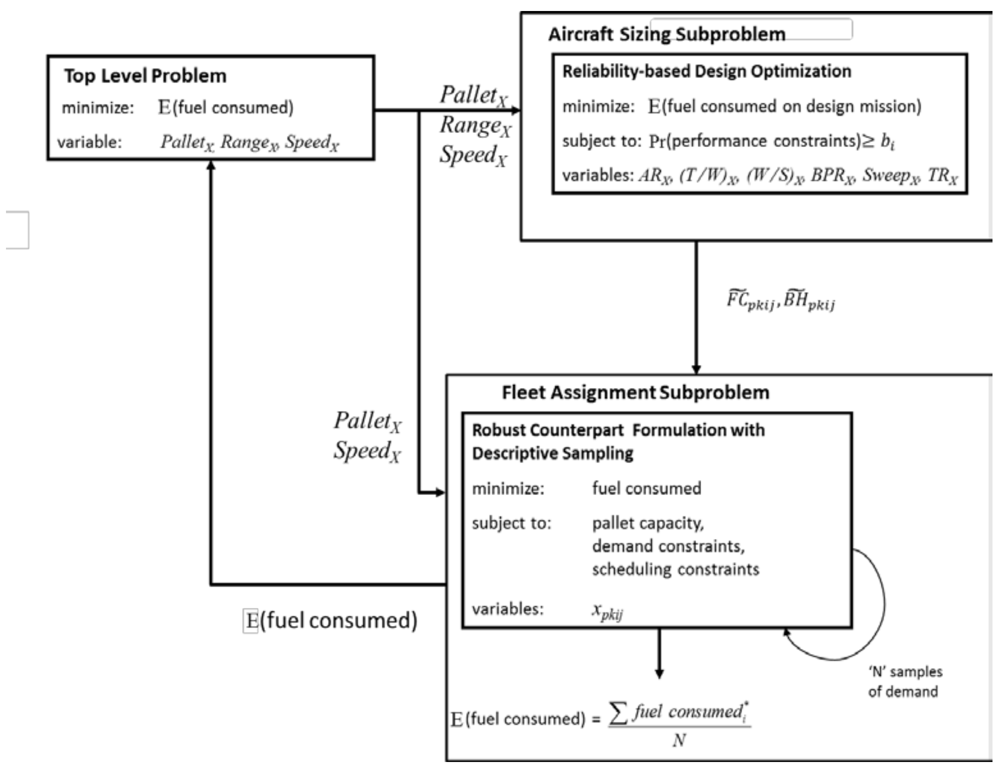


Figure 2. Subspace decomposition strategy for the USAF AMC application

Top-level subspace

The top-level optimization problem does not include any nonlinear constraints and only has bounds imposed on the top-level decision variables. Equations below describe the deterministic formulation of the top-level problem; the formulation

Minimize: $fleet_{fuel}$ (Fleet-level fuel consumption)

Subject to $14 \leq Pallet_X \leq 38$

$$2400 \leq Range_X \leq 3800 \quad \text{(Eq. 1)}$$

incorporating uncertainty appears in latter subspaces.

The objective function seeks to minimize the fleet-level fuel consumption using pallet capacity, range and cruise speed of the new, "yet-to-be-introduced" aircraft type X as decision variables. This subspace is subject to variable bound constraints. The values for the bounds were based on strategic airlift requirements, and characteristics exhibited by current cargo transport aircraft (Gertler, J. 2010, Graham, D., et al. 2003). Here, the design requirement decision variable describing payload capacity uses an integer number of pallets, while the design range and design speed decision variables are treated as continuous.

Aircraft sizing subspace

The conceptual phase of the aircraft design process relies upon semi-empirical equations and simplified physics models. The limited knowledge available about the system definition at this phase of the design process combined with the usage of low-fidelity modeling tools results in high uncertainty. Aircraft sizing typically determines the size, weight and performance of an aircraft to meet its design mission based on a set of nominal values on operating conditions (e.g. cruise altitude). However, when evaluating the "operating missions" to determine block time and fuel consumed on the flight, there might be a variation in assigned altitude, routing, speed, etc., which would alter the block time and fuel consumed. For instance, there is uncertainty in the prediction of the parasite drag coefficient.

Uncertainty in Design Parameters

The effort here uses a scaling factor k_{C_D} that follows a triangular distribution as appears in Figure 3 to represent the uncertainty in the parasite drag prediction, so that the "actual" coefficient relates to the "predicted" coefficient in the following manner:

$$C_{D_{0,actual}} = k_{C_D} . C_{D_0,predicted} \quad \text{(Eq. 2)}$$

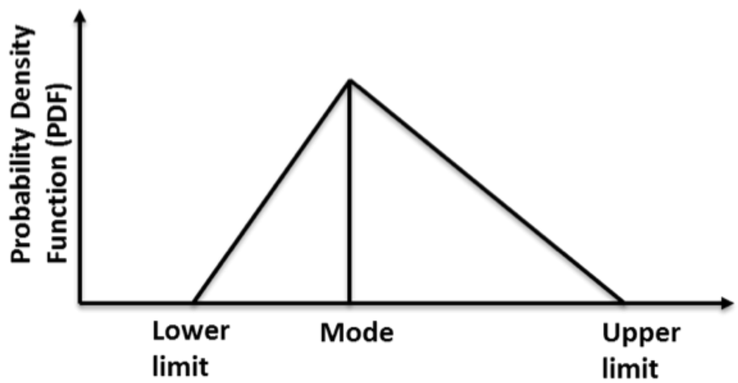


Figure 3. The triangular PDF distribution of the aircraft design uncertain parameters

To address the uncertainty related to operations and predictions of the new aircraft performance in the aircraft sizing subspace with reasonable computational expense, the Analysis of Variance (ANOVA) technique - a sensitivity analysis method, determined the subset of the most important parameters that influence the outputs under consideration (Montgomery, D. C., 2008). This investigation assumes triangular distributions (see Figure 3) for the scaling factors of identified parameters listed in Table 1.

Table 1. Triangular distributions of the ANOVA identified uncertain parameters in the aircraft sizing subspace

Uncertain Parameters (ξ)	Lower limit	Mode	Upper Limit
C_{D_0} multiplier, k_{C_D}	0.90	1.0	1.10
Specific Fuel Consumption, SFC [hr^{-1}]	0.45	0.5	0.55
Oswald efficiency multiplier, k_{e_0}	0.95	1.0	1.05
Cruise altitude [ft]	32000	35000	38000
Pallet mass [lbs]	7200	7500	7800

The aircraft sizing subproblem seeks to minimize the design mission fuel consumption of the new, yet-to-be-introduced aircraft for the values of design range ($Range_X$), pallet capacity ($Pallet_X$), and cruise speed ($Speed_X$) appearing as parameters from the top-level problem. With the top-level objective to minimize fleet-level fuel consumption, and the aircraft sizing objective to minimize the fuel consumed by the new aircraft for its prescribed design range, pallet capacity, and cruise speed, a slight disconnect exists between the objectives of these two levels. The difference in the objectives is that, at each aircraft sizing iteration, the minimization of fuel consumption uses a single combination of fixed values for design range, pallet capacity, and cruise speed—this is the typical case in aircraft design where these quantities are set as requirements for some 'representative design mission'. However, the top-level optimization problem drives the question of 'what requirements do we need to set in the first place?' by searching through the decision space of the top-level variables to find aircraft requirements that optimizes fleet-level operational aspects of how the aircraft is used.

For example, consider the dimension of design range—as the top-level problem searches across values of range, this naturally changes the set of feasible routes that the new aircraft can fly, thereby changing how the fleet comprising of existing and new aircraft serves the overall route network. By doing so, the top-level problem seeks additional fleet-wide fuel savings that these operational aspects reflect as a function of the decision variables. Therefore, the aircraft sizing objective can be viewed as a subset of the top-level problem objective. Because the type of aircraft assigned on individual flight segments drives the total amount of fuel consumed by the

$$Minimize: E(x, \xi)$$

$$Subject\ to: P[g(x, \xi) \leq 0] \geq b_i \quad \forall\ i = 1,2,..,m$$

(Eq. 3)

fleet, an aircraft designed for minimal fuel consumption will lead to improved fleet utilization that reduces fleet-level fuel consumption, when compared to fleet operations using only the fleet of existing aircraft. The approach in this work poses the aircraft design subproblem in the context of Reliability Based Design Optimization problem to account for uncertainty in the design phase. The Reliability-Based Design Optimization (RBDO) formulation (shown below) represents the aircraft design under uncertainty problem.

Aggregating the outputs for each realization (sample) of the uncertain parameter, allows for the estimation of statistical measures such as expectation and probability, which the objective and constraint function evaluations require. The objective of the aircraft sizing subspace is to minimize the fuel consumption of the new aircraft X using the decision variables listed in Table 2. For each function evaluation of the top-level problem, the current values of $Pallet_X$, $Range_X$, and $Speed_X$ become fixed parameters for the aircraft sizing problem. Table 2 summarizes the decision variables, constraints along with their bounds in the aircraft sizing optimization problem.

Table 2. Decision variables and constraints limits in the aircraft sizing optimization problem

Decision variables, (x)	Lower Bound	Upper Bound
Wing Aspect Ratio, AR_X	6.00	9.50
Thrust-to-weight Ratio, $(T/W)_X$	0.18	0.35
Wing Loading [lb/ft^2], $(W/S)_X$	65.00	161.00
Engine Bypass Ratio, BPR_X	4.50	14.50
Wing Leading Edge Sweep [deg], $Sweep_X$	10.00	35.00
Wing Taper Ratio, TR_X	0.10	0.40
Performance Constraints	**Value**	
Takeoff Distance [ft]	≤ 8500	
Landing Distance [ft]	≤ 5500	
Second segment climb gradient	≥ 0.025	
Top-of-climb rate [ft/min]	≥ 500	

The aircraft sizing sub problem includes performance constraints such as limits on take-off and landing distances, and also upper and lower bounds for the decision variables. The RBDO formulation optimizes the expected performance metric of interest and ensures that the probability of satisfying the performance constraints is greater than or equal to the user-defined reliability level, b_i, considering the uncertainty present in this subproblem.

Fleet operations subspace

This subspace mathematically represents the AMC's operations where the AMC fleet flies cargo missions to deliver pallets of supplies on an "as-needed" basis without a predetermined, long term schedule. The fleet assignment model here considers the multiple destination nature of the flight path for each aircraft, where an aircraft may fly from point A to B and then on to C – this in contrast is different to the airline case where airline aircraft are assigned to fly back and forth on specific segments points. This multiple destination travel path prompts the need to include tracking of "tail numbers" in the fleet operations subspace. Furthermore, the unscheduled and uncertain nature of demand for cargo transportation includes unknown origin and destination pairs of trips as well – this is modelled using random sampling of starting points for aircraft where the random sample mimics the end of the previous day flight termination point of the aircraft. The Interval Robust Counterpart (IRC) formulation addresses uncertainty in parameters within AMC fleet operations model; in this case the uncertainty associated with the fuel consumption rate $\widetilde{FC}_{p,k,i,j}$, and in the flight block hours $\widetilde{BH}_{p,k,i,j}$, on given routes in the service network. The optimization problem of the fleet operations model seeks to minimize the fleet level fuel consumption while enforcing a constraint on productivity. Mathematically, the formulation appears below.

