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A Systems Complexity-based Assessment of Risk in Acquisition and Development Programs

18 January 2019

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Stevens Institute of Technology

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Abstract

Development and acquisition programs of cyber-physical systems can often encounter cost or schedule overruns due to the complexity of the system. It has been shown that certain amount of system complexity is related to the system functionalities (effective complexity), whereas excessive complexity is related to unnecessary intricacies in the design (apparent complexity). While the former is necessary, the latter can be removed through precise local redesign. One of the major challenges of systems engineering today is the development of tools, quantitative measures, and models for the identification of apparent complexity within the system.

This technical report has the goal of presenting our research results during last year on evaluating and measuring the structural complexity of the engineered system, and does it through the analysis of its graph representation. The objective of this research has been to mathematically formulate and manage the relationship between the quantitative complexity level of an acquisition or engineering development program (at any point in lifecycle) and its relationship to the increased actual technical as well as programmatic risk respectively. The use of the concepts of graph energy and other spectral invariant quantities allow for the definition of an innovative complexity metric. This metric can be applied knowing the design of the system, to understand which areas are more in need of redesign so that the apparent complexity can be reduced.

Considering the positive correlation between complexity and risk, and complexity and cost, this technical research report presents quantitative measures of the complexity of the system of interest. A set of 12 metrics has been developed and applied to a software system and a defense system of systems. Validation of the metrics has been achieved through human experiments.

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Introduction

Complexity is the hallmark of many engineered cyber-physical systems and is a double-edged sword. Part of complexity is necessary due to the desired functionalities of the designed system, and part of it is due to the unnecessary and unwanted intricacies which deviate the final design from an elegant solution, the optimal one. For example, the first attempts at heavier-than-air flight were carried out by small teams of people that we would today call innovators. The goal of those systems was to achieve leveled flight, over a relatively short distance. As time passed, the requirements for airplanes increased in almost all the applications, from military to commercial flight. The need to carry cargo, payloads, or passengers over increasing distances, in shorter time, at a viable cost, safely and reliably has led to an increase in the complexity of these systems over the last one hundred years. As a result, today's airplane manufacturers employ tens of thousands of people, and have a hierarchy of suppliers with an even larger total workforce. In addition, the development time for a new program has also increased due to the overall increase in complexity.

Airplanes are only one of the many examples of engineered systems where an increase in complexity creates an increase in cost. The costs associated with larger complexity are justified only when they are dictated by system requirements. These design decisions can contribute to the functionality of the system (i.e. functional requirements), or increase system-level characteristics such as resilience, reliability, or safety (i.e. non-functional requirements). According to Carlson, robustness is the maintenance of desired characteristics despite the failure or partial performance of some components of the system, and is correlated with complexity. As long as there is a reason for a design decision, and there cannot be a simpler solution obtaining the same effect at the system level, then the increase in complexity is justified. When the increase in complexity is not justified, then the design solution is not optimal and should be avoided. Unfortunately, to determine the optimality of a design solution, it is necessary to have a deep knowledge of the specific application field, and to have a large set of possible solutions for comparison.

The Department of Defense (DoD) faces challenges in the integration and management of the network of systems that it has developed over the past thirty years due to its increased immense complexity. In 1996, the Vice Chairman of the Joint Chiefs of Staff proposed warfighting capability would be more reliant on systems of systems (SoS) and network centric operations. As such, DoD systems are becoming more and more interconnected and reliant on other systems to provide capability to the user. This creates a complex environment in which systems connect to each other through a variety of means that may not be initially evident to systems engineers. When these systems operate on the battlefield, they often cross service boundaries, but their development within the service makes collaboration difficult in traditionally hierarchal military structures. Additionally, the Government Accountability Office (GAO) found that the DoD lacked methods and tools for conducting portfolio management at the enterprise level for capabilities and noted that there were gaps in the DoD's ability to identify, understand, and assess the capability portfolio.

This technical report builds on complexity theory, network analysis, and systems engineering to propose a method to understand the complexity of complex systems. This report begins with a literature review on complexity theory and various measures of complexity and their shortcomings and merits. It also reviews the relationship between increased uncertainty/risk and increased complexity. Then our research methodology is presented and the formulations for twelve complexity measures are presented and discussed here. Next, we explore applying our introduced complexity measures to two case studies. The first case study is applied to a case of increase in complexity of a software system and its complexity is measured and studied over a period of time. The second case study examines how the addition of a new system to a network of legacy systems affects the complexity of the network. The second case study examines the addition of the F-35 Joint Strike Fighter (JSF) to the network of DoD systems and its effect on the complexity of the network, before the DoD fielded F-35A/B/Cs, during the transition to the JSF, and post deployment after the DoD replaced the legacy systems with the F-35 variants. The report continues to validation section for the methodology and complexity formulations. The last section presents a summary of results and conclusion of this report.

Literature Review

This section presents a brief review of the relevant literature on systems engineering, complexity theory, complexity measures and network analysis. The portion on systems engineering focuses on the foundation of systems engineering and the application of the ilities to help engineers manage complexity and the non-functional attributes of engineered systems. The literature review section includes a discussion on complexity theory and the impact of increases in technology and reliance on other systems. Finally, the literature provides an overview of network analysis techniques that serve as a basis for the quantification of complexity.

Systems Engineering

As a discipline, systems engineering faces increased complexity of systems as technology progress and systems are more interconnected. In 2006, a workshop consisting of thought leaders from a variety of disciplines met to discuss the issue of complex systems, and one area that received substantial attention was the modeling of complex systems with an emphasis on the dynamic, networked nature of systems. International Council on Systems Engineering (INCOSE) defines Systems Engineering as “an interdisciplinary approach and means to enable the realization of successful systems. It focuses on defining customer needs and required functionality early in the development cycle, documenting requirements, and then proceeding with design synthesis and system validation while considering the complete problem”.

Systems engineers differ from traditional engineers in that they consider the system in its entirety; lead the conceptual design of systems; and bridge the gaps between traditional engineering. As such, systems engineers have developed a variety of means: system architecture, system of systems analysis, and enterprise architecture, to deal with complexity. To manage complexity and the qualitative nature of systems engineering, systems engineers have developed the ilities as a construct for assessing nonfunctional attributes of a system. Systems engineers have begun to recognize the criticality of these non-traditional design criteria and have begun to include them in the design of systems. However, these properties and attributes of a system often manifest themselves after engineers have designed and put the system into operation. Further study of the ilities examines how system level ilities begin to emerge from the subsystem level, where systems engineers can design in these non-functional attributes.

Complexity Theory

Complexity theory has deep roots in various fields spanning from physics to biology. In engineering systems, Wade and Heydari categorized complexity definition into three major groups, according to the point of view of the observer. When the observer is external to the system and can only interact with it as a black box, then the type of complexity that can be measured is called behavioral complexity, since it looks at the overall behavior of the system. When the observer has access to the internal structure of the system, such as blueprints and source code for engineered systems, or scientific knowledge for natural systems, then the structural complexity of the system is the one being measured. If the process of constructing the entity is under observation, then is the constructive complexity to be measured, which is the complexity of the building process. This definition relates complexity to the difficulty of determining the output of the system.

Sheard and Mostashari developed a framework for the categorization of complexity types. Engineered systems have two types of complexity: structural and dynamic. Dynamic complexity can be short term or long term. Short term complexity is related to the operation of the system. System behavior can be unpredictable due to non-linear relationships among the system components. The environment can also play a major role on system behavior. Long term complexity is related with the evolution of the system, its growth, and its adaptation to its environment which plays an important role in shaping the new generations. Structural complexity instead, is interested in a snapshot of the system architecture, and can be divided into three components: size, connectivity, and topology.

Structural complexity metrics have been suggested over several decades in literature. The most common type of metrics is based on the concepts of entropy, information content, or logical depth. One of the most famous complexity metrics is Shannon's entropy. This metric measures how much information is contained in the output of a process, and is defined as

$$H = -\sum_i p_i \log_2 p_i \qquad (1)$$

where p_i is the probability of a specific character to be in the message. This formula can be seen as a weighted sum of the information content $-\log_2 p_i$, with weights p_i which make this weighted sum a weighted average. Since its introduction, entropy has been the basis of countless approaches to the measurement of system complexity in various domains, and it's nowadays still applied as a measure of graph complexity. The Jensen-Shannon distance is defined as the difference between the entropy of the system and the sum of the entropies of the subsystems, and has been used to determine regularities in the structure of DNA. Information content has been used by Rashevsky to measure the complexity of graphs representing chemical molecules. This application shows how the topology of a graph can have an effect on complexity, even when the nodes are physically indistinguishable. The type of nodes depends on the way they relate to each other, so that a set of homogeneous nodes can have a positive information content. This is the basis of topological information content, or topological complexity.

Willcox defined complexity as “the potential of a system to exhibit unexpected behavior in the quantities of interest, regardless of whether or not that behavior is detrimental to achieving system requirement”. She proposed an entropy-based metric

$$C(X) = e^{h(X)} \tag{2}$$

where X is the joint distribution of the quantities of interest, and h(X) is the differential entropy of X defined as

$$h(X) = -\int_{\Omega_X} f_X(x) \log f_X(x) dx \tag{3}$$

where Ω_X is the support of X.

