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Rethinking Government Supplier Decisions: The Economic Evaluation of Alternatives (EEoA)

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Rethinking Government Supplier Decisions: The Economic Evaluation of Alternatives (EEoA)

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Abstract

This paper offers an economic model to assist public procurement officials in ranking competing vendors when benefits cannot be monetized. An important defense application is "source selection"—choosing the most cost-effective vendor to supply military equipment, facilities, services, or supplies. The problem of ranking public investment alternatives when benefits cannot be monetized has spawned an extensive literature that underpins widely applied decision tools. The bulk of the literature, and most government-mandated decision tools, focuses on the demand side of a public procurement. The "economic evaluation of alternatives" (EEoA) extends the analysis to the supply side. A unique feature of EEoA is to model vendor decisions in response to government funding projections. Given a parsimonious set of continuously differentiable evaluation criteria, EEoA provides a new tool to rank vendors. In other cases, it offers a valuable consistency check to guide government supplier decisions.

Keywords: defense acquisition, decision analysis, multi-attribute auction

Introduction

As nations struggle to recover from a global pandemic that devastated lives and destroyed economic activity, massive government spending aimed at limiting the damage has shattered fiscal balance sheets. Record deficits and debt will place nations under enormous pressure to trim defense expenditures. To preserve capabilities, hard choices lie ahead that require a sober assessment of security challenges, and robust methodologies to prioritize defense and other public investments.

Defense procurement is big business. Recently the U.S. Department of Defense (DoD) spent over $300 billion on acquisition, research, development, test, and evaluation, most of it sourced to the private sector (Schwartz et al., 2018). The Organization for Economic Cooperation and Development reports member countries spend more than 12% of their cumulative GDP on public purchases (OECD, 2016). Significant academic effort has been focused on the defense acquisition process through an economic lens; these include theoretical studies (Cavin, 1995), experimental studies (Davis, 2011; Kirkpatrick, 1995), and empirical studies (Horowitz et al., 2016). Indeed, understanding and improving the efficiency and effectiveness of public procurement is of utmost practical and academic importance.

One of the biggest challenges for public procurement officials is to rank vendors when benefits cannot be monetized. Indeed, government benefits are often depicted as bundles of desirable characteristics or attributes that cannot easily be combined with costs into a single overall measure such as profitability. The problem of ranking public investment alternatives when benefits cannot be monetized has spawned an extensive literature generally referred to as multi-criteria decision-making (MCDM). A proliferation of applications of decision tools derived from this literature has appeared in the fields of management science, operations research, and decision sciences (prominent examples include Keeney & Raiffa [1976]; Kirkwood [1995, 1997]; Clemen [1996]; Parkes & Kalagnanam [2005]; Ewing et al. [2006]).

Today, widespread application of MCDM tools and techniques is mandated through various laws, rules, and regulations that govern public procurement, though the specific approach is not prescribed. For example, the main guide for federal procurement officials in the United States is the Federal Acquisition Regulation (FAR).[1]

> Evaluation criteria are the factors an agency uses to determine which of several competing proposals submitted in response to an RFP [Request for Proposal] would best meet the agency's needs. In establishing effective evaluation criteria, an agency must clearly identify the factors relevant to its selection of a vendor and then prioritize or weight the factors according to their importance in satisfying the agency's need in the procurement. …This allows the agency to rank the proposals received. (FAR, Proposal Development, Section M-Evaluation Factors for Award)

Similar source selection techniques are frequently applied in the United States at state and local levels, and in the private sector.

While demand side developments of MCDM models have been extensively studied in the academic literature, the literature is mostly silent about the supply side (vendor) problem. Vendor decisions (bid proposals) are generally treated as exogenous in the Decision Sciences and Operations Research literature. In contrast, the economic evaluation of alternatives (EEoA) captures both demand side—procurement official decisions—and supply side—vendor optimization decisions. Our model formulation is in the spirit of Lancaster's (1966, 1971) "Characteristics Approach to Demand Theory" as modified by Ratchford (1979), and closely corresponds to the third of six approaches to structure an EEoA introduced in Chapter 4 of "Military Cost–Benefit Analysis: Theory & Practice" (Melese, 2015, p. 96).

EEoA encourages public procurement officials to carefully consider the impact on vendor proposals of announced priorities (i.e., desired criteria, characteristics, or attributes for solicited quantities of products, services, or projects, such as computer systems, vehicles, weapon systems, logistics packages, and buildings). Officials should also consider the impact of anticipated future budgets. In response to government-issued priorities—evaluation criteria, quantities, and funding—competing vendors, with different input costs and technologies (described using "engineering production functions")[2] maximize their production offers—bid proposals that consist of bundles of non-price characteristics or attributes.

EEoA models public procurement official decisions in two stages. In the first stage, along with the requirement (quantity demanded), and funding guidance, the procurement official reveals desired evaluation criteria (characteristics or attributes) of the product or service (but not the relative importance/weights). Given this information, competing vendors engage in constrained optimizations based on their respective production technologies ("engineering production functions"), and input costs, to generate proposals that match anticipated future funding. Since input costs and production functions vary among vendors, they play a critical role in their bid proposals—interpreted as bundles of non-price characteristics or attributes embedded in each identical unit offered by a particular vendor. In the second stage, the

[1] Note the exclusive focus on the demand side in the FAR (i.e., ranking exogenously-determined bids received from vendors; see https://www.acquisition.gov/browse/index/far). Also note that standard practice for U.S. military (and other procurement officials) is to: 1) announce factors ("evaluation criteria") relevant to the selection, but then only after receiving vendor proposals, 2) assign specific relative importance/weights to those factors to rank vendors. This practice is modeled in the economic evaluation of alternatives (EEoA).

[2] For interesting discussions of "engineering production functions," see Chenery (1949), Kurtz & Manne (1963), Wibe (1984), Charnes et al. (1991), and Hildebrand (1999).

procurement official ranks competing vendors according to the government's utility function over the evaluation criteria[3] (see Figure 1).

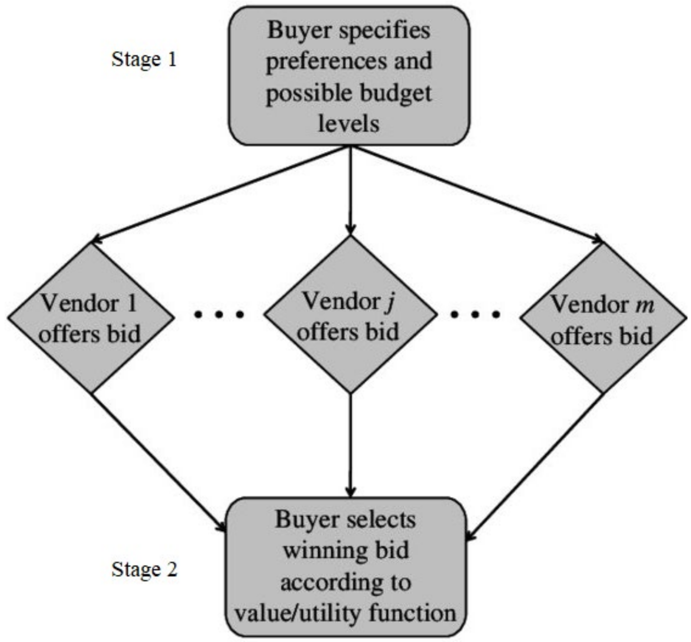

FIGURE 1. The Two-Stage Procurement Process

The dual objective of EEoA is to encourage governments: 1) to consider the supply side (i.e., to recognize the importance of modeling vendor responses to information provided or inferred in public procurements); and 2) to offer an alternative to the standard MCDM approach when benefits cannot be monetized. An attractive feature of EEoA is that it offers a novel technique to measure "benefits" that serves as a valuable consistency check for MCDM preference trade-offs among key attributes.[4] We explore assumptions under which the two decision models (MCDM and EEoA) are isomorphic from a procurement official's perspective. In practice, however, we demonstrate how EEoA can yield significantly different solutions (rank orderings of vendors) than the standard MCDM approach.