Minimize:

$$\sum_{p=1}^{P}\sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{N} x_{p,k,i,j} \times FC_{p,k,i,j} \qquad \text{(Fleet-level fuel consumption) (Eq.4)}$$

Subject to:

$$\sum_{p=1}^{P}\sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{N} x_{p,k,i,j} \times \left(Speed_{p,k,i,j} \times Cap_{p,k,i,j}\right) \geq L \qquad \text{(Fleet-level productivity limit) (Eq.5)}$$

$$\sum_{i=1}^{N} x_{p,k,i,j} \geq \sum_{i=1}^{N} x_{p,k+1,i,j} \quad \forall k = 1,2,3\ldots K, \quad \forall p = 1,2,3\ldots P, \quad \forall j = 1,2,3\ldots N \qquad \text{(Node balance constraints) (Eq.6)}$$

$$\sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{N} x_{p,k,i,j} \times BH_{p,k,i,j} \leq B_P \quad \forall p = 1,2,3\ldots P \qquad \text{(Daily utilization limit) (Eq.7)}$$

$$\sum_{p=1}^{P}\sum_{k=1}^{K} Cap_{p,k,i,j} \times x_{p,k,i,j} \geq dem_{i,j} \quad \forall i = 1,2,3\ldots N, \forall j = 1,2,3\ldots N \qquad \text{(Pallet demand constraints) (Eq.8)}$$

$$\sum_{i=1}^{N} x_{p,1,i,j} \leq O_{p,i} \quad \forall p = 1,2,3\ldots P, \forall i = 1,2,3\ldots N \qquad \text{(Starting location constraints) (Eq.9)}$$

$$\sum_{i=1}^{N}\sum_{j=1}^{N} x_{p,k,i,j} \leq 1 \quad \forall p = 1,2,3...P,\ \forall k = 1,2,3...K \qquad \text{(Trip limit) (Eq.10)}$$

$$x_{p,k,i,j} \in \{0,1\} \qquad \text{(Binary variable)}$$

Equation (4) is the objective function that seeks to minimize the fleet-level fuel consumption, where $FC_{p,k,i,j}$ indicates the fuel consumption coefficient of the k^{th} trip for aircraft *p* from base *i* to base *j*. The equation has two parts; the first product inside the square brackets, $x_{p,k,i,j} \times FC_{p,k,i,j}$, represents the fuel consumption of the existing fleet, while the rest of the terms inside the square brackets represents the fuel consumption of assigning the new, yet-to-be-designed aircraft. The fuel consumption characteristics of the new aircraft are a function of aircraft design variables (aspect ratio, thrust-to-weight ratio, etc.) and aircraft design requirements (pallet capacity, design range, and cruise speed). The term $x_{p,k,i,j}$ is a binary decision variable that takes a value of 1 if the k^{th} trip of aircraft type p is flown from base *i* to base *j*, and it takes a value of 0 otherwise.

Equation (5) accounts for the multi-objective nature of this problem. This forces the fleet-level productivity to be greater than a pre-defined limit, *L*; the limit is varied and the problem is re-solved for each varied value of the limit to generate a set of Pareto optimal solutions. The term, $x_{p,k,i,j} \times FC_{p,k,i,j}$, in Equation (4) refers to the productivity (speed of payload delivered) of utilizing aircraft type *p* for the k^{th} trip from base *i* to base *j*.

Equation (6) is the balance and sequencing constraint that enables the $(k+1)^{th}$ trip of an aircraft out of a base, *i*, to occur only after the k^{th} trip of that aircraft into base *i*. This constraint ensures that an aircraft needs is present at a base prior to completing a subsequent segment trip out of the same base.

Equation (7) limits flights to only occur within the daily utilization limit, B_p (here, this uses an assumption of 16 hours per day to account for loading, unloading, servicing, maintenance, etc.) of the aircraft, where $BH_{p,k,i,j}$ indicates the block hour of the k^{th} trip for aircraft *p* from base *i* to base *j*.

Equation (8) ensures that the carrying capacity of the combined trips meets or exceeds the pallet demand on each route, where $Cap_{p,k,i,j}$ indicates the pallet carrying capacity of the k^{th} trip for aircraft *p* from base *i* to base *j*.

Equation (9) ensures that the first trip of each aircraft *p* originates at its initial location (this is considered the aircraft's home or starting base for the day of operations); this initial location is randomly generated. Because the GATES dataset does not clearly indicate the starting location of aircraft each day, the problem formulation here uses a random distribution for each aircraft's starting location. The term $O_{p,i}$ is a binary variable that indicates if base *i* is the initial location for aircraft *p*.

Equation (10) ensures that each aircraft *p* flies at most one trip for its k^{th} segment.

The motivation for the "scheduling-like" formulation is to represent the scheduling and operations decisions made by Air Mobility Command; it does not explicitly consider pilot scheduling (this 16 hours per day of available aircraft time could represent this, in part) nor does it account for the prioritization of cargo (this is not addressed in this formulation). This formulation, using node balance constraints, allows individual aircraft to make multiple flight segments in one day (as long as these fit within a prescribed time limit), allows for pallets to be carried from their origin to destination on possibly multiple aircraft, and tracks each individual aircraft by "tail number". These features more directly model AMC operations than some of the previous models of the authors and their colleagues when considering passenger airline transportation (Mane, et al., 2007, Govindaraju, P., et al., 2013).

Uncertainty in Fleet Operations

The uncertainty associated with the performance of the newly designed aircraft (type *X*) propagates to the fleet assignment subspace through the distributions of the new aircraft's predicted fuel consumption, $\widetilde{FC}_{p,k,i,j}$, and flight block hours, $\widetilde{BH}_{p,k,i,j}$, on given routes in the network; only aircraft "tail numbers" *p* that are associated with type *X* aircraft have these distributions. Additionally, the AMC service network has inherent pallet demand uncertainty. Hence, the fleet assignment problem now includes uncertainty in both the performance of the new aircraft and the pallet demand in the service network. In this paper, a hybrid formulation that combines the interval robust counterpart formulation (Lin, X., et al., 2004) for user-defined tolerance parameters (δ), and the descriptive sampling technique (Saliby, E., 1990), solves the fleet assignment problem under uncertainty.

Lin *et al*. proposed a robust optimization approach for bounded uncertainty to overcome the large computational expense incurred by scenario/sampling-based frameworks. Their approach produces "robust" solutions that are immune against uncertainties in both the coefficients and right-hand-side parameters of the inequality constraints of the Mixed Integer Linear Programming (MILP) problems. Lin *et al*. term a solution to be robust, if it satisfies the following conditions:

- The solution is feasible for the nominal values of the uncertain parameters
- For any value of the uncertain coefficients in the objective function and the uncertain parameters in the right-hand side of the constraints, the solution must satisfy the i^{th} inequality constraint or, at worst, violate the constraint with an error of at most $\delta \times \max\ [1, |p_l|]$. In this expression, δ is a user-selected infeasibility tolerance coefficient, and p_l is the right-hand-side limit of the linear inequality constraint.

Applying the interval robust counterpart model to the deterministic formulation of the fleet assignment subproblem described above results in two additional set of constraints, and a modified objective function where an auxiliary variable (*Fleet fuel*) is introduced to enable introduction of the original objective function represented by Eq. 4 as a constraint – thereby making it amenable to robust optimization strategies. The reformulation of the original objective function (Equation 4) is now as follows:

$$\textit{Minimize} \quad \textit{Fleet fuel} \qquad \text{(Eq.11)}$$

$$\textit{Subject to} \quad \sum_{p=1}^{P}\sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{N} x_{p,k,i,j} \times FC^{U}_{p,k,i,j} \le \textit{Fleet fuel}\left(1+\delta\right) \qquad \text{(Eq.12)}$$

where, $FC^{U}_{p,k,i,j}$ is the upper bound of the fuel consumed by aircraft *p* on the k^{th} trip from base *i* to base *j*. Evaluating the performance of the new aircraft for different samples of the aircraft sizing uncertain parameters (ξ) generates distributions of the performance metrics such as the fuel consumption coefficient. The upper bound, $FC^{U}_{p,k,i,j}$, is then determined from the distribution of the fuel consumption coefficient $\widetilde{FC}_{p,k,i,j}$ applied to only aircraft *p* that are of the newly-designed type X. δ is the user-defined, infeasibility tolerance parameter that can take values between 0 and 1. For example, setting δ to 0.1 for a particular constraint indicates that 10% violation of the worst-case scenario of that constraint is acceptable. Using Eq. 12, if all of the uncertain fuel consumption coefficients for the new aircraft are at their upper bound (i.e., the aircraft burns the most possible fuel from the distribution, ($\widetilde{FC}_{p,k,i,j}$), then the total fuel consumed by the fleet is no more than 10% above the user-defined limit for fleet fuel consumption. The daily utilization limit constraint (Equation 7) is modified as follows:

$$\sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{j=1}^{N} x_{p,k,i,j} \times BH^{U}_{p,k,i,j} \leq B_P\left(1+\delta\right) \quad \forall p = 1,2,3\ldots P \qquad \text{(Eq.13)}$$

where, $BH^{U}_{p,k,i,j}$ is the upper bound of the distribution of block hours of aircraft *p* (restricted to only aircraft of type X) on the k^{th} trip from base *i* to base *j*. The deterministic robust counterpart fleet assignment problem now includes Eq. 11, 12, and 13 in addition to Eq. 4 - 10 from the original deterministic formulation of the fleet assignment problem.

The interval robust counterpart model is also applicable for the demand constraint (Equation 8) in the deterministic formulation, but this leads to a very conservative (protected against the maximum demand scenario) solution, because the right hand side constraint limit, $dem_{i,j}$, is set to its upper bound or maximum value, $dem^{U}_{i,j}$, for each route as shown in Eq. 14 below. For this constraint, the GATES dataset provides the values for the upper bound of the pallet demand on each route.

$$\sum_{p=1}^{P}\sum_{k=1}^{K} Cap_{p,k,i,j} \times x_{p,k,i,j} \geq dem^{U}_{i,j} \qquad \text{(Eq.14)}$$
$$\forall i = 1,2,3\ldots N, \forall j = 1,2,3\ldots N$$

Instead, because the AMC service network experience high fluctuations in pallet demand and the on-demand nature of military cargo transport, the approach here employs a descriptive sampling approach (Saliby, E., 1990) to incorporate the stochastic nature of the demand. The method of descriptive sampling involves a deliberate collection of sample values that closely describes the represented distribution. The descriptive sampling approach samples more values from regions of higher density and fewer values from regions of lower density. The purposeful collection of sample values at specific quantile levels helps to match closely the actual or reported discrete demand distributions using a reduced number of samples, thus reducing the computational expense.