Fischi proposed to measure the complexity of the system of interest with a complexity metric based on the information content, and defined as

$$C_t = \sum_{i=1}^{l} (-\ln(1 - p_{b_i})) \tag{4}$$

where p_{bi} is the probability that the i[th] system requirement is satisfied. Allen developed a series of metrics based on entropy, and on the classification of metric concepts provided by Briand. The concepts are size, length, complexity, cohesion, and coupling. According to Allen, the size of the system is a function

$$Size(S) = (n+1)H(S) - (-\log_2 p_L) \qquad (5)$$

Where $H(S) = \sum_{i=1}^{n} p_t(-\log_2 p_t)$ is the entropy of the graph and $-\log_2 p_{L(0)}$ is the information content of the environment surrounding the system. According to this framework, system complexity can be measured as

$$C(S) = \sum_{i=1}^{v} Size(S_i^{\#}) - Size(S^{\#}) \qquad (6)$$

where S# is the edge-only representation of the system, thus making this metric a measure of topological complexity.

Gell-Mann proposed a definition of effective complexity as the amount of information needed to describe a set of identified regularities of the system. This quantity is not to be confused with logical depth. Mandelbrot's set has high logical depth, since being a fractal, a simple rule is applied infinite times in a recursive fashion, but a low amount of effective complexity, since the formula used to describe it is relatively short. This is in general true for all fractals. The definition of regularities and randomness depends on the specific application.

Another common type of structural complexity metrics considers the spectrum (the set of eigenvalues) of the graph representation of the system. These metrics are known as spectral metrics and are the ones adopted in this research.

The first spectral metric has been proposed by Gutman in 1978, is known as Graph Energy and is represented by

$$E_A(G) = \sum_{i=1}^{n} |\lambda_i| \qquad (7)$$

where λ_i are the eigenvalues of the adjacency matrix of the graph G. A variation of this metric, proposed by Gutman as well, is the Laplacian Graph Energy, represented as

$$E_L(G) = \sum_{i=1}^{n} \left| \mu_i - \frac{2m}{n} \right| \tag{8}$$

where μ_i are the eigenvalues of the Laplacian matrix, n the number of nodes and m the number of edges of the graph G. Cavers provided a generalization of these two metrics that can be applied to any matrix representing a graph, which is represented by

$$\boldsymbol{E_M(G) = \sum_{i=1}^{n} \left| \lambda_i - \frac{tr(M)}{n} \right|} \tag{9}$$

where $\lambda_i(M)$ are the eigenvalues of the matrix M, and $tr(M)$ its trace.

Graph energy has been embedded in a structural complexity metric provided by Sinha, as a contribution of the topology of the graph. The formula

$$C(n,m,A) = \underbrace{\sum_{i=1}^{n} \alpha_i}_{C_1} + \underbrace{\left(\sum_{i=1}^{n} \sum_{j=1}^{n} \beta_{ij} A_{ij} \right)}_{C_2} \underbrace{\gamma E(A)}_{C_3} \tag{10}$$

where α_i represents the inner complexity of each node, and β_{ij} the complexity of the edges, is based on the idea that structural complexity has three contributions: components, connections, and topology.

Another type of spectral structural metric, proposed by Wu, considers the eigenvalues of the adjacency matrix as an exponential function, and adjusts the value through a logarithmic scale

$$\bar{\lambda} = \ln \left(\frac{1}{n} \sum_{i=1}^{n} e^{\lambda_i} \right) \tag{11}$$

The coefficient $1/n$ is a way of normalizing the graph according to the number of nodes, which allows to compare graphs of different sizes. This approach has been used by Sinha as well with the coefficient $\gamma = 1/n$. These metrics have been used as a starting point for the development of twelve metrics that consider the system as a graph and are based on the eigenvalues of a certain matrix representing this graph.

Research Methodology

This section presents the methodology that the authors adopted in the formulation of new spectral structural complexity metrics, and the data collection strategy for the characterization of the complex tactical aircraft system of systems. It begins with fundamentals of complexity metric design and is followed by the next section of the report on two case studies.

Formulation of innovative complexity metrics

The metrics presented in this report are all spectral complexity metrics, meaning that they are based on the eigenvalues of a certain graph representation of the system. To represent the graphs three different matrices are used:

- The adjacency matrix,
- The Laplacian matrix, and
- The normalized Laplacian matrix.

The adjacency matrix is the most frequently used representation of an architecture within the systems engineering domain. Also known as Design Structure Matrix (DSM), or N2 matrix, it is used to represent the interfaces and their arrangement, and allows the making of considerations on architectural modularity and clustering of components. The Laplacian matrix includes additional information with respect to the adjacency one, specifically regarding the degree of each component. The normalized Laplacian matrix has an interesting spectrum that is related to other graph invariants more than the spectra of the other two matrices. These three matrices are considered in their weighted variations, where edges and vertices of the graph carry different weights.

The metrics are based on two similar concepts, graph energy and natural connectivity, which as seen in the previous section are both functions of the eigenvalues of the matrix representation of the system. A corrective coefficient $\gamma=1/n$ to compare graphs with different number of nodes is included in the definition of natural connectivity and in Sinha's structural complexity metric.

The metrics are applied to two sets of random graphs, generated through Erdõs-Rényi (ER) and Barabási-Albert (BA) algorithms. The values of each metric are plotted against graph density, which is defined as

$$d = \frac{2m}{n(n-1)} \tag{12}$$

for undirected graphs, and as

$$d = \frac{m}{n(n-1)} \tag{13}$$

for directed graphs, where n is the number of nodes and m is the number of edges in the graph G.

Another graph indicator used in this research is graph diameter, defined as the maximum shortest path between all pairs of nodes in the graph. In absence of accurate information regarding the internal structure of nodes, which is usually the case in system of systems applications, where one organization cannot access data belonging to external actors, the complexity of the nodes can be approximated with the degree of the node $\alpha_i = deg\ v_i$, and $\beta_{ij} = \sqrt{\alpha_i \alpha_j}$.

The developed metrics are spectral, meaning that they consider the eigenvalues of a certain representation of the system. The general formula for the metrics is

$$C(S) = f\left(\gamma \sum_{i=1}^{n} g\left(\lambda_i(M) - \frac{tr(M)}{n}\right)\right) \tag{14}$$

Where M is the matrix representing the system, λ_i are its eigenvalues, g and f are generic functions, and γ is a scaling coefficient that considers the size of the system.

This representation collapses into graph energy for $(x) = x, \gamma = 1, g(x) = |x|, M = A$ (Gutman, 2001), Laplacian graph energy for $f(x) = x, \gamma = 1, g(x) = |x|, M = L$ (Gutman & Zhou, Laplacian energy of a graph, 2006) and natural connectivity for $f(x) = \ln x, \gamma = \frac{1}{n}, g(x) = e^x, M = A$.

These existing metrics can act as primitives to develop a set of twelve metrics. Table 1 shows the metrics that can be derived from this formula through combinations of these parameters. Two sets of functions, two values for the coefficient γ and three matrices, give twelve possible metrics. Throughout this paper, the metrics are referred to using acronyms: graph energy (GE), Laplacian graph energy (LGE), normalized Laplacian graph energy (NLGE), natural connectivity (NC), Laplacian natural connectivity (LNC), normalized Laplacian natural connectivity (NLNC), and where $\gamma = 1/n$, the acronym has a trailing n, such as in (GEn). These metrics will be applied in the next section to sets of random graphs, and to the TACAIR system of systems.

Table 1. Metrics that can be derived from this formula through combinations of these parameters

Adjacency Matrix	Laplacian Matrix	Normalized Laplacian Matrix
$E_A(G) = \sum_{i=1}^{n} \lvert\lambda_i\rvert$	$E_L(G) = \sum_{i=1}^{n} \left\lvert\mu_i - \frac{2m}{n}\right\rvert$	$E_{\mathcal{L}}(G) = \sum_{i=1}^{n} \lvert\nu_i - 1\rvert$
$N_A(G) = \ln\left(\sum_{i=1}^{n} e^{\lambda_i}\right)$	$N_L(G) = \ln\left(\sum_{i=1}^{n} e^{\mu_i - \frac{2m}{n}}\right)$	$N_{\mathcal{L}}(G) = \ln\left(\sum_{i=1}^{n} e^{\nu_i - 1}\right)$
$E_{An}(G) = \frac{1}{n}\sum_{i=1}^{n} \lvert\lambda_i\rvert$	$E_{Ln}(G) = \frac{1}{n}\sum_{i=1}^{n} \left\lvert\mu_i - \frac{2m}{n}\right\rvert$	$E_{\mathcal{L}n}(G) = \frac{1}{n}\sum_{i=1}^{n} \lvert\nu_i - 1\rvert$
$N_{An}(G) = \ln\left(\frac{1}{n}\sum_{i=1}^{n} e^{\lambda_i}\right)$	$N_{Ln}(G) = \ln\left(\frac{1}{n}\sum_{i=1}^{n} e^{\mu_i - \frac{2m}{n}}\right)$	$N_{\mathcal{L}n}(G) = \ln\left(\frac{1}{n}\sum_{i=1}^{n} e^{\nu_i - 1}\right)$

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Case Study 1: Application to System of Systems TACAIR

Capability Development in the DoD

The DoD generates requirements through the Joint Capability Integration and Development (JCIDS) process, which they then pass to the acquisitions community to develop and procure warfighting systems. As a part of this process, DoD systems engineers analyze the current state of legacy systems and determine how the new capability will integrate with these systems. The DoD designed the system to ensure validated military capability requirements support resourcing decisions for programs. The 2003 Joint Defense Capability Study first presented the concept of JCIDS and proposed a transition from requirements based acquisition to a capability-based approach. The JCIDS process supports the Chairman's and the Joint Requirements Oversight Committee's (JROC) statutory responsibilities to identify, assess, validate, and prioritize joint military capability requirements. The JCIDS process requires sponsors to generate three main documents: Initial Capability Document (ICD), Capability Development Document (CDD), and the Capability Production Document (CPD), that support different phases in the development and acquisition process by providing traceability from warfighter capability requirements to fielded systems.