The paper proceeds as follows. The next section develops the two-stage economic evaluation of alternatives (EEoA) model. On the supply side, two cases are presented to illustrate the model: 1) where vendors have identical attribute costs, but different production technologies ("engineering production functions"); and 2) where vendors have different attribute costs, but identical production technologies. A simple example serves to integrate procurement official (demand) considerations, with vendor (supply) decisions, under varying (probabilistic) scenarios. The next section contrasts an application of EEoA, with the standard textbook application of MCDM. The last section concludes with recommendations for future research.

[3] Note this is analogous to steps mandated in the FAR, except that, since funding is fixed in EEoA (i.e., the unit price is the same for each vendor), the second step involves the submission by vendors of sealed non-price bids for the announced level of funding, interpreted and evaluated by procurement officials as bundles of characteristics and attributes that respond to previously announced evaluation criteria (for example, see FAR 14.5).

[4] Both Australian and Canadian Ministries of Defence are considering implementing this consistency check for the MCDM component of their portfolio decision models. (Personal correspondence with fellow NATO SAS-134 Defence Official Panel Members studying Defence Portfolio Management for NATO; emails received 11/2018)

The Economic Evaluation of Alternatives (EEoA) Model

The challenge for our public procurement official is to select a competing vendor that delivers the best performance (combination of desired non-price attributes) for each identical unit of a requirement (e.g., 100 ventilators, or 50 computers, or 20 drones, or 2 hospital ships), at affordable funding levels. The EEoA framework can be thought of as a multi-attribute sealed bid procurement auction that extends traditional price-only auctions to one in which competition among $j \in [1, m]$ vendors (bidders) takes place exclusively over bundles of $i \in [1, n]$ non-price characteristics or attributes (a_{ij}).[5]

The EEoA model structures the problem as a two-stage optimization (see Figure 1). In the first stage, the public procurement official provides j competing vendors with the evaluation criteria, available funding, and the requirement (quantity demanded).[6] Given the anticipated budget, **B**, and their respective production technologies ("engineering production functions") and input costs, competing vendors offer their best possible non-price attribute packages bundled into each identical unit required.[7] Note that the greater the funding available, the greater the available funding per unit, which allows vendors to bundle more of the desired attributes into each identical unit (e.g., better ventilators, computers, drones, ships).[8]

The vendor (supply side) problem is formulated in the section titled First Stage EEoA: The Vendor's Problem (Supply Side). Competition takes place exclusively over non-price bid proposals from each vendor, evaluated by procurement officials as bundles of attributes offered by each vendor for a standard unit of the requirement. Whereas attributes for each unit of the requirement are identical for each vendor, the proposed bundles differ among vendors. Competing vendors' bid proposals (bundles of attributes) depend on a vendor's specific costs to generate each attribute, their individual engineering production function to combine those attributes, and anticipated future funding.

In the second stage, the procurement official's objective is to select the vendor j that maximizes the government's utility function, $U_j = U_j(a_{1j}, a_{2j}, \dots, a_{nj})$, subject to projected funding (i.e., the per unit affordability or budget constraint), **B**. For analytic tractability we assume the utility function is quasi-concave, and that attributes are continuous, non-negative, monotonic increasing variables (i.e., the domain of the buyer's utility function, and sellers' production functions and attribute cost functions) are the nonnegative real numbers. Non-satiation in the relevant range of attributes is also assumed, such that, $\partial U_j / \partial a_{ij} > 0$, or the greater the score of the $i \in [1, n]$ desired attributes, a_{ij}, the more value (utility/benefit) for the buyer, but the more costly it is for sellers to produce.

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[5] For example, in the case of military medical transportation of patients to receive emergency treatment, safe transport may require the use of a reliable ventilator. In evaluating ventilators, some key attributes include battery duration, gas consumption, and levels of leakage (L'Her et al., 2014, Blakeman & Branson, 2013).

[6] Since there is a fixed requirement (quantity demanded), the budget, **B**, can be interpreted as the unit funding/budget available to vendors to produce a unit of the required product or service. For example, if we anticipate $25,000 of funding is available for 50 computers, the budget (**B**) used by competing vendors to build their proposals would be $500 per unit.

[7] For example, suppose we have $25,000 of funding for 50 computers, or a budget, **B**=$500/unit. Then, for example, each of 50 identical Apple notebook computers offered at $500/unit would satisfy the basic evaluation criteria (screen size, memory, battery life, software), but consist of a somewhat different bundle of those characteristics/attributes, than each of 50 identical Microsoft (or Dell, or HP) notebook computers.

[8] The greater the funding available, the greater the funding per unit, allowing vendors to offer more of the desired attributes for each identical unit demanded by the buyer. For example, suppose for our 50 computers, instead of $25,000 (**B**=$500) of funding, it turns out $50,000 (**B**=$1000) will be available. Then each of the 50 identical notebook computers offered by Apple will have more and/or better characteristics/attributes, and so will each of the 50 identical notebook computers offered by Microsoft (e.g. bigger screens, more memory, longer battery life).

Following the literature, we allow the buyer's utility function (scoring/ranking rule) to be linear, additive, and separable across attributes (see Keeney & Raiffa, 1976; Kirkwood, 1997). The public procurement official's problem is to select a vendor $j \in [1, m]$ that maximizes the government's utility function:

$$(1)\ \boldsymbol{U_j} = \mathrm{U_j}\left(\boldsymbol{A}_j^T\right) = \boldsymbol{W}\boldsymbol{A}_j^{\mathrm{T}},$$

where desired attributes are known to sellers, and the bundle of attributes in vector $\boldsymbol{A_j}$ = $[a_{1j}\ a_{2j}\ ...\ a_{nj}]$ represents each vendor's offer (bid proposal) for each unit required. The relative weights for each attribute are the procurement official's private information, given by the vector:

$$\boldsymbol{W} = (w_1, w_2, w_3, \ldots, w_n \mid w_i \in \mathbb{R}^+, i \in [1, n]).$$

The procurement official maximizes (1) subject to a funding/affordability constraint:

$$(2)\ TC_j \ \leq \ \mathbf{B},$$

such that the total unit cost (price) of any vendor's bid proposal, TC_j, must fit within forecasted future funding (i.e., the per unit budget). **B.** Note that whereas the set of non-price attributes in the buyer's utility function are revealed to the $j \in [1, m]$ competing vendors, the ***relative*** (preference or "trade-off") ***weights***, w_i, are not.[9] This reflects practical application of the FAR:

> In government acquisition, procuring commands have their own best practices and priorities … but they all follow the [Federal Acquisition Regulation]. And in their selection of suppliers, they assign weights to their parameter criteria in accord with their priorities. … These weights for scoring of proposals do not have to be specifically revealed as an algorithm, but are typically communicated to offerors in terms of [rank ordering of] importance.
>
> Colonel John T. Dillard, U.S. Army (Retired),
> Past Program Manager for Advanced Acquisition Programs

In this formulation of the procurement problem, both buyer and seller suffer from imperfect and asymmetric information. While the seller does not know the specific relative importance/weights assigned to desired attributes (or "evaluation criteria"), the buyer (procurement official) does not know the vendors' costs of producing a particular attribute, nor the technology (engineering production functions) that combines those attributes into vendor proposals.[10] The supply side vendor problem is examined in detail in the next section, followed by the demand side procurement problem.

First Stage EEoA: The Vendor's Problem (Supply Side)

The first stage of the two-stage EEoA optimization framework focuses on the vendor's problem. The economic approach assumes vendors are strategic players, so that the anticipated/forecasted (per unit) funding/budget, **B**, for the procurement, impacts vendors'

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[9] For example, consider the following summary of Federal Acquisitions Regulations (FAR) Sections 15.1 and 15.3 "Evaluating proposals under the RFP [Request for Proposal] best value trade-off analysis criteria": In a negotiated bid there are factors [evaluation criteria] with varying weights assigned. The solicitation tells you the weight of each factor. However, government contracting agencies are not required to publicize the actual source selection plan [it is an internal document]. The agency has broad discretion on what it believes to be the best value. Note, however, the agency must be consistent in following their source selection plan in evaluating every vendor, or risk bid protests—e.g., see Melese (2018).