The deterministic robust counterpart formulation is solved times for each demand sample vector generated through the descriptive sampling approach. From these multiple solutions, the expected value of the fleet-level performance metrics (fleet-level fuel consumption and / or fleet-level productivity) now returns to the top-level optimization problem as the responses of interest. The robust counterpart formulation accounts for the propagation of uncertainty from the aircraft sizing to the fleet assignment subspace, while the descriptive sampling approach addresses the stochastic nature of pallet demand in the service network.

Problem Description: 25-Base Network Problem

The section demonstrates how the subspace decomposition approach can identify the best new aircraft requirements and subsequent aircraft design to address fleet-level metrics under uncertainty for the AMC – Military application. By treating this problem as a multi-objective problem, the approach can also generate tradeoffs between fleet-level metrics of interest; from these best tradeoff solutions, a decision-maker can also observe how the optimum design requirements for the new aircraft change for these different tradeoff opportunities.

Network Description

This study uses a subset of the AMC route network and fleet, comprising of 25 bases and 219 directional routes, to demonstrate the approach. Figure 3 depicts the geographical locations and routes of the 25-base network. For the 25-base network, the existing fleet of AMC comprises of 28 C-5s, 44 C-17s, and 21 chartered 747-Fs. The existing fleet serves as a 'baseline' to measure the improvements due to the introduction of the new aircraft. This study assumes that five, new, yet-to-be-designed-aircraft (all of type X) are introduced into the fleet. This assumption reflects an external decision made by the user or the decision-maker that specifies the number of new aircraft that are added to the fleet.

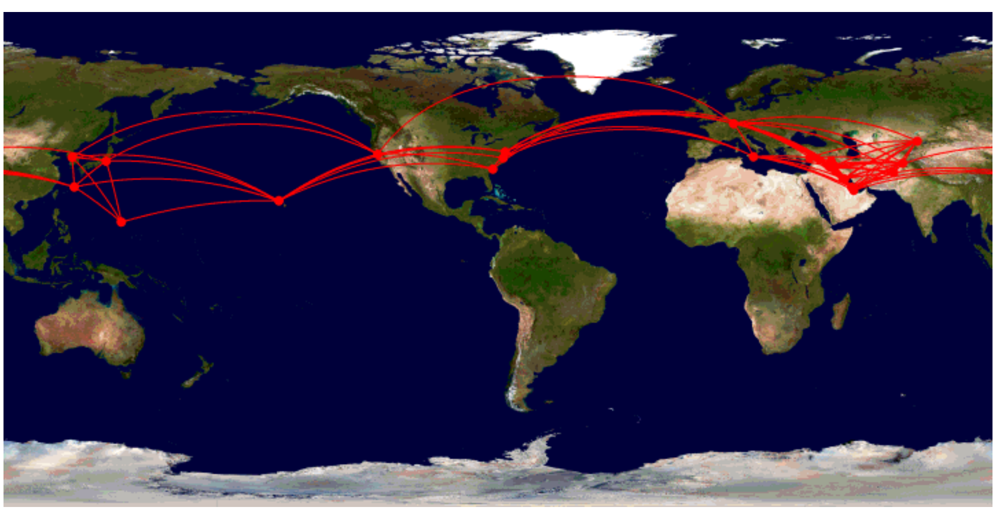
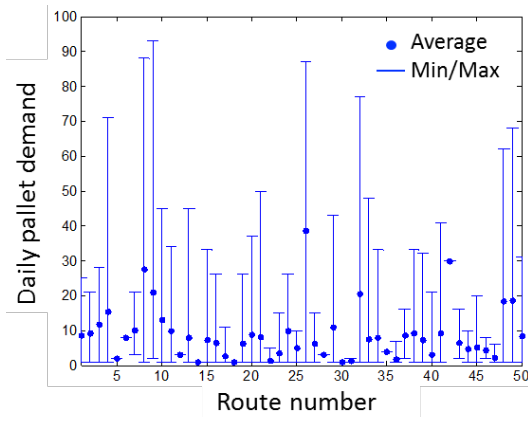
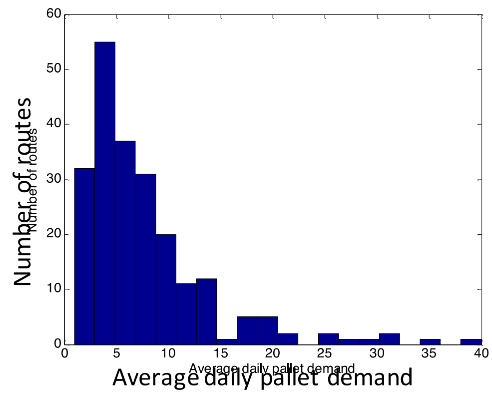


Figure 3: 25-base network (illustration generated using www.gcmap.com)

The 25 bases in the network are either the origin or the destination locations that transported the largest number of pallets in the AMC service network for the year 2006. The routes span the continents of North America, Asia and Europe. Figure 4a shows the average and the minimum/maximum of the directional daily pallet demand for 50 routes in the network. Figure 4b shows the distribution of the number of routes based on the average daily pallet demand. The histogram indicates that the demand distribution is right-skewed and that several of the routes have an average daily demand of less than 20 pallets.



(a) Directional daily pallet demand



(b) Histogram of average daily pallet demand

Figure 4: Pallet demand characteristics of the 25-base network

Result Summary

Figure 5 shows the results from the multi-objective analyses of the 25-base network problem, using the subspace decomposition approach for the AMC case study (refer Figure 2). The plot shows the normalized expected values of the fleet-level metrics. Using normalized fleet-level responses help to identify the trends, and help to show the relative variations in fleet-level responses for different solutions to the multi-objective optimization problem. The fleet-level responses have been normalized with respect to the lowest expected values from the results of the scenario labeled ‘Fleet with five new A/C’. Each point in the ‘Fleet with five new A/C’ scenario describes the optimal design of the new aircraft required to meet the specific fleet-level objectives. These results show the collection of optimal aircraft designs that would meet the fleet's operational needs at each level of permitted fuel consumption or at each level of required fleet-wide productivity.

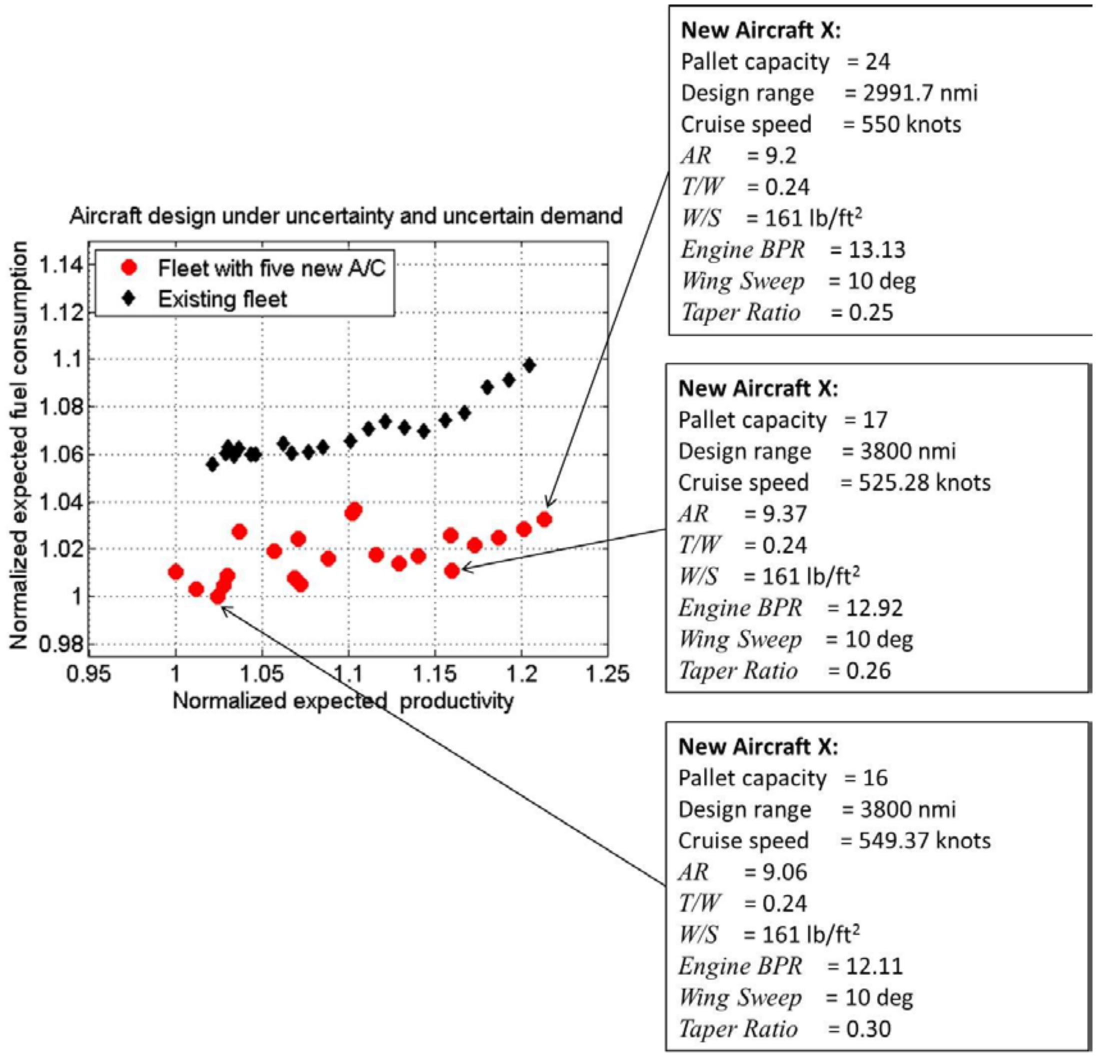


Figure 5. Results from multi-objective analyses of 25-base network problem

For three different solutions from the 'Fleet with five new A/C' results, Figure 5 contains callout boxes that describe the values of the new aircraft requirement decision variables along with the values of the aircraft design variables. The trends in the fleet-level responses are as expected, with fuel consumption increasing as productivity increases. There appears to be a trend in the 'size' of the optimal aircraft along the Pareto frontier for increasing productivity/fuel consumption values. For a normalized expected productivity and normalized expected fuel consumption value of 1.0, the optimal requirement decision variables of the new aircraft X are at the lower bounds for pallet capacity (16) and design range (3800 nmi). Moving from this point on the tradeoff plot towards solutions with increasing fleet-level productivity, the results suggest that larger pallet capacities for the new aircraft X can best meet the fleet-level objectives. There is not substantial evidence to determine whether these trends would generalize to other route networks or other similar design problems; however, the behavior is not unexpected, because the aircraft pallet capacity strongly drives the fleet-level productivity metric. Though it is intuitive that a larger aircraft would increase productivity, the optimal design features of the new aircraft X, such as, the aspect ratio (AR_X), the wing loading ($(W/S)_X$), the thrust-to-weight ratio ($(T/W)_X$), etc., are reflective of the specific existing fleet and demand characteristics of the service network. For each solution in the plot, the assignments of the fleet of aircraft to routes are different to meet the actual demands better. The introduction of the five new aircraft (of type X) results in fleet-level fuel savings between 2.79% and 6.48% for the same normalized expected fleet productivity values, when compared to the case where only the existing fleet operates in the network.