As part of the JCIDS process, the Joint Staff requires several DoD Architecture Framework (DoDAF) viewpoints to support the development of warfighter capabilities. Architecture frameworks assist decision makers by serving as a communication tool by presenting a manageable amount of information from a set of data to assist stakeholders in managing complex systems. System architects use DoDAF, one of several common frameworks, to capture multiple perspectives of a warfighting capability's system architecture. All architecture frameworks include specific taxonomies, artifacts, and terminologies for describing a system to ensure standardization across multiple individual architectures. DoDAF includes eight different viewpoints that capture data relevant capability requirements, integration, military operations, and program management aspects of a system. The DoD designed DoDAF to meet the needs of a diverse set of stakeholders and decision makers by abstracting essential pieces of information and presenting them in manageable pieces depending on their perspective. The required DoDAF products provide valuable data at the individual system level; however, they do not provide much insight into the larger, aggregated network of systems.

One shortfall of the DoDAF architectures used in capability development is that they do not capture a DoD wide perspective of the interactions between individual systems. Several efforts have attempted to aggregate independent DoDAF produces along mission threads; however, they still limit their approach to a subset of the entire DoD capability network of systems. Ring, et al. proposed the Activity-Based Methodology, which aggregates DoDAF architectures into an integrated architecture that captures the organization, system, and role aspects of DoD systems. Another effort proposed aggregating independent architectures through a system, capability, and mission perspective by utilizing independent DoDAF viewpoints.

F-35 Joint Strike Fighter

The F-35 JSF is a joint, multi-role fighter and attack aircraft that is entering service with the Air Force, Navy, and Marines to replace a variety of legacy systems. The F-35 is a fifth-generation fighter aircraft that incorporates stealth technology into the design of the aircraft and uses a common airframe across all three versions of the aircraft. The F-35A is the conventional take-off and landing version of the JSF that incorporates an advanced sensor package and situational awareness capability to drastically improve the effectiveness of the aircraft. The Air Force plans to replace both the F-16 and A-10 with the F-35A beginning in 2016 as it fields their version of the F-35 in air-superiority, suppression of enemy air defense, and close air support roles. The Marine Corps began fielding the F-35B short takeoff and vertical landing (STOVL) version of the JSF which provides the capability to take off and land on extremely short runways. The Marine Corps plans to use the F-35B to replace both the F/A-18 Hornet and the A/V-8B Harrier II with the JSF. The Navy's version of the JSF, the F-35C, includes increased wing area and structural enhancements to support carrier landings and take offs. The Navy plans to replace the F/A-18 with the JSF to serve as its primary air superiority and attack aircraft.

Application of Complexity Metrics to Case study

This section presents an overview of the methodology to develop three individual networks of systems that capture the “as-is”, “transitional”, and “to-be” networks. A variety of publicly available sources provides the necessary data to develop the network of systems and identify connections between the systems. The network captures interoperability connections between the systems that include information flows, shared resources, and physical connections.

Table 2. Excerpt from Adjacency Matrix

	A-10C	AIM-120	AIM-9X	F-16C	F-22	F-35A	GPS III	Link-16	JDAM	KC-46
A-10C			X				X	X	X	X
AIM-120				X	X	X				
AIM-9X	X			X	X	X				
F-16C		X	X				X	X	X	X
F-22		X	X			X	X	X	X	X
F-35A		X	X		X		X	X	X	X
GPS III	X			X	X	X				
Link-16	X			X	X	X				
JDAM	X			X	X	X				
KC-46	X			X	X	X				

The “as-is” network captures the systems that comprise the DoD’s tactical aircraft system and consists of aircraft, munitions, sensors, and communication systems prior to the fielding of the F-35. The “transitional” includes all the legacy aircraft as well as the JSF and its connections that represents the DoD network as the Air Force, Navy, and Marine Corps transition to the F-35 from their legacy aircraft. Finally, the “to-be” network depicts the DoD’s network of tactical aircraft and systems after the three services retire the systems the F-35 is scheduled to replace.

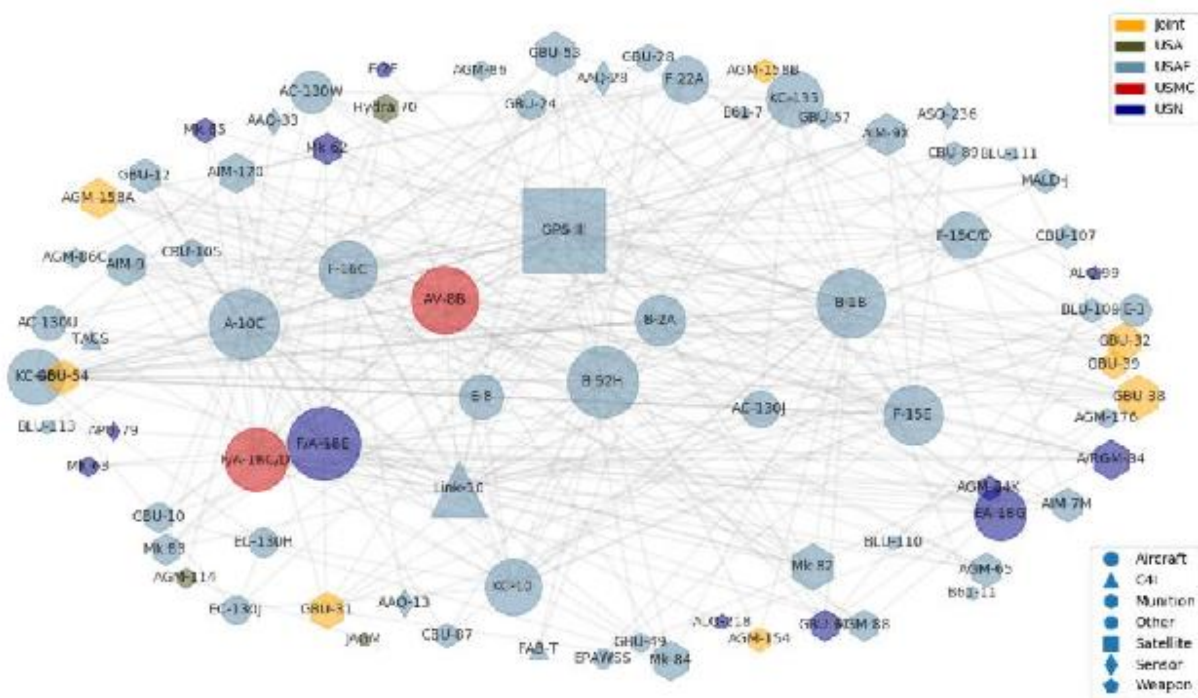

Figure 1. As Is Network of DoD Tactical Aircraft Systems

Table 2 presents an excerpt from the entire adjacency matrix for the tactical aircraft network of systems. A complete matrix for each of the networks captures the data required to analyze the complexity of the network.

Figure 1 presents the graphical depiction of the “as-is” network of DoD tactical aircraft systems and represents the past version of the network prior to the deployment of the F-35 JSF variants. This network captures various types of systems that operate together to provide the DoD with tactical aircraft capability to include the aircraft, munitions, sensors, satellites, weapons, and command, control, communications, computer, and intelligence (C4I) systems. In the graph, the colors represent the various services, the shapes of the nodes represent the type of system, and the size of the node represents its degree. This network represents the DoD’s tactical aircraft systems prior to the development of the JSF and provides the baseline for analyzing the complexity of the network.