[10] "Seller costs can be expected to depend on [the] local manufacturing base, and sellers can be expected to be well informed about the cost of (upstream) raw materials" (Parkes & Kalagnanam, 2005, p. 437).

formulation of their competing bid proposals. Vendor bid proposals consist of optimal attribute bundles, $\boldsymbol{A_j}$, from competing firms that maximize overall performance (output, $\boldsymbol{Q_j}$) given their respective engineering production functions, costs, and constraints.[11]

Specifically, given *n* desired attributes (a_{ij}), and anticipated future funding (the per unit budget, **B**), the m vendors each offer competing bid proposals (bundles of attributes), $\boldsymbol{A_j}$, based on their production technology,[12] and their unit costs of producing each attribute, $c_{ij}(\boldsymbol{B})$.[13] For any fixed requirement (quantity demanded) and funding level (per unit budget, **B**), a representative vendor's problem is to maximize the attribute output/performance of each (identical) unit required, subject to the vendor's costs of producing each attribute. Wise & Morrison (2000) observe that a multi-attribute auction allows competing vendors to differentiate themselves in the auction process and bid on their competitive advantages. Competing vendors offer their best possible non-price attribute bundle for the projected per unit funding/budget, **B,** given their idiosyncratic technology reflected in their respective "engineering" production functions given by Equation 3.

The vendor's problem can be expressed as selecting an attribute vector (bid proposal), $\boldsymbol{A_j}$ = $[a_{1j}, a_{2j}, \ldots, a_{nj}]$ that maximizes output or "product performance" given by their engineering production function:

$$(3)\ \boldsymbol{Q_j} = Q_j(\boldsymbol{A}_j^T),$$

subject to total unit costs (TC) not exceeding anticipated per unit funding (**B**) for the project,

$$(4)\ \boldsymbol{TC_j} = \sum_{i=1}^{n} c_{ij}\,(B)\,a_{ij} \leq \mathbf{B}.$$

Hollis Chenery was the first economist to introduce "engineering production functions" similar to Equation 3. In his pioneering article in the *Quarterly Journal of Economics*, he observes: "The engineer must usually resort to testing various sizes and combinations of equipment to determine the effect of such variables as size, speed, temperature, etc., upon total performance" (Chenery, 1949).[14] A detailed survey by Soren Wibe (1984) in *Economica* offers a useful contrast between "engineering" and "economic" production functions.[15] Similarly, but in a different context, Hildebrandt (1999) in this journal introduced what he calls a "technological military production function" derived from underlying technical relationships that relate military inputs to measures of effectiveness/performance. In his study, alternatives are scored along their important attributes to estimate a measure of effectiveness that reflects capabilities required to complete a mission. Although their theoretical foundations differ, engineering production functions parallel traditional MCDM approaches in the use of so-called "value" functions to estimate effectiveness.

For ease of exposition, the remainder of the study focuses on two vendors and two (non-price) attributes. We assume each vendor has a different technology (engineering production

[11] Note the supply-side development in this section generalizes a special case of the multi-attribute auction found in Simon and Melese (2011).
[12] Each vendor's bundle is a technologically-determined combination of attributes: for instance, a computer is a combination of screen size, memory, battery life, and others with unit costs associated with each attribute.
[13] For instance, with bigger budgets, a vendor's costs to provide more of a particular attribute (say computer memory) might enjoy increasing returns to scale because of quantity discounts.
[14] Chenery (1949) connects his engineering approach to production functions to studies that helped motivate Lancaster (1966), stating: "The use of multi-dimensional products has already been suggested in the field of consumption" (p. 514).
[15] Also see the extension of this survey offered by V. Kerry Smith (1986).

function) to combine the two attributes, and different attribute costs. The Lagrangian function for the vendor's problem is given by:

$$(5)\ \mathcal{L}_j = Q_j(a_{1j}\ , a_{2j}, \boldsymbol{B}) + \lambda_j[\boldsymbol{B} - \sum_{i=1}^{2} c_{ij}\ (\boldsymbol{B})\ a_{ij}],\ \text{for j=1,2.}$$

Since vendors compete on their product's quality/performance, we assume they will use the maximum expected per unit funding, **B**, to develop their bid proposals, so that Equation 4 is an equality. First order necessary conditions for an optimum are given by:

$$(5a)\ \partial\mathcal{L}_j/\partial a_{1j} = \partial Q_j/\partial a_{1j} - \lambda_j c_{1j}(\boldsymbol{B}) = 0,$$

$$(5b)\ \partial\mathcal{L}_j/\partial a_{2j} = \partial Q_j/\partial a_{2j} - \lambda_j c_{2j}(\boldsymbol{B}) = 0,$$

$$(5c)\ \partial\mathcal{L}_j/\partial \lambda_j = \boldsymbol{B} - \sum_{i=1}^{2} c_{ij}\ (\boldsymbol{B})\ a_{ij} = 0.$$

Solving Equations 5a–5c, yields optimal attribute bid proposals (performance outputs) for each vendor $j = 1{,}2$, for each identical unit required, for any given per unit budget, ***B***:

$$(6a)\ a_{1j}^{*} = a_{1j}^{*}(y_{1j}(\boldsymbol{B}), y_{2j}(\boldsymbol{B}), c_{1j}(\boldsymbol{B}), \boldsymbol{B}),$$

$$(6b)\ a_{2j}^{*} = a_{2j}^{*}(y_{1j}(\boldsymbol{B}), y_{2j}(\boldsymbol{B}), c_{2j}(\boldsymbol{B}), \boldsymbol{B}).$$

For purposes of illustration, we assume a standard Cobb-Douglas (see Cobb & Douglas, 1928; Douglas, 1976) engineering production function in the spirit of Charnes et al. (1991) and others, with two attributes $(a_{1j}\ , a_{2j})$ as inputs:

$$(6)\ Q_j\left(a_{1j}\ , a_{2j}\right) =\ a_{1j}^{y_{1j}} a_{2j}^{y_{2j}};$$

where the elasticities, y_{nj} (i.e., the % change in output/performance from a % increase in an attribute), are assumed to be independent of available funding (the budget, $\boldsymbol{B}$), and sum to 1. [16]

As stated by Charnes et al. (1986), the Cobb-Douglas engineering production function given by Equation 6 is "the simplest … case of static production with a single output [bundle of attributes] to be produced with a single function—one to a firm [vendor] or plant—from factors [yielding attributes] which are acquired at fixed prices per unit [i.e., fixed unit costs to produce each attribute]." In terms of our model, this suggests starting with the assumption that the unit costs for each vendor, j, are independent of available funding, or that: $c_{1j}(\boldsymbol{B}) = c_{1j}$, and $c_{2j}(\boldsymbol{B}) = c_{2j}$.

Two special cases help illustrate our model: 1) where vendors share common attribute costs, but have different production technologies (engineering production functions), and 2) where vendors share the same production technology, but have different attribute costs.

Vendors with Common Costs and Different Technologies

In the first case (illustrated in Figure 2), vendors $j = 1{,}2$ have identical, attribute costs (i.e., $c_{1j}(\boldsymbol{B}) = c_1$ and $c_{2j}(\boldsymbol{B}) = c_2$), but different, constant engineering production functions (i.e.,

[16] Interestingly, an article by Marsden et. al. (1972) in *Applied Economics* shows how a Cobb-Douglas production function for waste treatment plants can be derived from chemical and biological laws. Another notable engineering production function study by Kurtz & Manne (1963) in the *American Economic Review* estimates a Cobb-Douglas production function from engineering data for various metal machining processes. They also emphasize "it is the *characteristics* [or attributes] of the task that determine the input-output relationship" (p. 667).