The solutions to multi-objective analyses present a way to perform “fuel/cost as an independent variable” type of trade-space analysis; this might be more obvious by switching the axes in the plot from Figure 5. These types of plots can help decision-makers/acquisition planners to analyze the trade-space and select the optimal requirements and design of the new aircraft that would achieve the desired level of fleet fuel consumption and productivity. For instance, a decision-maker can determine the level of fleet productivity available for a specific level of fleet fuel consumption; this fleet-level productivity value can then be translated to a specific (or bounded) level for the mobility airlift requirements that are set by the DoD in terms of tonnage of cargo transported per day. Having established the goals for the fleet-level productivity and fuel consumption, the collection of optimal aircraft designs required to achieve these fleet-level goals can be determined from plots such as those shown in Figure 5.

Posterior Analysis

Figure 6 shows the results from a posteriori analysis (200 samples) of a few solutions from the multi-objective analyses of the 25-base network problem. The dispersion in fleet level fuel consumption does not show any discernible trend. However, the degree of dispersion in fleet-level productivity appears to decrease for solutions with increasing fleet productivity and fuel consumption values.

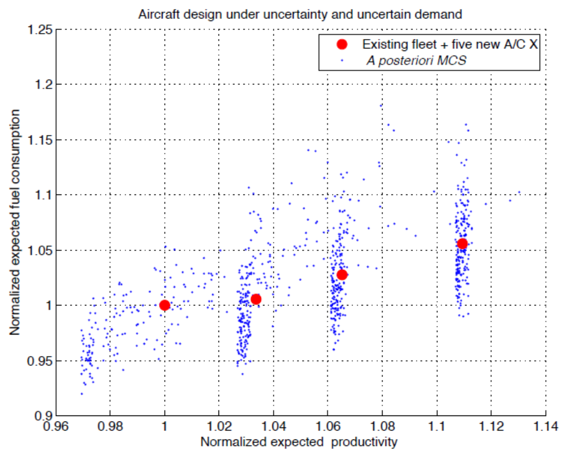


Figure 6. A posterior analysis for 25-base problem

Solutions with higher normalized fleet fuel consumption, in Figure 6, are more ‘robust’ (less variance) in terms of fleet productivity. A possible explanation for this behavior is because the multi-objective analyses (using the e-constraint formulation) vary the limit value of the fleet productivity constraint, while minimizing fleet-level fuel consumption. If solutions that are more ‘robust’ (less variance) to fuel consumption are desired, then the multi-objective analyses should vary the limit on the fleet-level fuel consumption constraint, while maximizing fleet productivity.

Decision-makers/acquisition planners can use such results to perform comprehensive exploratory analysis of the design space and identify regions in this design space that present significant viable or opportunities to reduce the fleet fuel consumption. For instance, AMC may need to incur “switching costs” (additional cost for training, maintenance and infrastructure due to the addition of a new aircraft type into the fleet) of integrating a new aircraft type into the fleet for relatively small decrease in fuel burn; however, the trade-space analysis (Figure 6 can help identify promising designs and ‘inflection points’, if they exist, where the decision to acquire a new aircraft type could provide significant benefits.

Case 2 - Commercial Airline

We apply the subspace decomposition approach, as a modified version of the AMC case, to the case of a commercial airline application. These modifications arise from the statistical differences in cargo demand between the AMC case study and passenger demand for a commercial airline, and, from the underlying business model where airlines will set and publish a schedule from which the traveling passengers select flights and purchase tickets. The highly uncertain nature of demand in the AMC case, versus the more symmetric and seasonal nature of demand in commercial applications, prompts different computational strategies within the approach presented here. The detailed subspace decomposition framework for the commercial airline application appears in Figure 7 below (also appears in Roy et al. 2017). For the commercial airline application, the fleet operation subspace is further sub-divided into two subspaces – airline allocation and a profit evaluation block.

Top Level Subspace – Commercial Application

The top-level optimization problem for the commercial airline application, seeks to maximize the expected fleet level profit of a representative airline based on the choices made about the design requirements for the new, yet-to-be designed aircraft; here, the range and passenger seating capacity are the design variables in this top-level problem. Like the AMC formulation, the top-level optimization problem is

$$\underset{x}{\text{Maximize:}} \qquad E[Fleet\ Profit]$$

$$Subject\ to: \qquad 75 \leq SeatCapacity_x \leq 250 \qquad \text{(Eq.15)}$$

unconstrained except for bounds imposed on the decision variables. The following equations describe the formulation of the top-level problem; consideration for uncertainty, as reflected in the expectation of profit appears later in the aircraft sizing and airline operations subspace.

The objective function here seeks to maximize fleet level profit using passenger seating capacity and range, of the yet-to-be-introduced aircraft type X, as decision variables. The two constraints describe the bounds for the top-level design variables of aircraft passenger seating capacity and range. The values for the bounds on these design variables were based on typical characteristics of current class of aircraft. Here, the design requirement decision variable describing passenger seating capacity and the design range are both integer variables. While the expectation term appears in the objective function of the top-level formulation, the source of uncertainty associated with the expectation term comes from the aircraft design and fleet allocation subspaces. Our discussion in these latter sections will make clear the evaluation of the expectation term for the top-level objective function.

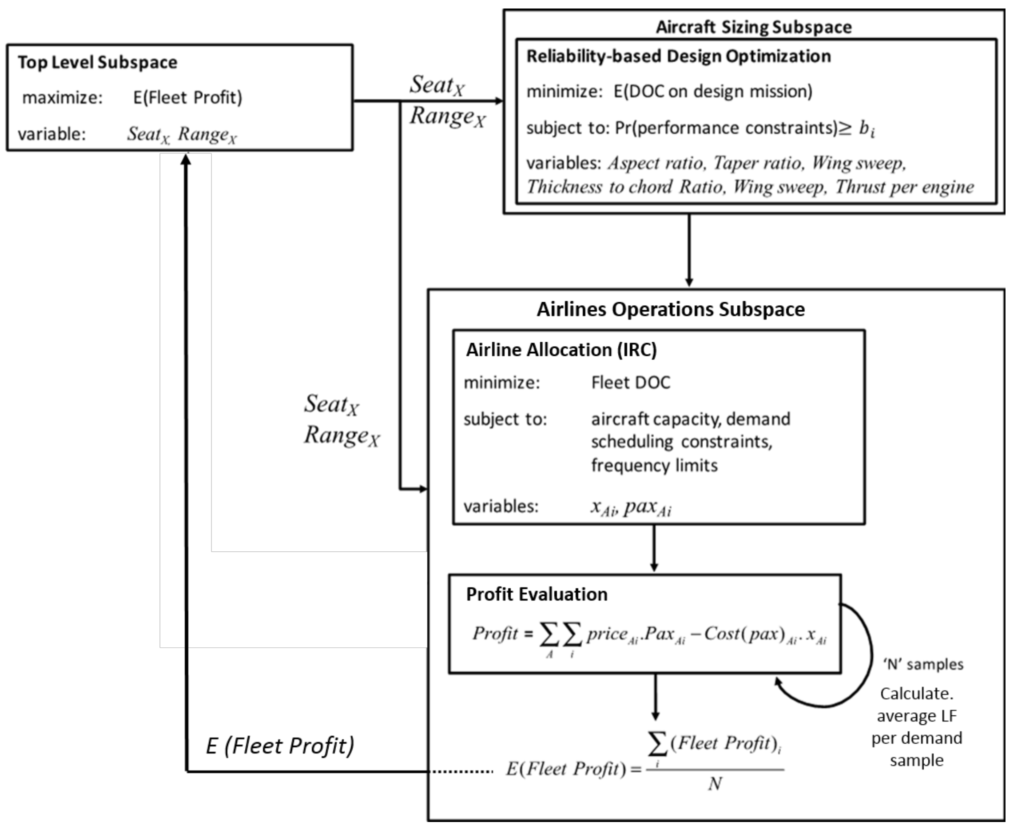


Figure 7. Subspace decomposition strategy for the commercial airline application

Aircraft sizing subspace

This subspace is similar to the AMC work as described before (see Eq. 3). However, to accommodate different number of seats as required by the top-level problem formulation for the commercial applications, the sizing code needs to vary the size of the fuselage and the tail using an empirical relation established using the existing aircraft data. For this work, the uncertain parameters of choice, as appears below in Table 3, are selected based on subject matter expert opinion for illustrative purposes. A more formal approach of identifying most relevant factors would involve an Analysis of Variances (ANOVA) and a Design of Experiments (DOE) approach to identify the most statistically relevant design parameters influencing the aircraft design.

Table 3. Uncertain parameters in the commercial aircraft sizing optimization problem

Uncertain Parameters (ξ)	Lower Bound	Default	Upper Bound
C_{D_0} Multiplier [non-dim]	0.95	1	1.05
Oswald Efficiency Factor Multiplier [non-dim]	0.95	1	1.05
Thrust Specific Fuel Consumption Multiplier [non-dim]	0.95	1	1.05
Passenger Weight [lbs]	90	165	220

The RBDO formulation optimizes the expected performance metric of interest and ensures that the probability of satisfying the performance constraints is greater than or equal to the user-defined reliability level, considering the uncertainty present in this subproblem. Here, we assume a triangular distribution for the uncertainties in each parameter; this will facilitate demonstration of the method, but better characterization of these distributions would improve the quality of the results. The aircraft sizing sub-problem includes performance constraints such as limits on takeoff and landing distances, second segment climb gradient, top of climb rate and also upper and lower bounds for the decision variables.

As mentioned earlier, at the solution of the RBDO problem, the resulting aircraft design has uncertain responses because of the input uncertainties (Table 3). Of interest for the airline operations subspace – the cost to fly the new aircraft on any route, the block hours needed to fly any route, the maximum number of passengers that the aircraft can carry on each route, and the takeoff distance of the aircraft – all follow probabilistic distributions.

Airline Operations

This subspace mimics an airline's operational behavior. The Interval Robust Counterpart (IRC) formulation recognizes and obtains the performance characteristics of the uncertain aircraft for the nominal and worst-case values of the uncertain aircraft design parameters of Table 3. We use these performance data in our allocation formulation to minimize the airline's fleet-level direct operating cost, while satisfying maximum predicted passenger demand on the route network. Here, the maximum predicted passenger demand comes from historical data available from the Bureau of Transportation Statistics; this provides a credible demand distribution for the problem, as if this historical demand were actually a prediction of future demand. Solving the allocation solution represents setting the airline's schedule, and then the approach samples the uncertain passenger demand that would fly on the set schedule and evaluates an expected profit considering the uncertain demand. To further capture seasonal variation in passenger demand, we set four different quarterly allocations. The purpose of considering each quarter's worth of data is to capture better the impact that seasonal fluctuations will have on the observed maximum number of passengers traveling on each route for a representative travel day. Average profit (or the expected profit) over all sampled demand for all the quarters then returned to the top level and appears as the top-level objective function. A detailed description of the airline operations subspace appears below.