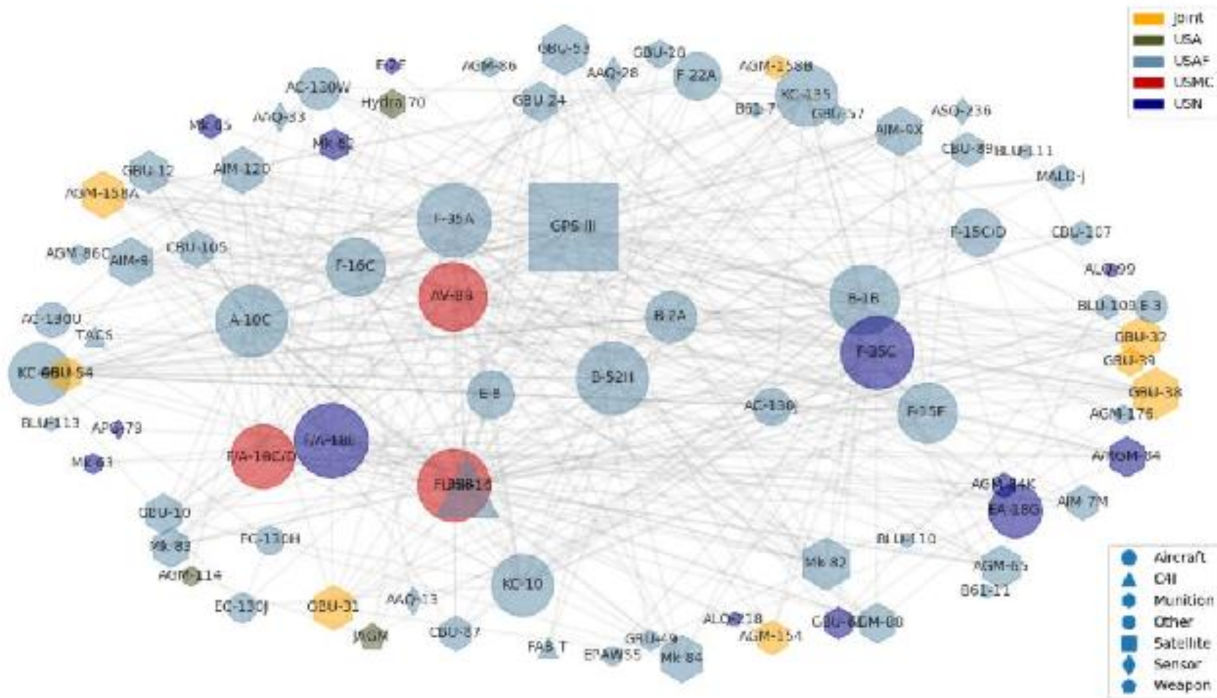

Figure 2. "Transitional" Network of DoD Tactical Aircraft Systems

Figure 2 presents the graphical depiction of the network and the connections that will be present during the transition from the legacy aircraft to the JSF variants. In this case, both the JSF and the aircraft the services plan to replace with the F-35 variants are included in the network along with any of their connections to other systems in the network. The colors, shapes, and sizes are consistent with the previous graphical representation of the network. This version of the network provides a means to evaluate the complexity of the network during the transition period to the JSF which could impact resource expenditures, maintenance, supply operations, and tactical operations of the DoD.

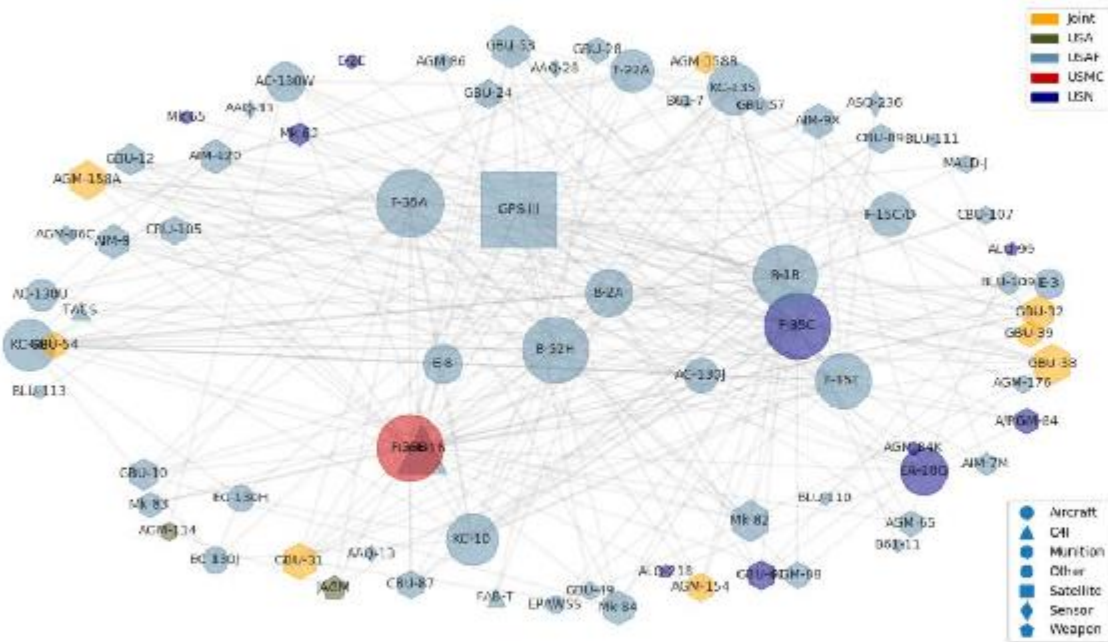

Figure 3. "To-be" Network of DoD Tactical Aircraft Systems

Figure 3 presents the final version of the network and represents the “to-be” tactical aircraft network after the services retire the legacy systems that they are replacing with the F-35. In this case, the network does not include the retired A-10, F-16, F/A-18, and AV-8B systems from the Air Force, Navy, and Marine Corps. In addition to the four retired systems, the network removes systems that may not be retired but no longer connect to the network to include the Hydrai Rockets, AN/APG-79 AESA Radar, and Mk 63 Sea Mine. This does not indicate that these systems could also be retired as they may be used by other systems; however, it does affect the complexity of the tactical aircraft network. This version of the network provides the means to calculate the complexity of the network after a complete transition to the JSF and can determine if the DoD increased or decreased the complexity of its tactical aircraft network.

Results

The metrics have been applied to two sets of random graphs, generated with ER and BA models respectively. The sets of graphs contain approximately 23,000 and 38,000 unique labeled graphs.

Figure 4 represents the values that the twelve spectral structural metrics assume when applied to the ER set of random graphs. Most of the metrics have a positive correlation with the number of nodes in the graph, meaning that the metric value is higher when the number of nodes is higher. This is the expected behavior for a complexity metric, and the two metrics that do not follow it, NLGEn and NLNCn, are not suitable as complexity metrics.