$y_{11} \neq y_{12}$ and $y_{21} \neq y_{22}$). From the first order necessary conditions for an optimum ((5a) – (5c)), and (6), competing vendors' optimal attribute bundle bid proposals, for the expected per unit funding/budget level **B**, are given by:

$$\text{(6a')}\ a_{1j}^{*} = [y_{1j}/(y_{1j} + y_{2j})\ c_1]\ \mathbf{B}, \text{ and}$$

$$\text{(6b')}\ a_{2j}^{*} = [y_{2j}/(y_{1j} + y_{2j})\ c_2]\ \mathbf{B}.$$

Figure 2 illustrates optimal attribute bundle bid proposals for each vendor for a specific unit funding/budget level, **B**: $A_1 = (a_{11}^{*}, a_{21}^{*})$ and $A_2 = (a_{12}^{*}, a_{22}^{*})$. The optimum for each vendor is determined graphically by the tangency of each vendor's isoquant (derived from their separate production functions), with the common budget constraint.

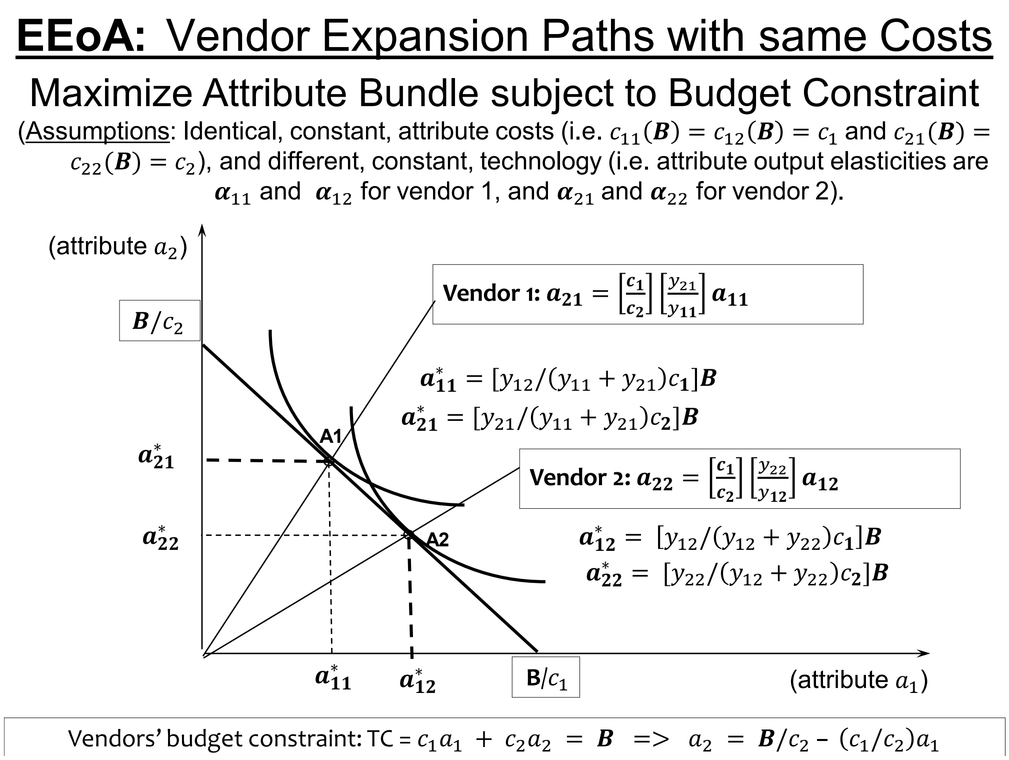

FIGURE 2. Common Attribute Costs but Different Technologies

Suppose instead of a single funding forecast, the buyer (procurement official) reveals a range of possible budget estimates for the procurement (say optimistic, pessimistic, and most likely).[17] Then Equations 6a' and 6b' can be combined to yield each vendor's expansion path, given by:

$$\text{(7)}\ a_{2j} = [(c_{1j}(B)/c_{2j}(B))\,(y_{2j}/y_{1j})]\ a_{1j}\ , \text{ for } j = 1{,}2.$$

The two expansion paths defined by Equation 7 reveal optimal attribute bundles offered by each vendor at different possible funding levels, *B*. Each point on the expansion paths derived for each vendor reveals optimal attribute bundle offers (bid proposals) for each identical unit required, over different possible budgets.

[17] For example, see Simon & Melese 2011.

Given this formulation, if attribute costs and technology parameters are constant (i.e., independent of funding levels), then the expansion paths are linear.[18] Expansion paths for the first case, where vendors' share common costs but different technologies, are given by:

$$(7a)\ a_{21} = [c_1/c_2][y_{21}/y_{11}]\ a_{11}, \text{ for vendor 1, and}$$

$$(7b)\ a_{22} = [c_1/c_2][y_{22}/y_{12}]\ a_{12}, \text{ for vendor 2.}$$

This is illustrated as two straight lines from the origin in Figure 2. For the specific per unit budget level, **B**, the two competing attribute bundle bid proposals offered by each vendor (from Equations 6a' and 6b') appear as points $A_1 = (a_{11}^*, a_{21}^*)$ and $A_2 = (a_{12}^*, a_{22}^*)$ on the competing vendors' expansion paths.

Vendors With Common Technologies and Different Costs

Turning to the second example (illustrated in Figure 3), suppose vendors have different (constant) attribute costs, but identical (constant) engineering production functions (i.e., in Equation 6: $y_{1j} = y_1$ and $y_{2j} = y_2$ for j=1,2), together with constant returns to scale (such that: $y_1 + y_2 = 1;\ i.e. if\ y_1 = y\ then\ y_2 = 1 - y$). In this case the two vendors' optimal bid proposals for unit funding/budget level, **B**, are given by:

$$(6a'')\ a_{1j}^* = [y/c_{1j}]\ \mathbf{B}, \text{ and}$$

$$(6b'')\ a_{2j}^* = [(1-y)/c_{2j}]\ \mathbf{B}, \text{ (j=1,2).}$$

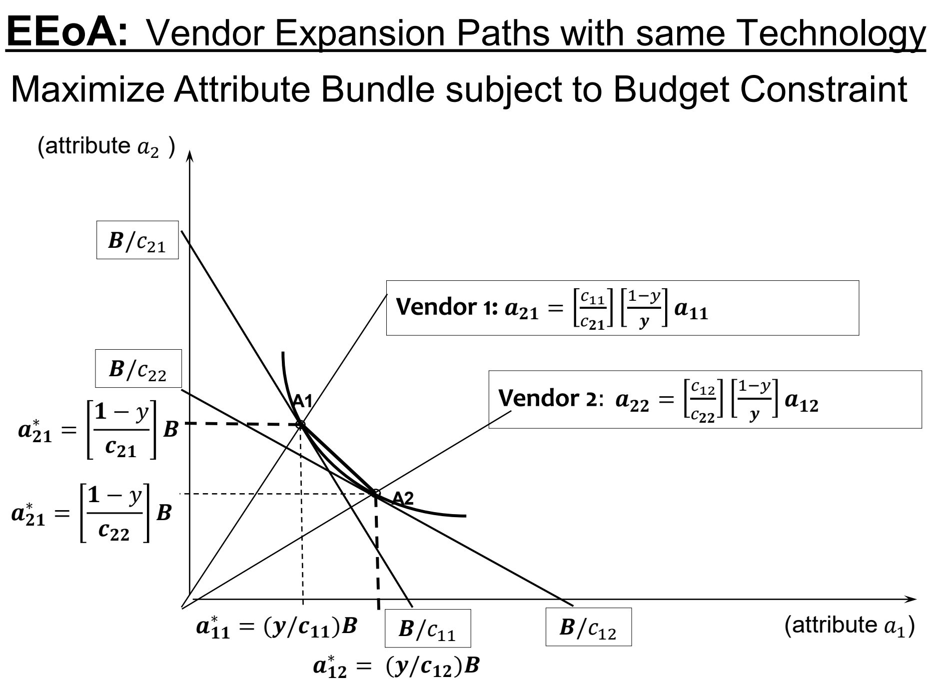

FIGURE 3. Common Technology but Different Attribute Costs

Similar to the first case, Figure 3 illustrates competing optimal attribute bundle bid proposals for each vendor, for the unit funding/budget level, **B**: $A_1 = (a_{11}^*, a_{21}^*)$ and $A_2 =$

[18] For example, see Nicholson & Snyder (2017), pp. 330–333.