Airline Allocation via IRC

The purpose of the fleet allocation problem is to determine the best allocation of the fleet's aircraft (including existing aircraft along with the new aircraft) on routes in a way that minimizes fleet-level operational costs. This includes pseudo-scheduling considerations of number of minimum number of flights required on each route, so as to capture the typical schedule conveniences that airline would offer to passengers. The allocation of aircraft is naturally subject to the performance constraints of the aircraft (e.g., range of aircraft dictates which routes can be serviced by various types of aircraft in the fleet). It becomes apparent here that the impact of setting design requirement values at the top-level optimization problem begins to manifest operational impacts at the allocation stage of the decomposition framework.

Passenger Travel and Airline Network Data

The allocation of the fleet assumes satisfying the need to carrying an estimated number of passengers on each route that is based on historical data. In our commercial application, we use data from the Bureau of Transportation Statistics (BTS) T-100 segment passenger data (Bureau of Transportation Statistics (BTS) n.d.); the data contains a full report of the number of passengers that directly travel between a given origin-destination point, and, contains number of flights, aircraft types operated, among other categories. In this report, we use data for the case of a thirty-one-route subset of the data that is representative of passengers travelling with a notional carrier on trips within the continental United States. Figure 8 shows the routes that we have selected for our concept problem, with Memphis being the central hub of notional airline service network. This network resembles a portion of the Northwest Airlines network before the merger with Delta. Figure 9 displays a map of the subset of the passenger travel data for the thirty-one-route network.

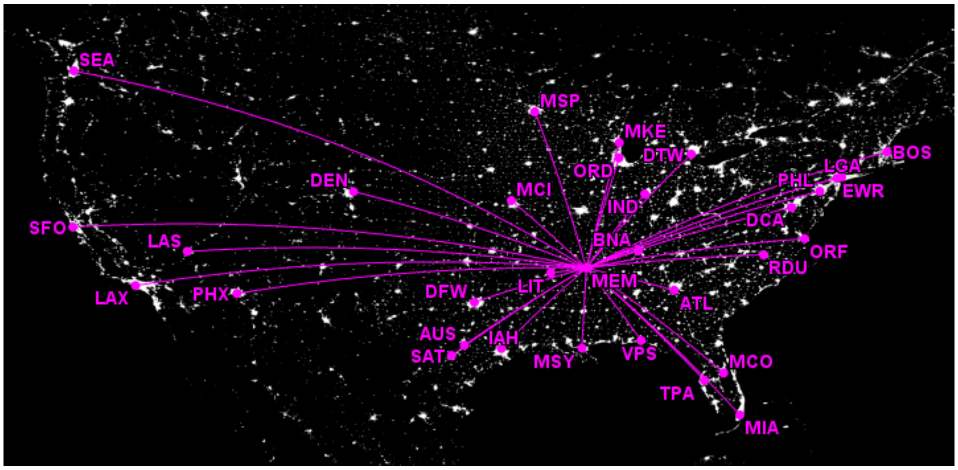


Figure 8. Notional thirty-one-route network with hub at Memphis (illustration generated using www.gcmap.com)

We use passenger demand data over the span of four years starting from 2004 to build the passenger demand distributions. Example data for the Memphis-Los Angeles route appears in Figure 9. A more realistic application of this approach to determine the right new aircraft requirements for an airline and its network would have predicted future demand distributions on each route; without a readily available means to generate these predicted future demands, using the historical demands provides a credible distribution to demonstrate the approach. The resolution available in the BTS T-100 data is monthly, and hence monthly bins appear in Figure 9. Effects of seasonal demand are evident in the near periodic appearance of high travel demand, and changes in schedule are also evident where there was no demand carried between MEM and LAX, indicated no direct flights operated those months.

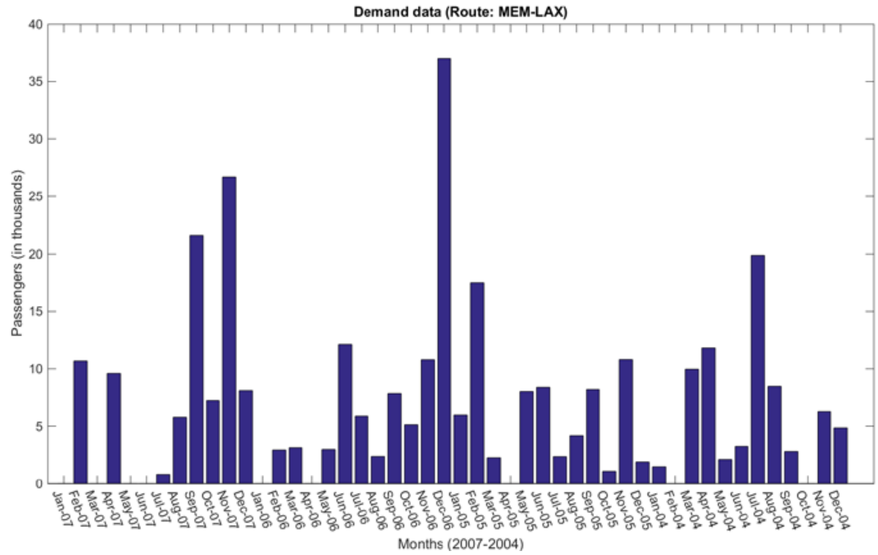


Figure 9. Passenger demand characteristics for the MEM-LAX route

Aircraft Allocation Formulation

In this work, we consider quarterly demand distributions to capture seasonality effects, resulting in four fleet allocation sub-problems that are solved – one for each quarter. The fleet allocation problem in this report is same as the AMC work and is posed as a mixed integer program (MIP) with a linear objective function and linear constraints. The Generic Algebraic Modeling System (GAMS) software package, accessed through a MATLAB® interface, is used to solve the allocation problem using the CPLEX solver option (Brooke, et al. 1998). Given the ability of the CPLEX solver to handle large-scale MIP problems with linear objective and linear constraints as in our case, this increase in the number of design variables, even in the case of considering four seasonal quarters, is computationally tractable.

Because the cost and performance of the aircraft in the fleet are uncertain, like the AMC work, we adopt the IRC formulation to accommodate the uncertainty associated with the performance of the new aircraft (of type X). The resulting mathematical formulation of the fleet allocation problem for the commercial airline application appears as the following set of equations:

Minimize DOC (Eq. 16)

Subject to:

$$\sum_{a}^{A}\sum_{j}^{J} \mathrm{Cost}_{\mathrm{nominal},aj} x_{aj} \leq \mathrm{DOC} \quad \text{(Eq. 17a)}$$

$$\sum_{a}^{A}\sum_{j}^{J} \mathrm{Cost}_{\mathrm{worst},aj} x_{aj} \leq \mathrm{DOC}(1+\delta) \quad \text{(Eq. 17b)}$$

$$\sum_{a}^{A} \mathrm{pax}_{aj} = \mathrm{dem}_j \quad \forall\, j = 1,2,\dots,J \quad \text{(Eq. 18)}$$

$$\sum_{j}^{J} [x_{aj}(\mathrm{BH}_{\mathrm{nominal},aj} + \mathrm{MH}_{aj} + \mathrm{T}_{aj})] \leq 12 \times \mathrm{fleet}_a \quad \forall\, a = 1,2,..,A \quad \text{(Eq. 19a)}$$

$$\sum_{j}^{J} [x_{aj}(\mathrm{BH}_{\mathrm{worst},aj} + \mathrm{MH}_{aj} + \mathrm{T}_{aj})] \leq 12 \times \mathrm{fleet}_a(1+\delta) \quad \forall\, a = 1,2,..,A \quad \text{(Eq. 19b)}$$

$$\mathrm{pax}_{aj} \leq x_{a,j}\mathrm{Cap}_{\mathrm{nominal},a} \quad \forall\, a = 1,2,\dots,A;\; j = 1,2,\dots,J \quad \text{(Eq. 20a)}$$

$$\mathrm{pax}_{aj} \leq x_{aj}\mathrm{Cap}_{\mathrm{worst},a}(1+\delta) \quad \forall\, a = 1,2,\dots,A;\; j = 1,2,\dots,J \quad \text{(Eq. 20b)}$$

$$x_{a,j}\left(\mathrm{TD}_{\mathrm{nominal},a} - \mathrm{RunwayLength}_j\right) \leq 0 \quad \forall\, a = 1,2,\dots,A; j = 1,2,..,J \quad \text{(Eq. 21a)}$$

$$x_{a,j}\left(\mathrm{TD}_{\mathrm{worst},a} - \mathrm{RunwayLength}_j(1+\delta)\right) \leq 0 \quad \forall\, a = 1,2,\dots,A; j = 1,2,..,J \quad \text{(Eq. 21b)}$$

$$\sum_{a}^{A} x_{aj} \geq \mathrm{Freq}_j \quad \forall\, j = 1,2,\dots,J \quad \text{(Eq. 22)}$$

$$x_{aj} \in \mathbb{Z}^+, \mathrm{pax}_{aj} \in \mathbb{Z}^+ \quad \text{(Eq. 23)}$$

The airline has *A* different aircraft types flying on *J* different routes (*J* = 31) allocated for a representative day within each quarter of a year. The purpose of considering each quarter's worth of data is to capture better the impact that seasonal fluctuations will have on the observed maximum number of passengers traveling on each route for a representative travel day. The objective function and first constraint of the fleet resource allocation problem seeks to minimize the fleet-level Direct Operating Cost (DOC) from all aircraft that are allocated to each route on the notional airline service network. Here, the accounting for uncertainty in the cost coefficients of flying aircraft type *a* on route *j* are included in the objective function, through introduction of an additional variable (*DOC*), and, relegation of the cost summation term to the first set of constraints (Eq.17a and 17b); such a strategy is common for introducing robustness in an objective function. The two constraints Eq.17a and

Eq.17b, represents the nominal and the worst-case scenario. Our uncertain aircraft has a distribution of its cost and performance characteristics due to the presence of uncertain aircraft parameters (Table 3). To recognize this uncertain aircraft in the allocation formulation, we then use the nominal and worst-case values of the uncertain aircraft parameters and evaluate the aircraft performance and cost at these two points, thereby obtaining two sets of cost and performance data as represented by $Cost_{nominal}$ and $Cost_{worst}$. We use the same δ tolerance of 0.1 as the AMC work and relax the upper bound by this tolerance for the worst-case scenarios to account for robustness in the IRC formulation. The second constraint ensures the total number of passengers carried on a given route j equals total demand in that route (here, demand per quarter). The third set of constraints limits the number of hours available for each aircraft for service to meet the daily passenger demand (this includes block hours, maintenance hours and flight time) to a 12-hour work day schedule. The fourth set of constraints ensures that the number of passengers transported by any aircraft type on any individual route is less than the combined seating capacity of that aircraft type. The fifth set of constraints enforces the condition that take-off distances never exceed runway lengths due to the varying runway lengths at each airport. The sixth constraint ensures that the allocation problem allocates a minimum number of flights (as reported in the BTS database) for each route, and is reflective of frequency of service conditions. The last constraint ensures the variables **x** and **pax** are integer types. However, to reduce the computational runtime we have relaxed the integrality constraint on the **pax** variables and treated them as continuous.