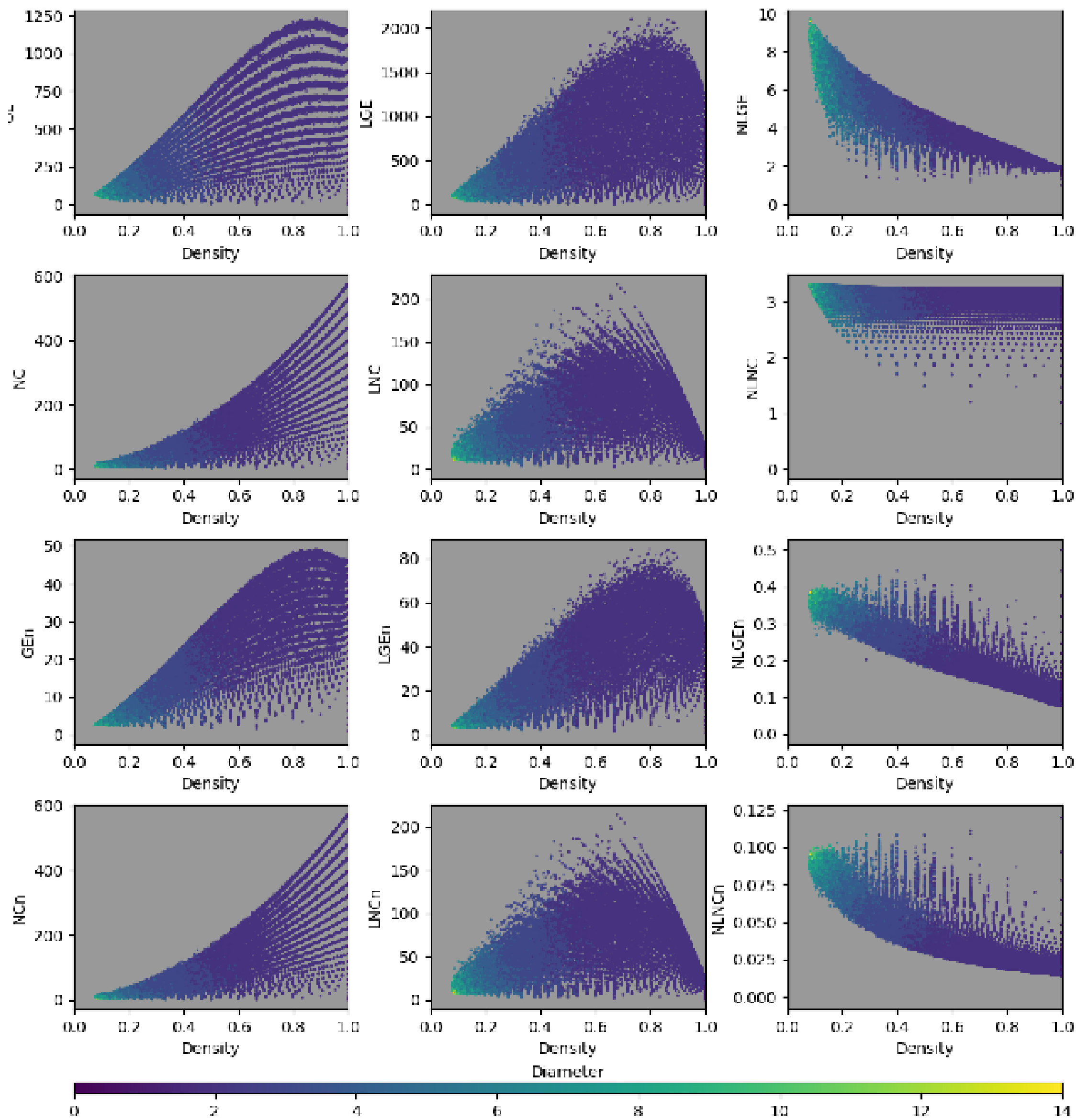

Figure 4. Metric vs. density plots with color scale according to diameter, for each metric, for graphs generated using Erdos-Renyi algorithm

From Figure 4 it is possible to see that for ER random graphs the diameter is high with low density graphs, and low when the density is high. This relationship is expected since the complete graph has diameter one and removing edges creates an increase of the shortest path between pairs of nodes. Although valid for ER graphs, the relationship between density and diameter is not general, since star graphs and path graphs with the same number of nodes have the same density, but the former have diameter 2 while the latter have diameter $n-1$. This means that for high n, the diameter of these two types of graphs is very different. This is one limitation of the ER algorithm, which will not generate star graphs, or graphs with highly skewed degree distributions, given its uniform probability of edge creation.

The metrics have been evaluated, as reported in Figure 5.

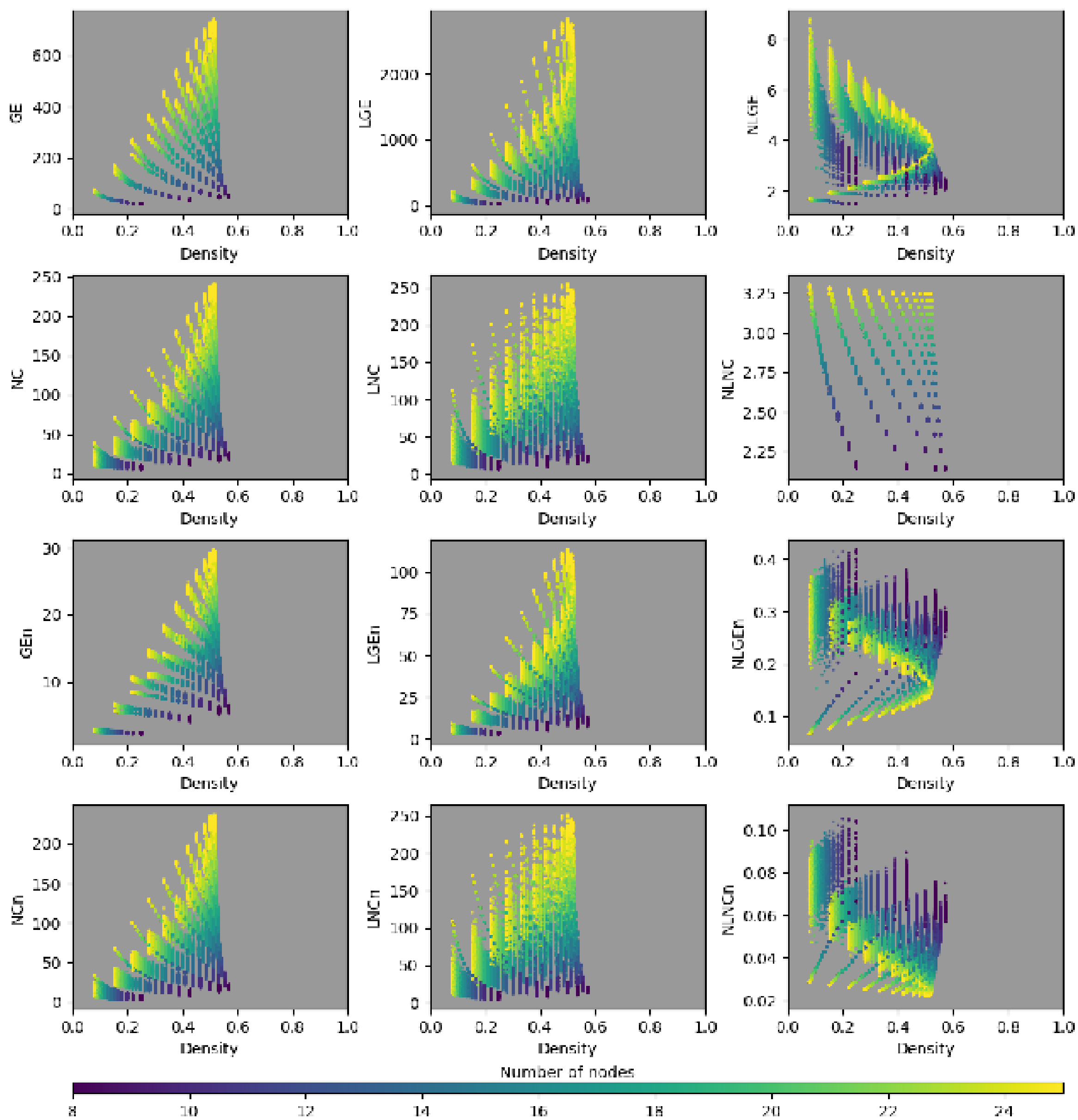

Figure 5. Metric vs. density plots with color scale according to number of nodes, for each metric, for graphs generated using Barabasi-Albert algorithm

To overcome the limitations of the ER model, and to better mimic the topology of engineered systems with heterogeneous components, a set of graphs has been generated using the BA model. These graphs have a more skewed degree distribution, given by the preferential attachment strategy.

Figure 5 shows the metrics evaluated for the set of BA random graphs. Given the way the algorithm works, these graphs do not span the whole density range, but stop at $d = 0.57$. The main feature of these point clouds is a folding, a bifurcation, so that graphs with the same density will belong to two distinct sets with a high and low value of each metric respectively. This bifurcation gives meaning to the metrics, highlighting the fact that they are responsive to topological changes.