(a^*_{12}, a^*_{22}). Now the optimum for each vendor occurs at the point where their respective budget constraints are tangent to their common isoquant. If vendors' technology and attribute cost parameters are constant (i.e., independent of funding levels), both expansion paths are again linear. Expansion paths for this second case (where vendors share a common technology, but have different attribute costs), are illustrated as two straight lines from the origin in Figure 3, given by:

$$\text{(7a')}\ a_{21} = [c_{11}/c_{21}][(1-y)/y]\, a_{11}, \text{ for vendor 1, and}$$

$$\text{(7b')}\ a_{22} = [c_{12}/c_{22}][(1-y)/y]\, a_{12}, \text{ for vendor 2.}$$

Focusing on this second case (where vendors share a common technology, but have different attribute costs), for any unit funding/budget level, **B**, connecting the two optimal vendor attribute production points (A_1 and A_2) creates an attribute "production possibility frontier" (PPF), illustrated in Figure 3. The slope of this PPF reflects attribute trade-offs possible in the marketplace by switching from one vendor to another. This technical (or engineering) trade-off is given by the slope: $\Delta a_2/\Delta a_1 = (a^*_{21} - a^*_{22})/(a^*_{11} - a^*_{12})$.

The first stage vendor optimization problem in the two-stage EEoA framework highlights the importance of modeling the supply side (i.e., vendor decisions in response to anticipated future funding). The second stage focuses on the demand side (i.e., the procurement official's source selection problem).[19]

Second Stage EEoA: Procurement Official's Problem (Demand Side)

For any given requirement (quantity demanded), and forecasted per unit funding/budget, **B**, the procurement official (decision-maker) must rank the vendors' (optimized) bid proposals. For example, consider attribute bundles such as those illustrated in Figure 3: Vendor 1=>(a^*_{11}, a^*_{21}) and Vendor 2=>(a^*_{12}, a^*_{22}). Recall the lens through which the government evaluates competing vendors is the utility function given by Equation 1.[20] In EEoA, the government supplier decision ("source selection") depends on the public procurement official's (decision-maker's) preferences revealed through explicit trade-offs for any pair of attributes that leave decision-maker's indifferent in any given scenario. These explicit pair-wise comparisons elicited from a public procurement official (or expert decision-makers) generate relative weights assigned to the desired attributes.

The public procurement official's problem is to select a vendor $j \in [1, m]$ with a bid proposal (per unit attribute bundle) $\boldsymbol{A}_j = [a_{1j}, a_{2j}, \ldots, a_{nj}]$) that maximizes the government's utility function given by Equation 1. Recall, following the standard assumption in the literature (see Keeney & Raiffa [1976]; Kirkwood [1997]), the utility/benefit provided by any vendor j is given by the linear, separable utility function:

$$\text{(1')}\ \boldsymbol{U_j} = \mathrm{U_j}\left(\boldsymbol{A}_j^T\right) = \boldsymbol{W}\boldsymbol{A}_j^{\mathrm{T}} = \sum_{i=1}^{n} w_i\, a_{ij},$$

where the vector $\boldsymbol{A_j} = [a_{1j}\ a_{2j} \ldots a_{nj}]$ represents the bundle of attributes (performance) of each unit, offered by each of the $j \in [1, m]$ competing vendors. As discussed earlier, specific relative trade-off weights for every attribute are the procurement official's private information, given by the vector:

[19] Note this second stage demand-side problem is the exclusive focus of most textbooks, the majority of the related decision sciences and operations research literature, and standard support tools and algorithms.
[20] An interesting extension of Equation 1 is developed later to address uncertainty when different possible scenarios (states of nature) impact the government's utility function (for example, due to possible future changes in the political, economic, or threat environment).

$$\boldsymbol{W} = (w_1, w_2, w_3, \dots, w_n \mid w_i \in \mathbb{R}^+, i \in [1, n]).$$

The procurement official is also fiscally informed, with a forecasted funding/budget (affordability) constraint for the procurement given by Equation 2. So the per unit price (total unit costs) of any vendor proposal, TC_j, must fit within forecasted future funding (the anticipated per unit budget, $\mathbf{B}$), or $TC_j \leq \mathbf{B}$. The next step is to combine demand and supply (i.e., the procurement official's source selection problem) with vendors' (optimization-generated) bid proposals. The following simple source selection example demonstrates how EEoA integrates demand and supply.

Demand and Supply: A Two Scenario, Two Vendor, Two Attribute Example

For purposes of illustration, suppose a public procurement official responsible for UN peacekeeping missions is asked to select a vendor for a new fleet of Autonomous Electric Off-road Light Armored Transport Vehicle (AEOLATV). Assume the anticipated (per unit) budget, **B**, for the program allows two competing vendors to offer the required set of vehicles, and that there are only two evaluation criteria in the government's utility function: ***Top Speed*** of each vehicle measured in miles per hour (a_1), and ***Range*** measured in miles (a_2).[21] In Figure 3, this involves a choice between vendor 1 that offers less speed but more range (a_{11}^*, a_{21}^*), and vendor 2 that offers more speed, but less range (a_{12}^*, a_{22}^*).

In EEoA, the source selection decision (vendor ranking) depends on the procurement official's (decision-maker's) preferences revealed through pair-wise comparisons (i.e., explicit acceptable trade-offs between pairs of attributes within a particular scenario). This generates relative weights assigned to the desired attributes within a particular scenario.

A straightforward modification of Equation 1' allows us to extend the analysis to address different possible scenarios (states of nature) that could impact the procurement official's pair-wise comparisons.[22] Equation 8 accounts for k possible scenarios (or "states of nature"), Ns, $\forall s \in$ [1,k], with corresponding probabilities, $P(Ns)$. This linear, separable ***expected*** utility function captures the differing relative weights, derived from explicit preference trade-offs among pairs of attributes that depend on specific scenarios (states of nature). Now the procurement official's problem is to select the vendor (e.g., bidder or investment alternative), $j \in$ [1,m], that maximizes the government's ***expected*** utility given by:

$$(8)\ \mathbf{E}(\boldsymbol{U_j}) = \sum_{s=1}^{k} P(Ns) \sum_{i=1}^{n} w_{is}\, a_{ij}.$$

Consider a simple case with two possible states of nature N1 & N2, (e.g. Scenario s=1 a High Tech Threat environment, vs. Scenario s=2 a Low Tech Threat Environment), with corresponding probabilities, P(N1) and P(N2).[23] From Equation 8, the government's expected utility function (scoring rule) for the two scenario, two attribute case is:

$$(8')\ \mathbf{E}(\boldsymbol{U_j}) = P(N1)[w_{11}a_{1j}+w_{21}a_{2j}] + P(N2)[w_{12}a_{1j}+w_{22}a_{2j}].$$

Totally differentiating the procurement official's (government's) utility function (8') and setting the result equal to zero in each scenario (N1 & N2), generates two sets of relative weights (or indifference curves). In general, relative weights for any two pairs of attributes (a_1,a_2) in each of the k scenarios in Equation 8 are given by:

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[21] For example, we could assume all other characteristics (or attributes) of the vehicles offered by the vendors are the same, so top speed and range are the only differentiating factors.
[22] For example, different possible threat environments in which the United Nations might operate.
[23] In the AEOLATV example, scenario N1 could represent the possibility of facing a fast adversary with limited range with probability P(N1), and scenario N2 a slower adversary with greater range with probability P(N2); where P(N1)+P(N2)=1.

$$(9)\ \partial a_2/\partial a_1 = -(w_{1s}/w_{2s}) = -X_s,\ \forall s \in [1,\mathrm{k}].$$

The last term in Equation 9, $X_s > 0$, represents the acceptable trade-off determined by a decision-maker (procurement official) between any pair of attributes (a_1,a_2) for a specific scenario: $w_{1s} = (w_{2s})$x(X_s). It reflects acceptable pair-wise trade-offs for the government over the relevant range of attributes in each scenario. These preference trade-offs define linear indifference curves between any two pairs of attributes in each scenario (or piecewise linear approximations over specific ranges of attributes). The slopes of these indifference curves are the relative weights for each pair of attributes, in each state of nature, over relevant ranges of each attribute.