Initial Fleet Composition

Figure below shows the initial fleet composition of the airline in our model. The red bar shows the number of new "yet-to-be-deployed" aircraft that the airline seeks to purchase. To obtain a better estimate of the current fleet composition, we use the "BTS T100 segment" monthly data to obtain the total number of departures scheduled by Northwest Airlines (representative airline in our model) with flights originating from Memphis in the span 2004 – 2007. By doing this, we have assumed our airline serves a 31-route network with a hub at Memphis, TN; this is much like our approach to extract a subset of Air Mobility Command routes from the GATES database.

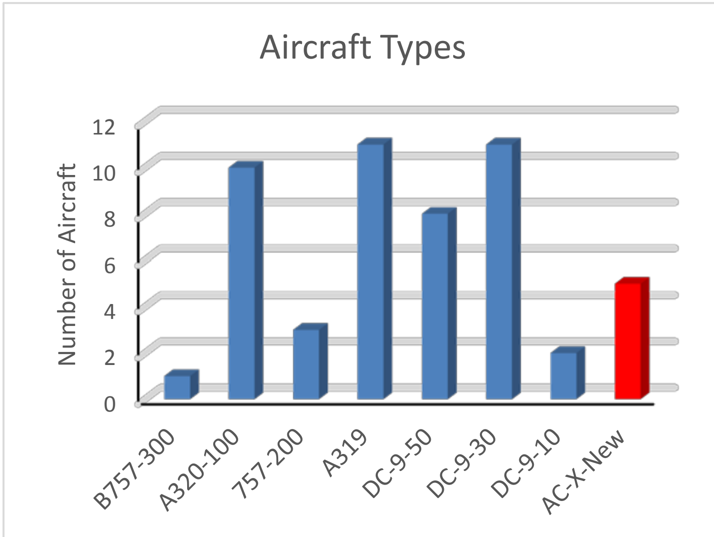


Figure 10. Initial fleet composition of the airline with eight different aircraft types (includes the new "yet-to-be-deployed" aircraft)

We then divide the number of departures by the total number of days in that month to obtain an estimate of the daily departure by each aircraft type present in the fleet. BTS also reports the ramp-to-ramp time for each aircraft type for a given city pair. We assumed in this study, ramp-to-ramp time closely represents the block hours of the aircraft for the route under consideration. With the daily departure data and the approximate block hours, we now can estimate the number of aircraft for a particular type using the following aircraft utilization equation.

$$\sum_{j=1}^{J} x_{k,j}.\left(BH_{k,j} + MH_{k,j} + 1\right) \leq 12.ACtype_k \quad \text{(Eq. 24)}$$

Evaluating the above equation for the monthly departure data obtained using BTS for the year 2004 to 2007, gives a plot similar to Figure 11. This figure shows the number of aircraft needed to meet the daily trips reported in the BTS data for which Northwest Airlines operated a Boeing 757-200 originating from Memphis. For our initial fleet size estimate, we pick the highest value from the plot and round it up to the next higher integer value to get an estimate of the number of that aircraft type. The highest bar in this plot shows just over 1.8. This value gives an estimate of the initial number of B757-200 aircraft type present in the fleet. We carry out a similar strategy to obtain an estimate for the other aircraft types in fleet (see Figure 10).

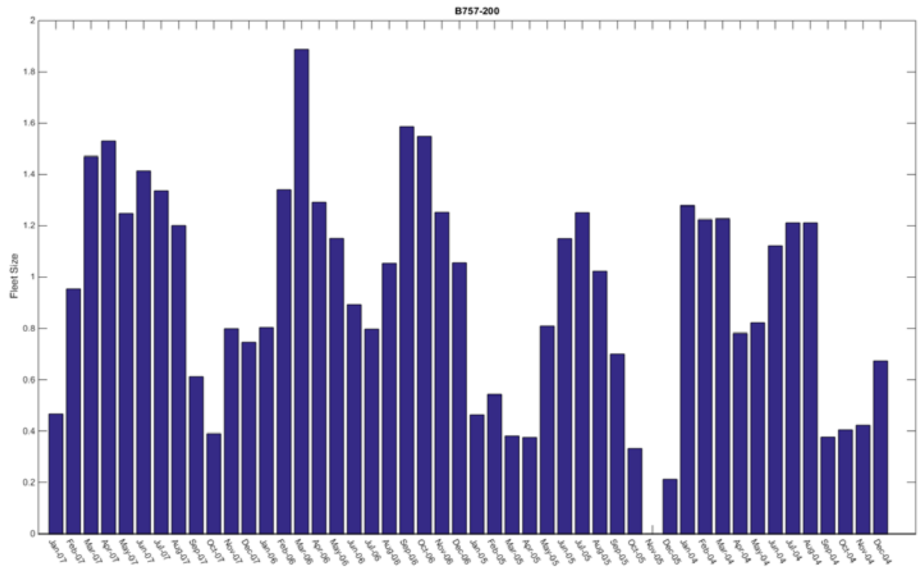


Figure 11. B757-200 monthly utilization by North West Airline (2004-2007) for flights originating from Memphis, TN

Profit Evaluation

An airline typically assigns aircraft to various routes and to corresponding schedules ahead of time, and then sells tickets for the seats on these aircraft to the public. The airline then executes various revenue management strategies to increase yield in selling the tickets and to capture passenger market share. We have a simplistic approach to capture this dynamics of "allocate-first then sell the seats" by separating the allocation and revenue management segments in our decomposition framework. First, the airline allocation problem seeks to minimize the fleet level costs by efficiently allocating the airline's fleet of aircraft across the routes within the airline's service network, given specific assumptions on demand data; this is reflective of the 'allocate-first' phase of planning the number of seats to be sold on a route. Here, the maximum observable demand on each route per quarter is used as the projected demand data for the fleet allocation phase. After performing the airline allocation problem, we then mimic the 'revenue management' portion of an airline's operations. However, for this work, we have used publicly available average ticket prices on these 31 routes from the *flightaware* website[‡]. The ticket prices are kept constant for each route and there is no fare class distinction for a given departure.

Here, there is a need to run scenarios of sampling various passenger demand conditions efficiently, and, under different aircraft performance conditions. Our strategy is to generate the fleet wide expected profit that results from a combination of realized demand samples and varying performance conditions, for each top-level optimization value of range and passenger capacity. The strategy involves the following steps:

Step 1- Generate samples of demand realization η_{ij} for each route. This is the uncertain passenger demand instance *i* that shows up on route *j*.

Step 2- Use the allocation results **x** and **pax** to distribute this uncertain passenger demand η_{ij} across various flights scheduled on route *j*. This gives as an estimate of the load factors on each flight.

Step 3- Obtain the cost coefficients for both new and existing aircraft at these load factors.

Step 4- Evaluate fleet-wide profit for this instance of the sampled demand *i*.

Step 5- Save current net profit resulting from Step 3-4

Step 6- Repeat step 1-5 for N=500 samples.

Step 7- Calculate expected profit for current top-level selection of range and passenger capacity, as the average profit over all the *N* samples

In the sampling phase of Step 1, each sample represents a realized value of the number of passengers traveling on each route, where each route has a set of pre-allocated aircraft. The cost of operating on each route, is dependent on the number of passengers travelling on each of the allocated aircraft on the route – therefore, there is a need to establish the exact number of passengers who travel on each aircraft. We adopt a weighted sum approach to allocate each sampled instance of passengers to aircraft on each route. The average load factor per departure is calculated as follows:

[‡] http://www.flightaware.com/

$$Pax\_distribution_{aj} = \frac{pax_{aj}}{\sum_a^A pax_{aj}} . \left(sampled\_demand_j\right) \qquad \text{(Eq. 25a)}$$

$$Average\ Load\ Factor_{aj} = \frac{Pax\_distribution_{aj}}{x_{aj} .\ Cap_a} \qquad \text{(Eq. 25b)}$$

Eq. 25a calculates the total number of passengers carried by aircraft type *a* on the route *j,* and then Eq. 8b calculates the load factor for each trip made by aircraft type *a* on route *j*. Using this average load factor per aircraft, and the cost look-up table, we then determine the cost coefficients of flying each aircraft type on each route in Step 3. The cost calculation includes consideration for available seat capacity on each route, as dictated by the aircraft's payload-range diagram; the further the aircraft must travel above the nominal design range (designed with a load factor of 80%), the less it can carry due to physical limits imposed by flying further. Figure 12 shows the available seats for each aircraft type available in our notional example problem's fleet on each route in the network. In this plot, the routes appear as categories, so the horizontal axis does not use a scale of nautical miles. The curve for the DC-9-10, which has the shortest range in the fleet, has a slow drop in load factor up to the 760-nmi route; this corresponds to the MTOW line in the payload range diagram. Then, there is an abrupt drop in available load factor from the 760-nmi route to the 991-nmi route; this corresponds to the maximum fuel volume limit on the payload range diagram. Several other aircraft, including the AC-X-New, also exhibit these tradeoffs at ranges within the values covered in this current network. The performance of the AC-X-New shown here corresponds to the aircraft from the results discussion that appears later in this report.