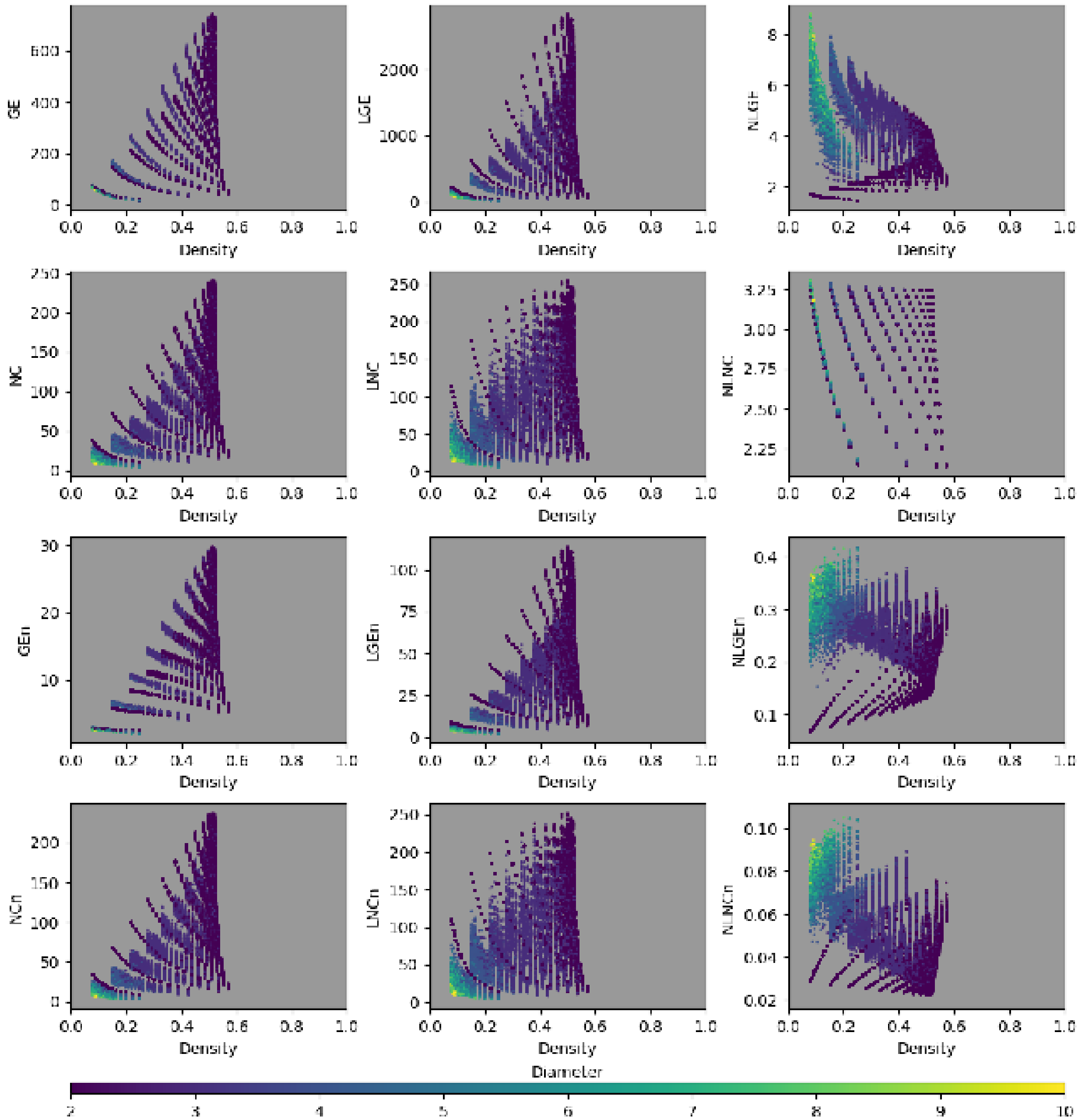

Figure 6. Metric vs. density plots with color scale according to diameter, for each metric, for graphs generated using Barabasi-Albert algorithm

Figure 6 shows that this bifurcation in BA random graphs is related to the diameter of the graphs. The diameter does not have the same trend as in ER graphs. There exist graphs with low density which have high diameter and low diameter. These two sets are represented by trees with high depth, and stars respectively. While a star topology is not common in engineered systems, since it is subject to bottlenecks and the complexity of the central node would tend to be too high, trees are common structures for engineered systems, where a certain level of decentralization is in order. Even in the presence of cycles, when the graph is not a tree anymore, a diameter value of 10 in a graph of 25 nodes is representative of engineered systems.

The TACAIR system of system, in its three versions presented earlier is undergoing radical changes. The introduction of the F-35 in the operational scenario and the subsequent retirement of legacy systems is causing modifications to the network topology. The number of nodes went from 82 to 85 and will go down to 77, and the number of interfaces went from 384 to 466 and will be 347 once the transition is complete. This leads to a density value going from 0.115 to 0.130, and to 0.118 in future. This density variation is not accompanied by a change in diameter which remains constant to 5, due to the centrality of the nodes that are being added and removed from the network. In this case, the metrics are beneficial to the network analysis, since they can tell more than the diameter about the topology of the network.

Figure 7 shows the metrics applied to the TACAIR system of systems. Other than NLGEn and NLNCn, which we have already ruled out as reliable complexity metrics, and NLGE, the other metrics agree that the introduction of the F-35 represents an increase in the complexity of the network. Most of the metrics, other than NC and NCn, also agree that the retirement of the legacy systems is beneficial for the network, and will lead to a simplification of the overall network.

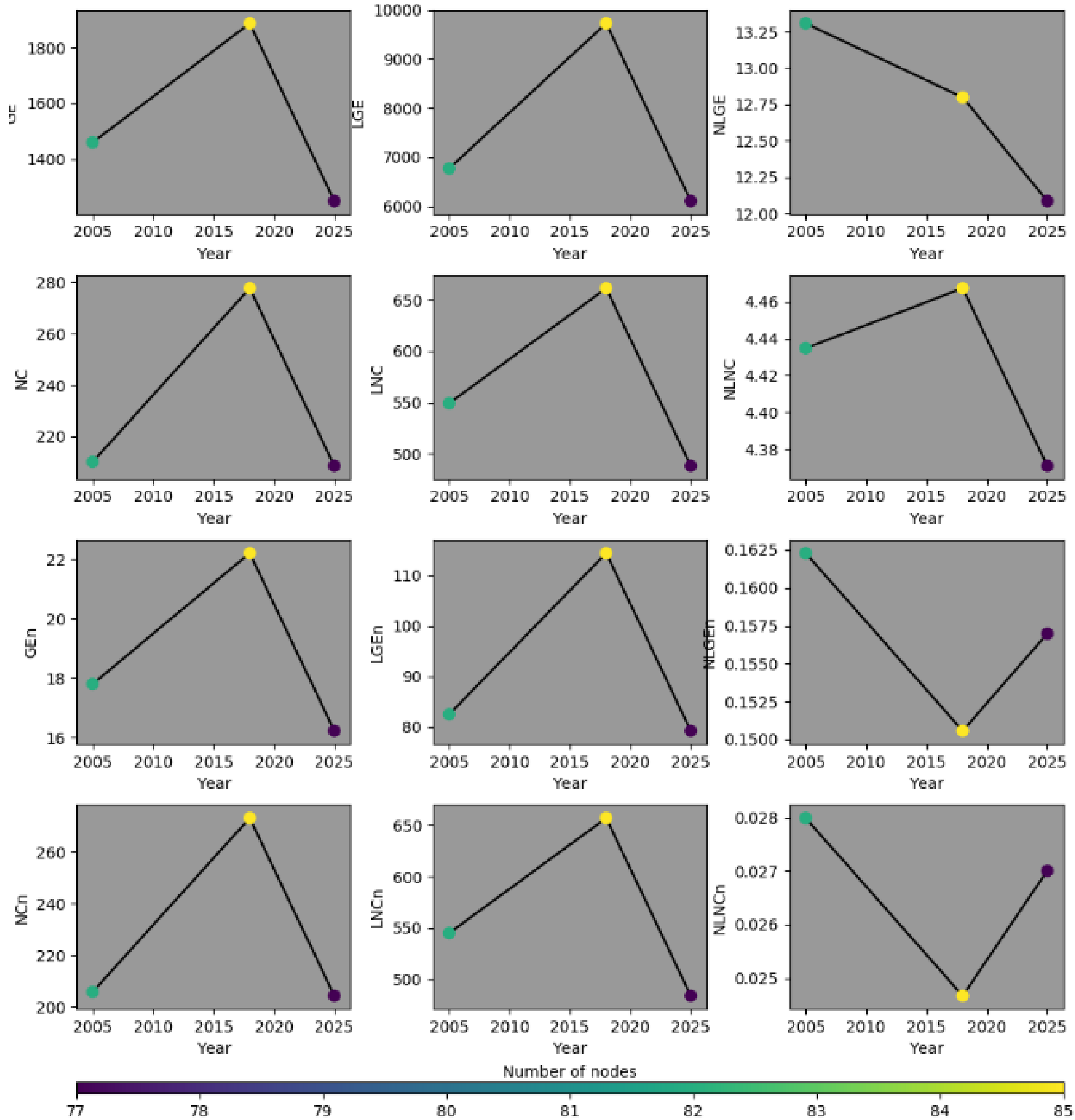

Figure 7. Application of the metrics to the TACAIR system of systems.

Case Study 2: Application to Software System

The approach to the generation of the software system architecture is the same implemented by MacCormack, but instead of using software analysis tools available on the market, a specific tool has been developed to analyze Python software. The software system is the source code of Reddit, available on Github, and represented as a hierarchy of files and folders in Figure 8. When designing a software system, the modularization of the source code is achieved through grouping of related files in a folder hierarchical structure. This grouping should reflect the actual dependencies between files, so that clusters of connected files reside in the same folder, and the modification of the code in one such file can have effects only on the files directly connected to it, which ideally reside in the same folder. This theoretical modularization is in practice never achieved, since the overall source code is connected, and no hierarchy of folders can cluster the files in a proper manner. Figure 9 shows the connections between the files in the source code. Each connection represents a functional call, which in the case of Python code is an import statement. The Python subset of files and the connections created through functional calls define a connected graph in which nodes are files and edges are connections, as shown in Figure 10. This graph represents the state of the source code at the end of 2017, and the coloring according to the folder in which each file resides shows that the hierarchical representation of the source code does not map to the clustering given by the functional calls.