Optimal vendor rankings in EEoA can be determined by comparing the slope of the government's (buyer's) revealed preferences (indifference curves), with the competing vendor-proposed bundles of attributes (production possibility frontiers). For example, Figure 4 illustrates two different sets of indifference curves (dashed lines) that reflect two different scenarios. In turn, these yield two different vendor rankings.

For a given per unit budget, **B**, if the slope of the indifference curve is steeper than the slope of the production possibility frontier (where the PPF reflects technical/engineering trade-offs available between competing vendors), or if from Equation 9, -X = $-(w_1/w_2) < -(a^*_{21} - a^*_{22})/(a^*_{11} - a^*_{12})$, then vendor 2 is selected, since U^*_2**>**U_1. If the reverse is true, then vendor 1 wins, since U^*_1**>**U_2 (see Figure 4).

Suppose a government decision-maker (public procurement official) is willing to trade off relatively more range (a_2) for the same incremental increase in top speed (a_1) in scenario N1, than in scenario N2. For example: 20 miles of range for an extra 10 mph top speed in N_1, versus only 10 miles for an extra 10 mph in N_2. In this case, $-X_1 = -2 < -X_2 = -1$, implies the slope of the indifference curve is steeper (more negative) in Scenario N_1 than in N_2.[24] From Figure 4, vendor 2 is ranked higher (offers greater utility) in scenario N1, and vendor 1 in scenario N2. This is consistent since the decision-maker revealed a stronger relative preference for top speed in scenario N1 (i.e., was willing to trade off more range), and vendor 2 offers relatively higher top speed (a^*_{12}) than vendor 1 (a^*_{11}).

[24] In this case, under scenario N1 vendor 2 ranks higher (offers greater utility) than vendor 1, and there is a rank reversal under scenario N2.

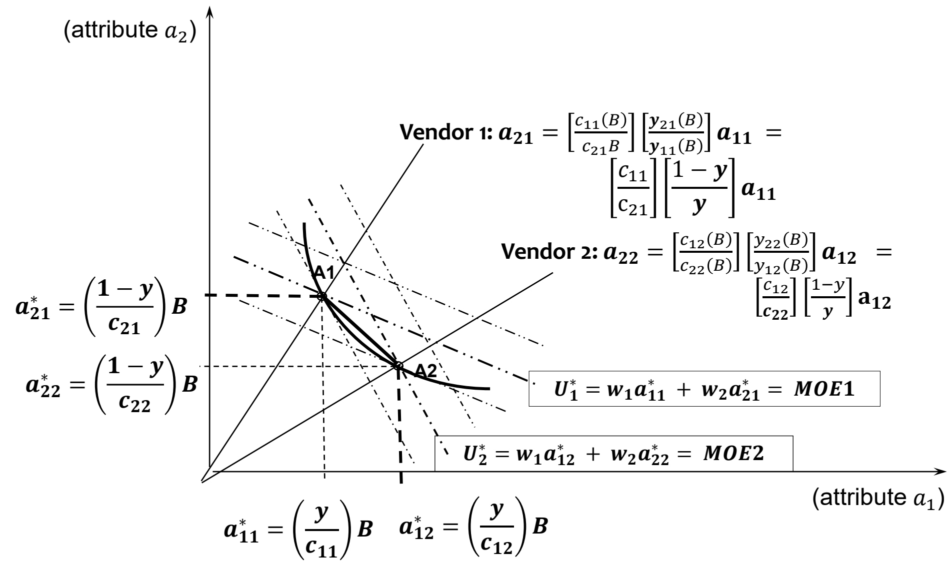

FIGURE 4. Procurement Agency Vendor Selection

In general, probabilities assigned to each scenario in Equations 8 or 8' generate an ***Expected Utility*** vendor ranking metric that consists of a probability-weighted average of pair-wise attribute trade-offs (-Xs) that define expected utility functions in each of the $s \in$ [1,k] scenarios. For example, in the two scenarios, two vendors, two attributes case, this determines the slope of a new indifference curve that is a combination of the two indifference mappings illustrated in Figure 4. For any specified budget, the tangency (or corner point) of this new indifference curve with the PPF reveals the optimal Expected Utility ranking of the two vendors. The next section contrasts this EEoA with the standard textbook MCDM model commonly applied by public procurement officials to guide government supplier decisions.

Comparison of EEoA and MCDM Models

The topic of multi-criteria decision-making (MCDM) has spawned a rich literature with many variations to account for decision-making in complex scenarios. This section presents a standard textbook MCDM model frequently applied to guide government supplier decisions as a baseline (see Keeney & Raiffa [1976]; Kirkwood [1997]). We contrast this MCDM model with the EEoA approach within a single scenario. The MCDM additive value function typically used to rank vendors is given by:

$$(10)\ \boldsymbol{V_j} = \mathrm{V_j}\left(\boldsymbol{A}_j^T\right) = \boldsymbol{\lambda} v_i(a_{ij}) = \sum_{i=1}^{n} \lambda_i\, v_i(a_{ij}).$$

This value function is the sum of individual value functions, $v_i(a_{ij})$, defined over relevant ranges of each attribute $i \in [1, n]$, for any vendor j. The vector of preference weights is given by:

$$\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3 \ldots, \lambda_n \mid \lambda_i \in \mathbb{R}^+,\ i \in [1, n]).$$

The individual value functions $v_i(a_{ij})$ are typically monotonic and scaled (normalized), while the preference weights (λ_i) reflect the importance of each attribute. While these weights ($\boldsymbol{\lambda}$) are analogous to the relative weights ($\boldsymbol{W}$) in EEoA, they are only equivalent if raw attribute measures are used in MCDM instead of normalized values to determine pair-wise trade-offs (i.e., iff $v_i(a_{ij}) = a_{ij}$). For purposes of comparison with EEoA, it is convenient to assume procurement officials (decision-makers) are subject to the same funding/affordability constraint given by (2): $TC_j \leq \mathbf{B}$. Implications of this MCDM model are explored in the next section under the usual assumption that attribute measures are normalized using individual value functions with preferential independence.

Implicit Trade-Offs in MCDM vs. Explicit Trade-Offs in EEoA

From Equation 10, the only theoretical difference between the procurement official's objective function (1) or (1') in EEoA, and MCDM is an additional step in Equation 10 that involves normalizing attribute measures through individual value functions. In fact, the demand side of EEoA can be thought of as a special case of MCDM, where $v_i(a_{ij}) = a_{ij}$.

In theory, any value function, v_i, in conjunction with the appropriate attribute weights λ_i, can recover the EEoA utility function for any given vector of attributes $\boldsymbol{A_j}$.This is clear when we consider a procurement official's value function with two attributes as before:

$$(10') \; \boldsymbol{V_j} = \sum_{i=1}^{n} \lambda_i v_i(a_{ij}) \Rightarrow [\lambda_1 v_1(a_{1j}) + \lambda_2 v_2(a_{2j})].$$

Totally differentiating Equation 10 or 10' and setting the result equal to zero yields ***implicit*** trade-offs in the MCDM approach between any two pairs of attributes (a_1, a_2) (i.e., the first two terms in Equation 11). For sake of consistency given a particular decision-maker's preferences, this should precisely correspond to the explicit trade-offs (revealed preferences) obtained from that decision-maker in EEoA (i.e., represented by the last two terms in Equation 9).

$$(11) \; \partial a_2/\partial a_1 = -[\lambda_1 v_1{}'(a_1)]/[\lambda_2 v_2{}'(a_2)] = -\frac{w_1}{w_2} = -X_s.$$

While the MCDM approach adds a degree of freedom for procurement officials and expands the decision space, it risks obscuring explicit trade-offs between attributes revealed in the EEoA approach. From Equation 11, we see that:

$$\lambda_1 / \lambda_2 = X_s[v_2{}'(a_2)/v_1{}'(a_1)], \text{ or}$$

$$Z = [v_2{}'(a_2)/v_1{}'(a_1)],$$

where the constant Z= $\lambda_1 / (\lambda_2 X_s)$. So in general, for any pair of attributes, and alternatives (i.e., vendors $j \in$ [1,m]),

$$(12) \; Z v_1'(a_{1j}) = v_2{}'(a_{2j}).$$

Integrating both sides of (12) yields:

$$(13) \; v_2(a_{2j})/v_1(a_{ij}) = Z = \lambda_1/(\lambda_2 X_s).$$

That is to say, if the goal is to ensure EEoA and MCDM approaches generate the same rank ordering, procurement officials must set individual attribute value functions v_i's and attribute weights λ_i's in the precise ratio specified in Equation 13.