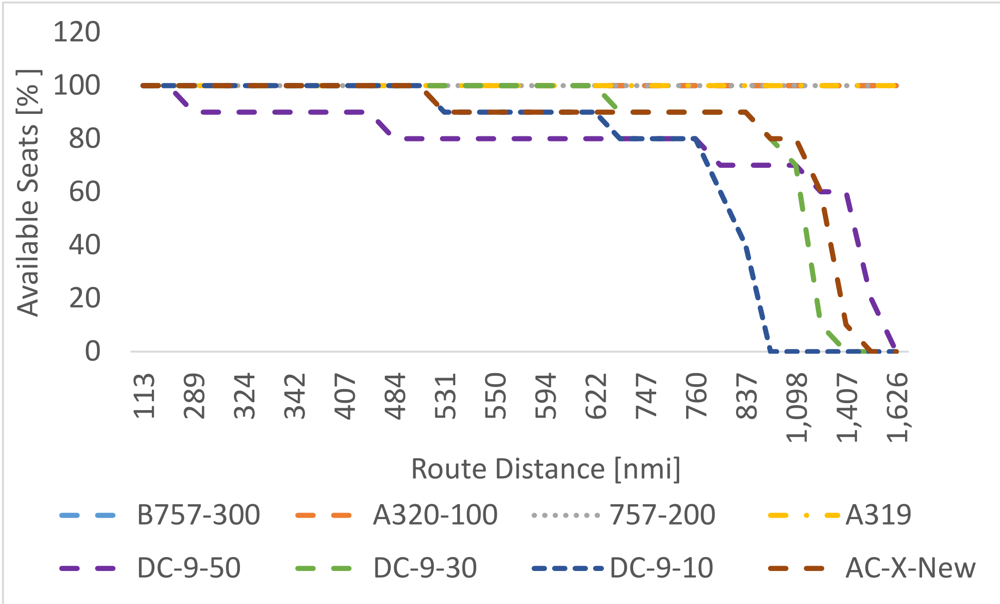


Figure 12. Available seats for each route and aircraft type

Figure 13 shows the associated cost per seat for each route, again using the route distances as axis label categories rather than in linear scale of nautical miles. This uses our own cost predictions; the trends are credible, but these should not be viewed as exact predictions of cost. In this figure, the lowest cost aircraft for each route will appear at the bottom of the plot. As aircraft start to reach their passenger-carrying limits at MTOW, the cost per passenger begins to increase. For the few aircraft where the maximum fuel volume limit is active for some of the routes in the network, this cost per passenger makes a dramatic increase as the range increases. As might be expected, the new AC-X-type aircraft, which uses models of current technologies, has the lowest cost-per-seat for a large number of the routes, until routes where the new aircraft would start to trade-off fuel for passengers at MTOW.

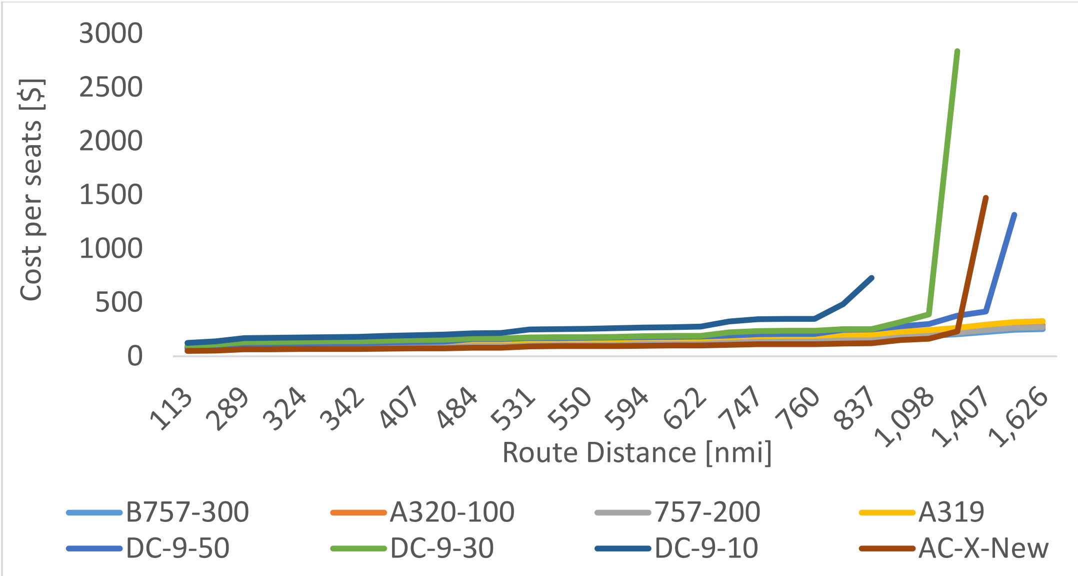


Figure 13. Cost per seat for each route and aircraft type

Result Summary

In this conceptual study, we used a pseudo-enumeration approach to address the top-level problem that uses the following range of discrete choices, as appears in Table 4. The interval values within the range for each of the variables is selected to more rapidly generate reasonable solutions at this stage of development in our approach – refinements in the grid space for the top-level enumeration scheme can be selected as required for more realistic problems.

Table 4. Design variable values of top-level problem for enumeration

Range [nmi]	Seat Capacity
500	75
1200	150
1900	250
2600	

For each combination of design variables (4 range variables × 3 seat capacity variables = 12 enumerations points), we execute the overall subspace decomposition methodology shown in Figure 7. Figure 14 below shows the profit data for all possible combinations of the enumerated top-level design variables from Table 4.

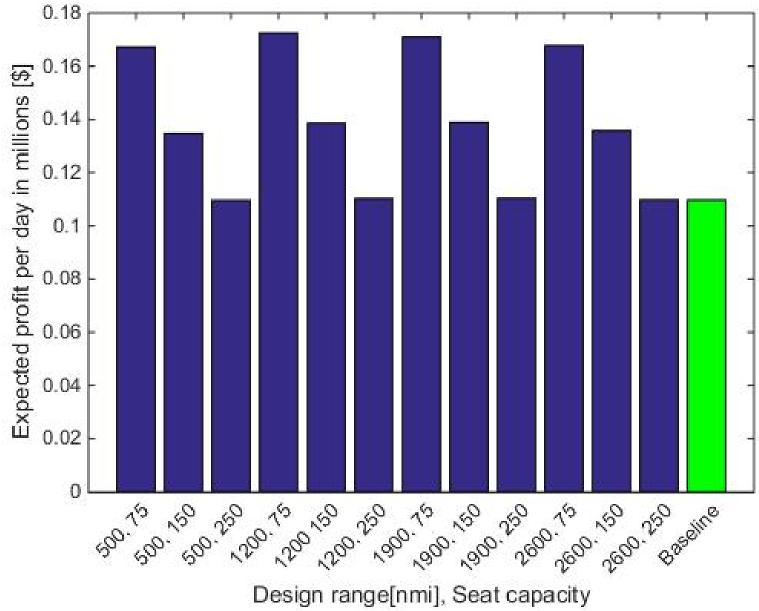


Figure 14. Expected fleet profit values for the combination ('Test cases') of the top-level design variables (green denotes baseline fleet with no new aircraft type X use)

The results show that the optimal seating capacity is 75 seats for the new aircraft, because the new aircraft is allocated on routes with average passenger demand of less than 110 passengers. Also, because the route distances of these routes in which the new aircraft is allocated are less than 1000 nmi (the longest route in the network is 1626 nmi), the optimal design range of the new aircraft corresponds to a distance of 1200 nmi. Further physical details of the optimal aircraft are retrieved from the aircraft design subspace problem that corresponds to the optimal range and passenger capacity values [1200nm, 75seats] and appear in Table 5 below.

Table 5. Optimal aircraft design solution

Range [nmi]	1200
Passenger capacity	75
Aspect ratio	12
Taper ratio	0.3
Thickness-to-chord ratio	0.095
Wing sweep [deg]	10.43
Wing area [sq.ft]	664.76
Thrust per engine [lbs]	9351

Figure 15 below shows the utilization of each aircraft type in the fleet, over each quarter. In these plots, we note that most flights of the new aircraft design are allocated around the 500 nmi range to fill in the travel needs. Given the number of aircraft available for each aircraft type, it is desired (as seen from the allocation results) to have a 1200 nmi range aircraft, as it provides the option to be used on fewer long-range routes.

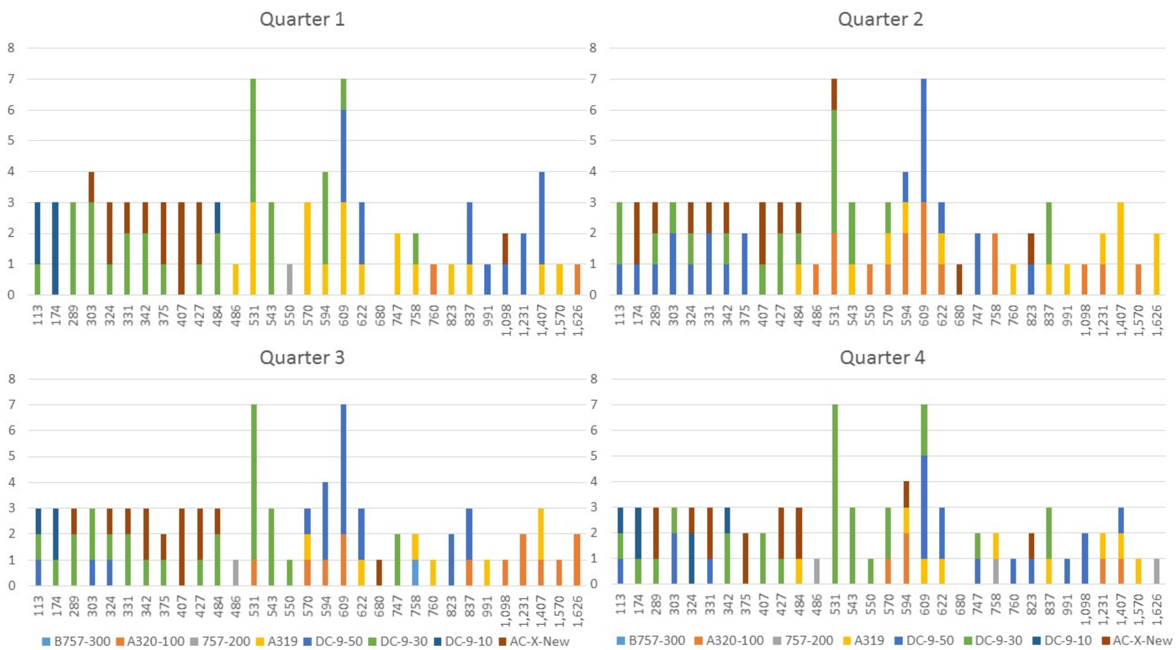


Figure 15. Distribution of fleet allocation over routes per quarter

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Posterior Analysis

To validate the commercial application of our framework, we performed a posterior analysis with a different set of 1000 samples. To generate this set of 1000 samples, we pick one sample for each uncertain parameter in the aircraft design subspace and performed an off-design mission analysis across all the routes in the network, keeping the aircraft design variables fixed to values obtain from the RBDO formulation. We then evaluate the performance characteristics of the aircraft and determine how many occasions these performance constraints are satisfied. Figure 16 below shows out of these 1000 samples how many times the aircraft performance constraints are met. Take-off distance seems to violate the most, as 78 of the 1000 samples did not meet the take-off distance criteria. The take-away from this plot is all the constraints are satisfied well within the 10% tolerance limit, set in the RBDO formulation at the time of designing the aircraft.