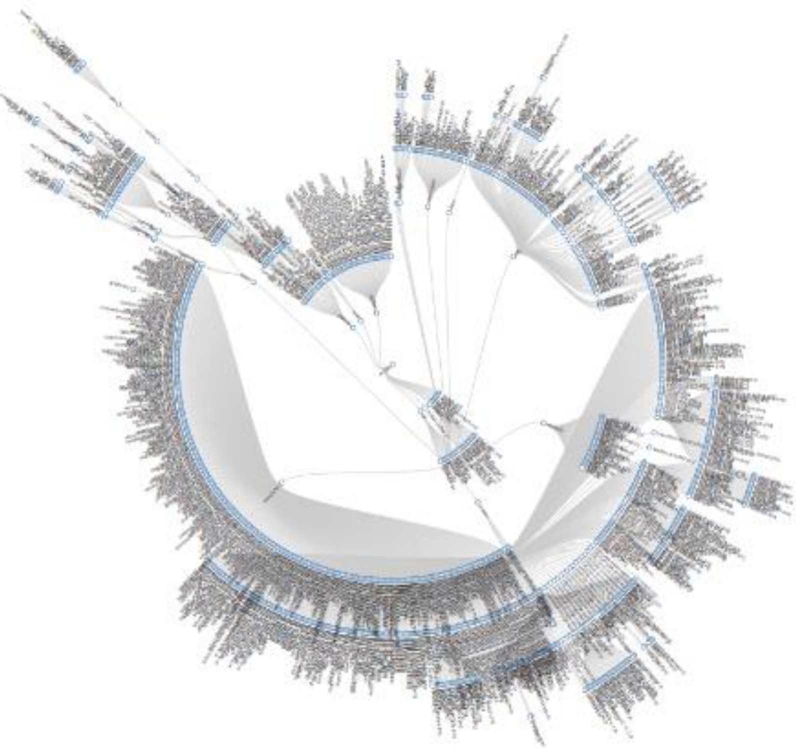

Figure 8. Hierarchical representation of the source code of Reddit. Data from Github and visualization code from d3js.org

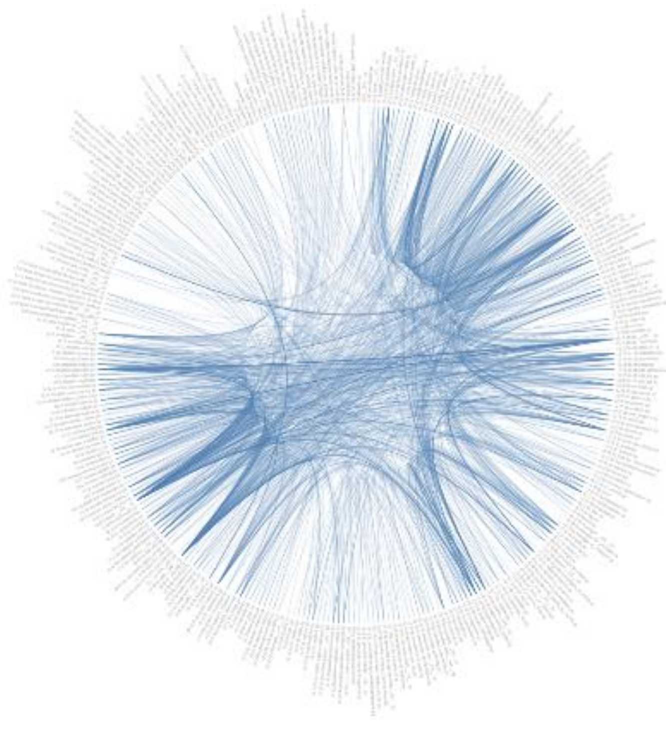

Figure 9. Representation of the import dependencies between files in the source code of Reddit. Data from Github and visualization code from d3js.org

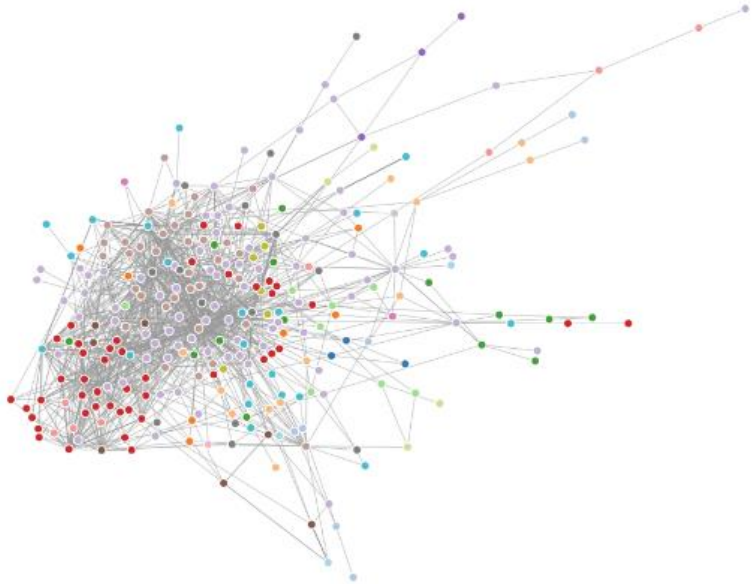

Figure 10. Graph representation of the source code of Reddit with node coloring according to file location in folder. Data from Github and visualization code from d3js.org

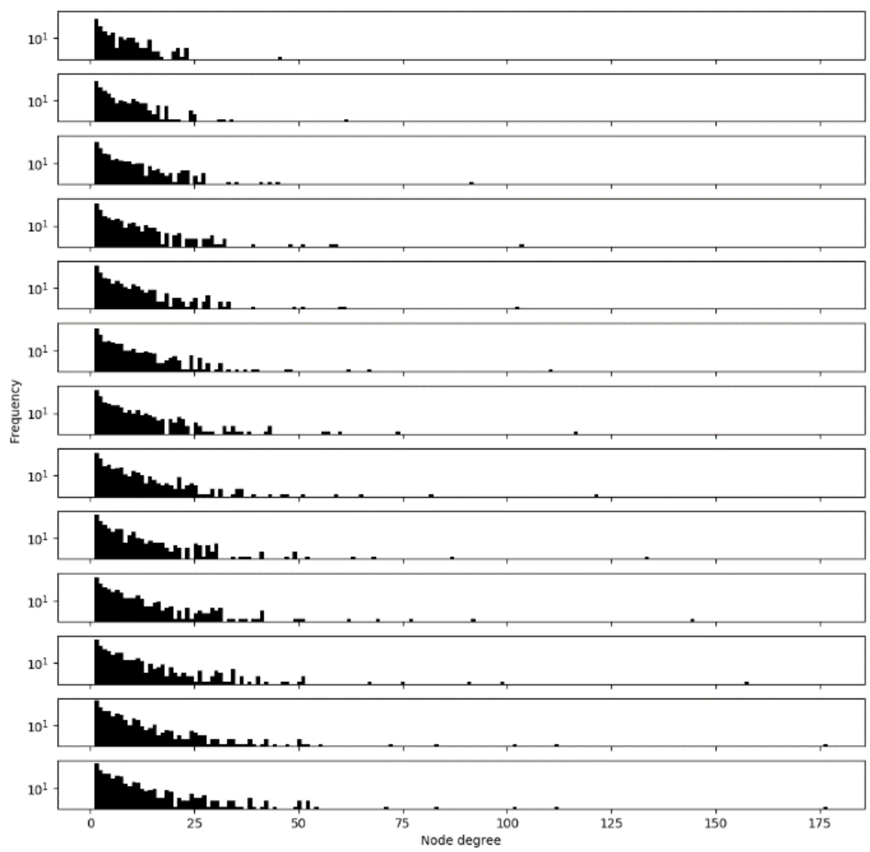

Figure 11. Histograms of node degree frequency for various versions of the source code.

The evolution of the Reddit source code is analyzed from the point of view of its structural complexity. The Git repository contains a total of 7,956 commits. A series of thirteen versions has been selected to represent various stages in the evolution of the software system, nine spaced of roughly 1,000 commits, three spaced each 250 commits, to better sample the initial stages of development in which commits were less frequent and contained more changes, and one to identify the end of development, roughly 100 commits before the last one.