In practice, there is no reason to assume this happens, and reconciling the two approaches to generate the same rank ordering is non-trivial. While a procurement official may have a certain trade-off in mind between pairs of measurable attributes when developing the MCDM value function, normalizing each attribute with individual value functions, and selecting appropriate weights to assign to those value functions, can easily yield *implicit* pairwise trade-offs among attributes that generate different rank orderings than the *explicit* pairwise trade-offs determined in EEoA.[25] Which decision support model best elicits public procurement officials' (decision-makers') preferences remains an important empirical question and warrants further research. We now turn to another important contribution of EEoA: the importance of modeling the supply side; specifically, accounting for vendor responses to anticipated future funding.

Accounting for Vendor Responses to Anticipated Future Funding

Traditionally, MCDM models focus on the demand side of a public procurement and treat supply side vendor decisions as exogenous. This section demonstrates the potential value of explicitly accounting for vendor responses to anticipated future funding (affordability or procurement official's budget constraints).

Since each vendor's expansion path represents their optimal attribute bundle bid proposals for any given budget (see Figures 2, 3, and 4), these expansion paths can easily be converted, through the buyer's utility function (1'), into cost-effectiveness (or Budget-Utility) ***functions*** for each vendor. For example, substituting each vendor's optimal attribute bundle (6a'') & (6b'') into Equation 1' for any specific scenario yields two points in cost-effectiveness space that represent the utility of each vendor's bid proposal for the per unit funding/budget, **B**: $(\mathrm{U}_1^*, \mathbf{B})$ and $(\mathrm{U}_2^*, \mathbf{B})$. Different budgets represented along the expansion paths generate different utility. For example, the cost-effectiveness/utility relationships illustrated in Figure 6 reflect the value to the government of each vendor's offers at different funding levels.

There is an important contrast between the endogenously derived EEoA cost-effectiveness ***functions*** for each vendor, and the exogenous cost-effectiveness ***points*** generally used to represent vendor offers in MCDM.[26] This becomes especially apparent when vendor costs depend on anticipated future funding. For instance, with bigger budgets, a vendor's costs to provide more of a particular attribute (say computer memory) might enjoy increasing returns to scale because of quantity discounts, learning curves, the ability to employ just-in-time inventory techniques, or the possibility of adopting other process improvements that reduce a vendor's costs of incorporating/producing a desired attribute.

Consider the case illustrated in Figure 5, where vendor 1's costs of producing attribute 1 are assumed to depend on the funding level or anticipated per unit budget, **B** (i.e., $c_{11}(\boldsymbol{B})$). For ease of exposition, suppose both vendors $j = 1, 2$ have identical, constant production technologies (i.e., $y_{1j} = y_1$ and $y_{2j} = y_2$), and constant returns to scale $y_1 + y_2 = 1$. The difference between them is in their individual attribute costs. As before, let $c_{12}(\boldsymbol{B}) = c_{12}$; $c_{22}(\boldsymbol{B}) = c_{22}$; and $c_{21}(\boldsymbol{B}) = c_{21}$, but now suppose vendor 1's costs for attribute 1 depends on the budget. For example assume the following relationship: $c_{11}(B) = c_{11} - k\boldsymbol{B} > 0$. Also let $\boldsymbol{B} < c_{11}/k$, $c_{11} > c_{12}$, and $k \in [0,1)$.[27] In this case (from (6a'') and (6b'')), each vendors' optimal attribute bundle proposals for a unit funding/budget level $\boldsymbol{B}$ is given by:

[25] Note: Linear normalization combined with careful swing weighting in MCDM could recover similar trade-offs to those explicitly revealed in EEOA (see Equation 9), resulting in an identical rank ordering of competing vendors. (An example is available upon request.)

[26] For an example of the latter, see the U.S. Defense Acquisition Guidebook, which states: "Cost-effectiveness comparisons in theory would be best if the analysis structured the alternatives so that all the alternatives have equal effectiveness (the best alternative is the one with lowest cost) or equal cost (the best alternative is the one with the greatest effectiveness). Either case would be preferred; however, in actual practice, in many cases the ideal of equal effectiveness or equal cost alternatives is difficult or impossible to achieve due to the complexity of AoA [Analysis of Alternatives] issues. *A common method for dealing with such situations is to provide a scatter plot [of points representing competing vendor proposals] of effectiveness versus cost*" (emphasis added; DoD, n.d., ch. 2–2.3., 2.7).

[27] These simple assumptions help illustrate our point. A model with quadratic costs could add another dimension (a "knee of the curve," i.e., monotonic increasing with a single inflection point) to the cost-effectiveness function, which could offer an interesting extension of the model.

$$(14a)\ a_{11}^{*} = [y/c_{11}(\boldsymbol{B})]\ \boldsymbol{B} = [y/(c_{11} - k\boldsymbol{B})]\boldsymbol{B},$$

$$(14b)\ a_{21}^{*} = [(1-y)/c_{21}]\boldsymbol{B}, \text{ and}$$

$$(15a)\ a_{12}^{*} = [y/c_{12}]\boldsymbol{B},$$

$$(15b)\ a_{22}^{*} = [(1-y)/c_{22}]\boldsymbol{B}.$$

Figure 5 illustrates each vendor's optimal attribute bundle bid proposals (given by Equations 14a and 14b and Equations 15a and 15b) for a specific budget, **B** (i.e., points $A_1: (a_{11}^{*}, a_{21}^{*})$ and $A_2: (a_{12}^{*}, a_{22}^{*})$).

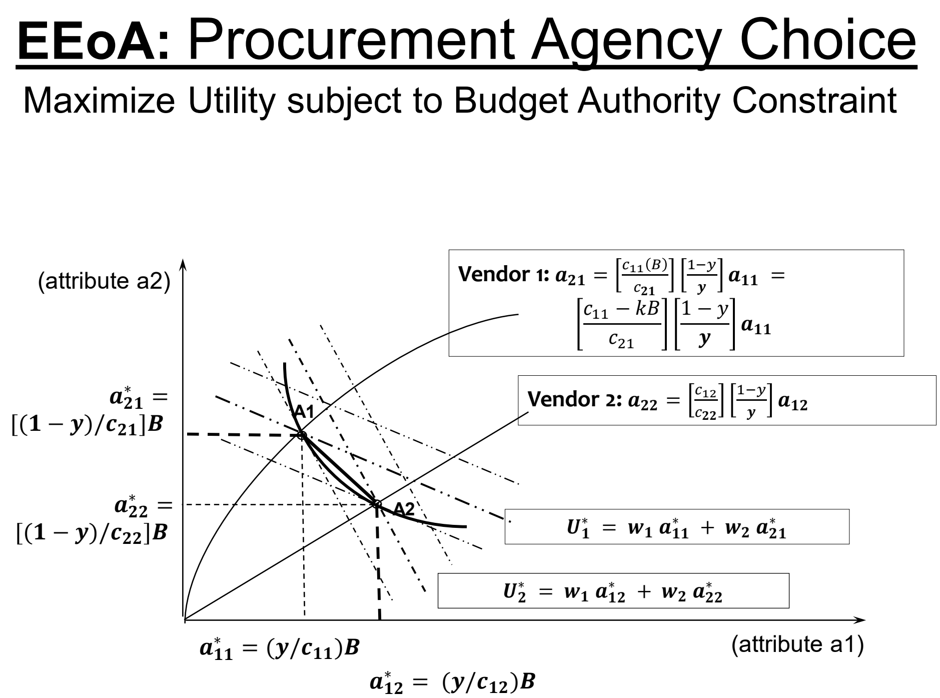

FIGURE 5. Vendor Selection When Vendor 1's Attribute Costs Depend on Budget

The expansion path for vendor 2 is again linear, with the same positive, constant slope for any budget (i.e., identical to (7b')). However, since vendor 1's attribute costs now depend on the anticipated per unit funding/budget, B, vendor 1's expansion path is nonlinear, increasing at a decreasing rate as illustrated in Figure 5 and given by:[28]