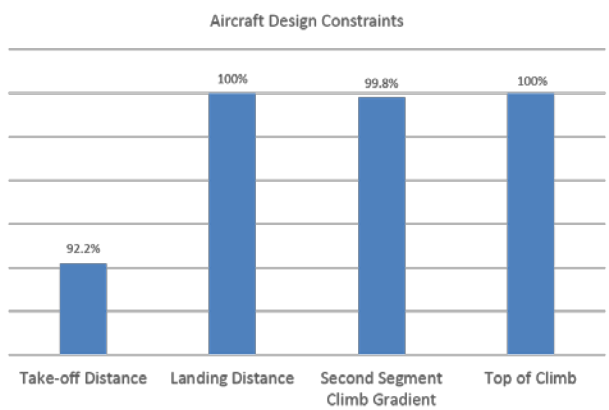

Figure 16. Percentage satisfaction of the aircraft performance constraints for the 1000 samples

It is important to note that even though we see few aircraft designs are infeasible, we do not see any reduction in the fleet-level profit as seen in Figure 17. This is because those infeasible aircraft design solutions do not get allocated on those routes for which these designs cannot meet any one or more of the performance constraints (see Eq. 21a). However, the allocation subspace allocates the aircraft (including the existing one) in way that does not affect the fleet-level profit.

The bigger question here, given the broad-level goal we seek to achieve, is the acquisition practitioner willing to take the risk of accepting a design that meets all the performance constraints over 90% of the time or should it re-solve the problem with a reduced tolerance limit to improve the percentage of constraint satisfaction?

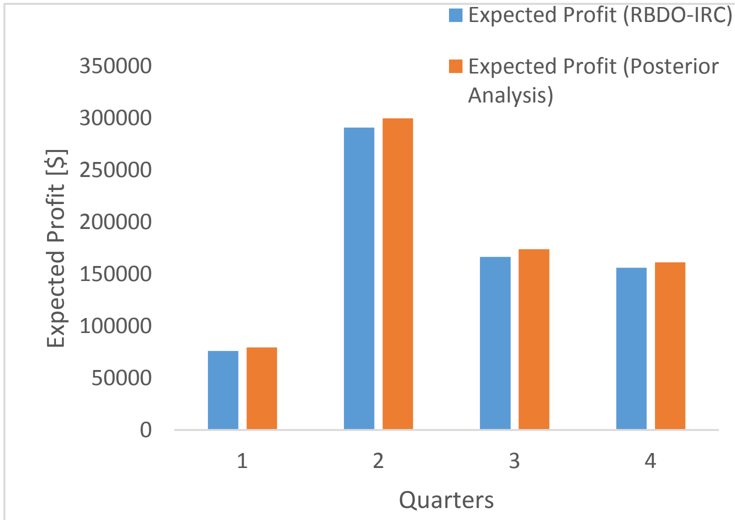

Figure 17. Expected profit comparison [Posterior analysis]

The expected profit calculation uses one sample of demand for every route. This demand sample combined with the extrinsic sample of the aircraft design subspace, both drawn independently, constitutes one sample for the posterior analysis. We repeat this step a thousand times. Intuitively, one can say the expected profit from the posterior analysis should be around the same value as the original RBDO-IRC formulation run, if both of these methods handling the associated uncertainties well. This is confirmed in the plot below. We feel confident of our framework to address this type of problems, as attested via posterior analysis with 1000 independent samples.

Further, we compared these results (previous sections) based on our approach to a fully deterministic approach with no uncertainty included. Our comparison shows the value of our approach that incorporates uncertainties, relative to the current, deterministically driven decision-making process. An overview of the deterministic subspace decomposition approach, based on Crossley et al. 2004, Mane et al. 2007, appears in the Figure 18 below.

The deterministic approach assumes nominal values for the uncertain parameters in the aircraft design subspace and solves a Nonlinear Programming (NLP) problem to design the new aircraft. The objective function for this deterministic case is the direct operating cost, subject to the aircraft performance constraints. The designed aircraft is then allocated along with the existing aircraft in the airline operations subspace with the objective to minimize the fleet-level direct operating cost, subject to the demands and operational constraints. Lastly, in the profit calculation block, passenger demand is represented by the mean demand value on a given route as a realization of the passenger demand on a given day.

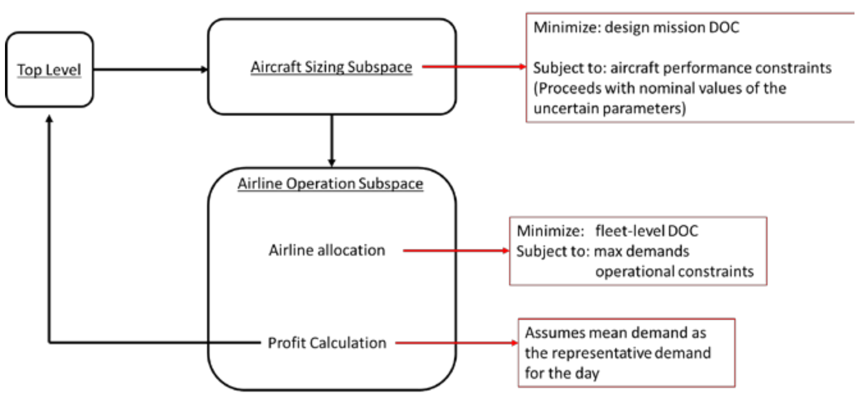

Figure 18. The deterministic approach without considering the uncertainties in the aircraft design and airline allocation subspace

We now use this deterministically designed aircraft and then perform a posterior analysis with 1000 set of samples of the uncertain parameters to mimic a typical day of “uncertain operations”. The aircraft design parameters (the design variables in the aircraft design subspace) are kept fixed. However, the aircraft is “sized” for each different sample in the posterior analysis. It appears the aircraft designed using the deterministic approach is unable to meet the performance related constraints on several instances of the uncertain parameters. Figure 19 below compares the aircraft designed using RBDO-IRC approach (considers uncertainties) and the fully deterministic case (no uncertainties). The figure reveals the deterministically designed aircraft is unable to meet the take-off field length constraint for more than 40% of the time.

The above deterministic run assumes the nominal values of the uncertain parameters in the aircraft design subspace that leads to a design that cannot meet the performance constraints on several instances of the samples in the posterior analysis. Alternatively, one can overdesign by setting a high factor of safety margin to satisfy the performance constraints. However, such over-designed aircraft may lead to drastic reduction in the fleet-level profit.

Summary of Subspace Decomposition Approach in USAF AMC vs. Commercial Airline Applications

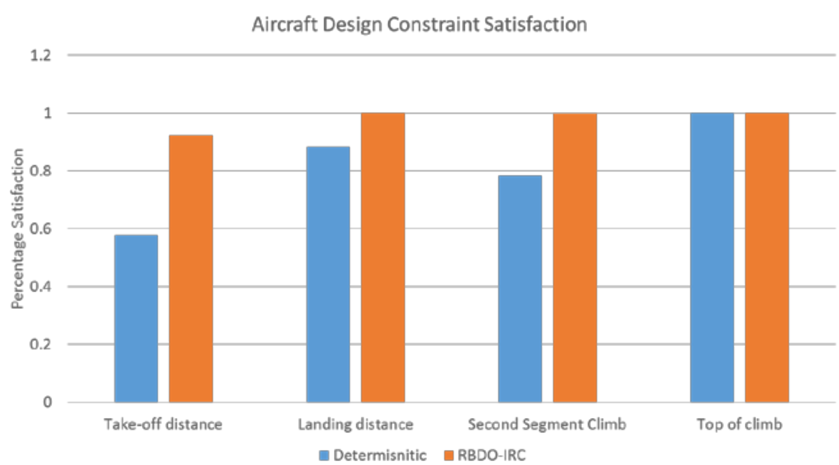

Figure 19. Aircraft performance constraint satisfaction for the aircraft designed with and without considering uncertainties

The main difference between the use of the subspace decomposition approach to the AMC and commercial airline cases are dictated by the nature of the payload for each aircraft type (pallets vs passengers), and, the statistical nature of the demand for transport (uncertainty, unstructured cargo vs. scheduled commercial flights). The details of differences in subspace modeling in both cases, and, as discussed in earlier subsections, are summarized in Table 6.

Table 6. Comparison of subspace decomposition formulation between the AMC and the commercial applications

Subspace Level	Subspace Decomposition Approach Application	
	USAF AMC - Military Cargo	**Commercial Airline**
Top-Level	Requirements are number of pallets, range and cruise speed of aircraft. Use of Global Optimizer (NOMAD) to search design space	Top level requirements are number of seats and range of aircraft Perform Pseudo enumeration to search the design space
Aircraft Sizing	Fuselage sizing rules based on number of standardized pallets	Fuselage sizing rules based on number of seats
Fleet Operations	USAF AMC flight operations based on 'as needed' basis for demand for cargo transport IRC formulation minimizes fuel consumption and enforces constraint on productivity. External demand sampling loop. Single IRC solution for each sampled demand set. Average fuel consumption of sampling returned to top level problem Use of aircraft assignment that tracks 'tail numbers' of aircraft Demand sampling done by random sampling of starting locations for aircraft	Fleet operations based on BTS (BTS 2015) data to model future prediction of demand (assumes demand in symmetric) IRC formulation minimizes operating cost while meeting the ever recorded maximum demand on a route for travel BTS data on historical airline data used to predict future demand distribution Scheduling done to meet maximum demand on all routes at same time Profit calculated through statistical sampling schemes on demand, and, includes ticket pricing model

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Conclusions and Recommendations

In this report, we have presented application of a subspace decomposition approach, that better enables identification of design requirements of a new, “yet-to-be introduced” system (here, aircraft) towards improving fleet-wide performance metrics. The approach explicitly accounts for the impact that the new system will have on fleet wide performance, when used alongside existing systems within a fleet, and, also accounts for various data uncertainty that manifest in the problem. We have presented an application of the approach for commercial airline and military cargo airlift cases, demonstrating domain agnosticism of the approach. The approach is envisioned to be useful to relevant decision-maker within the general acquisition community (government, military, commercial) by enabling trade-off analyses between performance metrics of interest, and, under conditions of data uncertainty, thereby enabling a framework for robust decision-making on setting design requirements of a new, yet-to-be introduced system. Future work may encompass extension of the approach to include additional relevant forms of domain driven data uncertainty, and, further improvements in computational efficiency.

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Outreach and Collaboration

Work documented in this report has resulted in three conference publications at the 11th , 12th & 13th NPS Acquisition Research Symposium (2015, 2016 & 2017) (Govindaraju et al. May 2015, Govindaraju et al. May 2016 & Roy et al. May 2017). Resulting interactions have produced very valuable feedback on the merits of our current results and potential further development of the portfolio approach. The symposium especially allowed us to foster closer ties and exchanges with various members of the NPS community. The work in this report has also been presented at the SciTech 2016 Conference, January 7th - 11th, 2015 in Kissimmee, FL (Govindaraju et al. Jan, 2015) and at the SciTech 2017 conference, January 7th – 11th, 2017 in Grapevine, TX (Roy et al. Jan, 2017). These conference presentations generated useful feedback from attending practitioners from operations research and financial engineering communities, focused on ways to improve and further develop the framework presented in this report. We also anticipate that the work proposed here, building upon our previous efforts, will result in an archival journal article describing the approach and its results for both the air cargo and commercial airline application

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