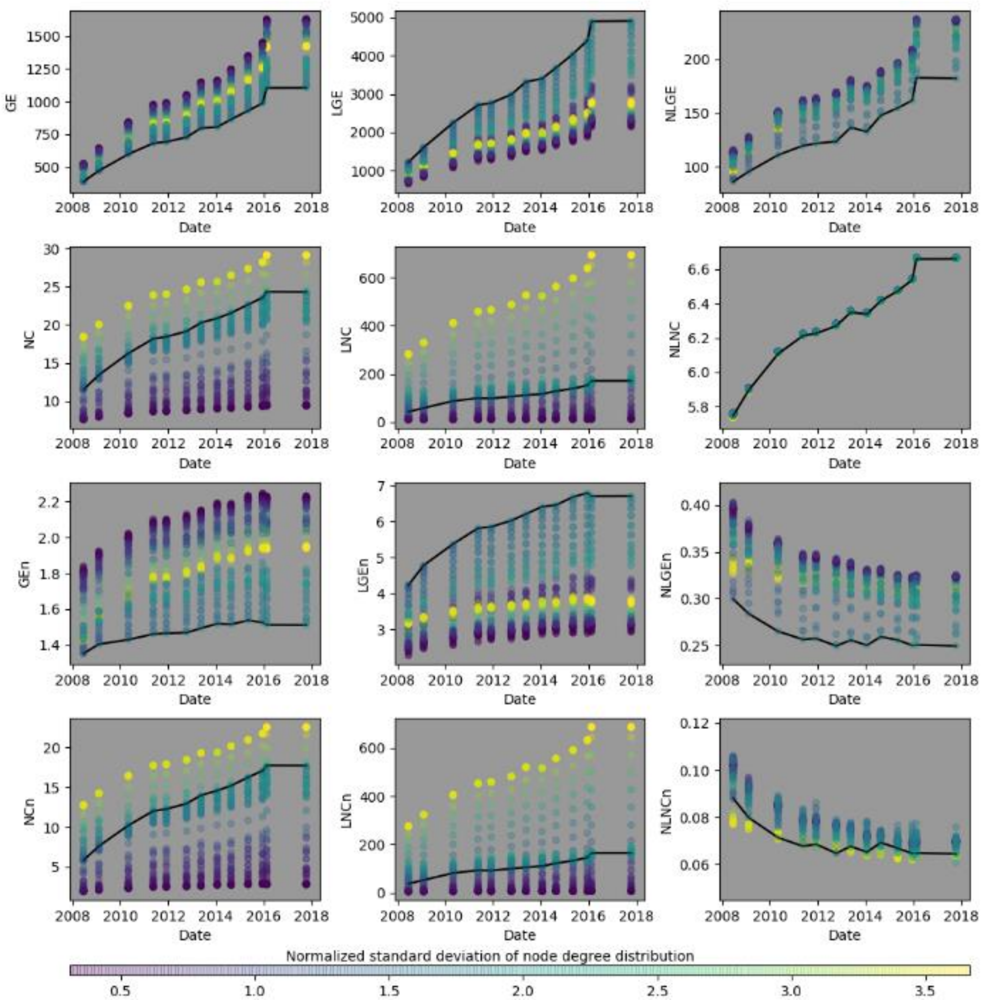

Figure 12. Evolution of metric values over time. Coloring according to normalized standard deviation of node degree.

The evolution of this system through ten years of development has been tracked, and thirteen versions have been selected to represent the system at a certain point in time. Figure 11 represents the normalized distribution of node degree and Figure 12 the evaluation of the metrics throughout development.

Validation

Validation has been achieved through a human experiment. Subjects were asked to assemble models in a 3d environment. The relationship between the assembly time and the value of the metric for each assembly, has indicated that some of the metrics have a positive correlation with the integration time.

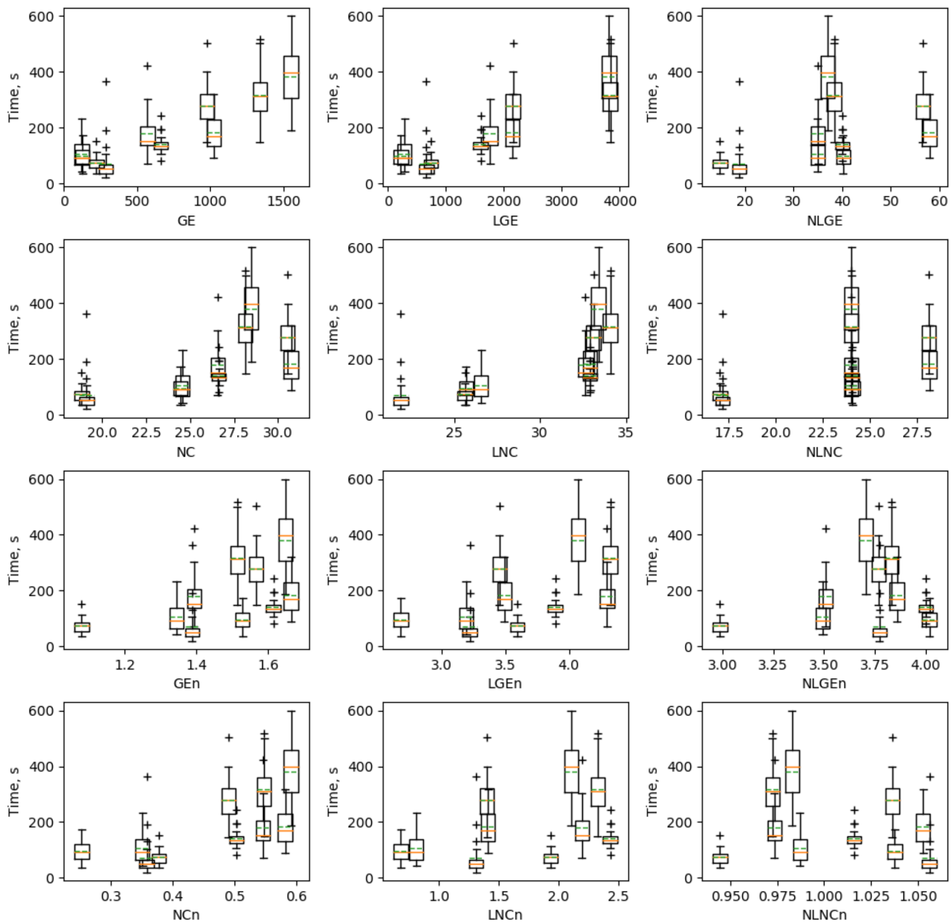

Figure 13. Box-plot representation of integration times vs. metric values for each metric.

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Analysis and Results

This research provided a new approach to the development of structural complexity metrics. The introduction of embedded weights for nodes and edges within the matrix representing the system, helps the metric pass decomposition tests which were not passed by previous metrics. This feature allows for a complexity budget to be carried out across multiple organizations, where suppliers provide their products accompanied by a complexity evaluation, without the need to share information about the architecture of the products.

The feature-based approach to the development of new metrics, together with the application to random graphs and real systems, and a human study focused on integration time, allowed us to determine the following findings about the behavior of each metric and the whole development process:

1. Spectral metrics are related to the diameter of the network. This relationship has been missed by previous researchers who focused on sets of graphs generated through the Erdõs-Rényi algorithm, and can be seen through the application of the metrics to graphs built using the Barabási-Albert algorithm, which have a large variation in diameter.
2. The combination of a graph representation based on the normalized Laplacian matrix and a normalization coefficient inversely proportional to the number of nodes lead to an inverted behavior of the metric with respect to the number of nodes within the system. This means that the metric will return lower values of complexity for systems with more number of components.
3. The metrics have different relationships with respect to the shape of the node degree distribution. The expected behavior is for the metric to have a directly proportional relationship with the standard deviation of the node degree distribution, since this means that more specialized networks are more complex. The metrics that satisfy this condition are based on natural connectivity, with the graph represented through either the adjacency or Laplacian matrix.
4. Representing a graph with a normalized Laplacian matrix for the purpose of measuring its structural complexity might be the wrong approach, even when the normalization coefficient $\gamma = 1.$. The extrapolation of results from the human study at low complexity levels suggests that the relationship with integration time would not increase monotonically at higher complexity values. Further tests could disprove this finding.
5. The findings presented by (Sinha & others, 2014) regarding a super-linear relationship between graph energy and integration time are not confirmed when the complexity of the components and interfaces are taken into account. In this

case, graph energy has a close to linear relationship with the integration time. Despite these interesting findings, the search for a definitive structural complexity metric that encompasses all the features of complexity is not over yet. Future research will focus on the application of the most promising metrics to various representations of the same system. Changing the definition of dependency creates a different graph for the same source code. Good metrics should be agnostic to the specific representation, and present consistent values independently of the dependency definition. Proper metrics should also be internally consistent, meaning that they return the same value for any the level of decomposition. This means that when the system is represented at various levels of decomposition, with three, five, or ten hierarchical levels, a good metric would return the same value regardless.

Conclusion

This report presented an approach to the measurement of structural complexity that involves the measurement of the eigenvalues of a matrix representation of the system. Twelve spectral metrics have been created, based on features of existing metrics. The metrics have been applied to two sets of graphs, generated using the Erdõs-Rényi (ER) and Barabási-Albert (BA) algorithms respectively. It is argued how the application of these algorithms to the generation of graphs representing engineered systems should be carried out together with considerations about the heterogeneity of the components of the system and the expected distribution of node degree. ER models having a close to uniform distribution of node degree are applicable to the representation of homogeneous graphs, such as networks of routers, in which all the components have the same tasks and functionalities. When specialization arises, and the components of a system are wildly heterogeneous, the degree distribution if highly skewed, and BA models are more appropriate.

The application to the TACAIR system of systems is an example of how the operational scenario can become complex thanks to the relationships between different types of systems, and how the introduction of new systems and the retirement of legacy ones can be beneficial to the management of the network, by streamlining the supplying of common resources and reducing the diversity of systems that achieve the same functionalities. Of course, this type of analysis can be improved when details about the architecture of each system are available, and the interfaces can be modeled with high fidelity regarding the timing and range of connections.

Limiting the approach to publicly available data, allowed us to assume the point of view of an external actor who is interested in introducing a new system in an already existing environment. Examples of such systems can be the introduction of a new type of transportation system, such as the hyperloop concept, within the already existing network of air, sea, and land transportation systems, or the introduction of a new surgical tool to be used in conjunction with the existing set of operation room equipment.

In the future, if detailed data is available regarding one of the existing systems in the network, it would be possible to analyze the network and yield more insightful considerations about the retirement of such systems and the effect on the overall network.

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