$$(16)\ a_{21} = [c_{11}(B)/c_{21}][(1-y)/y]\ a_{11} = [(c_{11} - kB)/c_{21}][(1-y)/y]\ a_{11},$$

where the slope (first derivative) is given by:

$$(16')\ \partial a_{21}/\partial a_{11} = [c_{11}(B)/c_{21}][(1-y)/y] = [(c_{11} - kB)/c_{21}][(1-y)/y] > 0,$$

[28] The illustration of the two expansion paths assumes that throughout the relevant range of budgets (funding levels), $(c_{11}(B)/c_{21}) > (c_{12}/c_{22})$.

and change in slope with a change in the budget (second derivative) given by:

$$(16'')\ \partial(\partial a_{21}/\partial a_{11})/\partial B = [c_{11}'(B)/c_{21}][(1-y)/y] < 0.$$

Substituting vendor 1 and 2’s optimal attribute bundle offers Equations 14a and 14b and Equations 15a and 15b into the procurement official’s (buyer’s) utility function for any given scenario in Equation 8’ yields:[29]

$$(17)\ \mathrm{U}_1^* = w_1 a_{11}^* + w_1 a_{21}^* = w_1\,[y/c_{11}(B)]\,B + w_2\,[(1-y)/c_{21}]\,B$$

$$(18)\ \mathrm{U}_2^* = w_1 a_{12}^* + w_1 a_{22}^* = w_1\,[\mathrm{y}/c_{12}]\,B + w_2\,[(1-y)/c_{22}]B.$$

Equations 17 and 18 represent functions that can be plotted in cost-effectiveness (Budget-Utility) space over a relevant range of funding scenarios (see Figure 6). In this case, assuming identical, constant costs for attribute 2 (i.e., $c_{21} = c_{22} = c_2$), from Equations 17 and 18,

$$(19)\ \mathrm{U}_1^* \gtreqless \mathrm{U}_2^* \text{ as } c_{12} \gtreqless c_{11}(\boldsymbol{B}) = c_{11} - k\boldsymbol{B} \text{ or as } \boldsymbol{B} \gtreqless (c_{11} - c_{12})/k = B'.$$

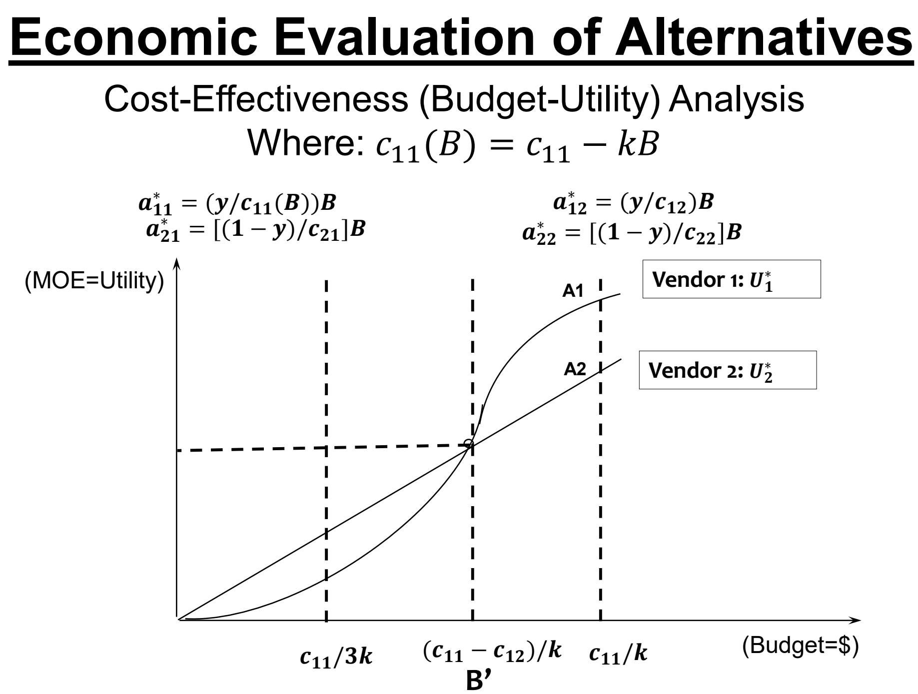

FIGURE 6. Vendor Selection in Cost-Effectiveness (Budget-Utility) Space

What is revealed in Figure 6 is an *optimal rank reversal*. The relation given by Equation 19 indicates it is optimal for the buyer to switch vendors at B’. For any unit funding/budgets **B**>B’, vendor 1 is ranked higher than vendor 2. The two are ranked the same for the budget, **B**=B’, and for budgets **B**<B’, vendor 2 is ranked higher than 1. As expected, evaluating the slopes of the two vendors’ cost-effectiveness functions at the switch point, B’=$(c_{11} - c_{12})/k$, yields:

$$(20)\ \partial\mathrm{U}_1^*/\partial\boldsymbol{B} > \partial\mathrm{U}_2^*/\partial\boldsymbol{B} \text{ or } (c_{11}(\boldsymbol{B}) - c_{11}'(\boldsymbol{B})\boldsymbol{B})/c_{11}(\boldsymbol{B})^2 > 1/c_{12} \text{ since } c_{11} > c_{12}.$$

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[29] For a specific unit funding level **B**, this represents two optima that can be compared that represent the maximum utility a buyer can obtain from each vendor. This is illustrated in Figure 4 as the highest indifference curve attainable given the corresponding point on the attribute production possibility frontier.

> This highlights the importance of modeling the supply side. Specifically, this example emphasizes the importance for public procurement officials to obtain realistic budget forecasts for government programs, and to offer those as guidance to vendors. As two pioneers in defense economics Hitch and McKean (1967) wisely counseled: "As a starter ... several budget sizes can be assumed. If the same [vendor] is preferred for all … budgets, that system is dominant. If the same [vendor] is not dominant, use of several … budgets is nevertheless an essential step, because it provides vital information to the decision maker."

Instead of plotting procurement alternatives (vendor bid proposals) as single points in cost-effectiveness (budget-value) space, EEoA encourages procurement officials in fiscally constrained environments to solicit bids over a range of possible budget scenarios.[30]

From a practical standpoint, the biggest limitation of the EEoA approach is that as the number of attributes (n) under consideration expands, it is increasingly burdensome to generate required pairwise comparisons. For example, assuming each alternative (vendor proposal) includes a set of n attributes, applying EEoA requires $\frac{n(n-1)}{2}$ pairwise comparisons to fully flesh out the decision-maker's preferences. Interestingly however, EEoA could be applied in combination with MCDM as a consistency check for important attributes. That is to say, if $\partial a_2/\partial a_1 = -(w_{1s}/w_{2s}) = -X_s$ is the explicitly determined trade-off that a public procurement official is comfortable with in a particular scenario, given specific ranges for each attribute, then weights developed in MCDM should reflect this relative preference (trade-off).[31] The test simply involves application of Equation 11.

In other words, procurement officials can generate pairwise comparisons for the most critical attributes as a consistency check. For example, when comparing options for AEOLATV procurement, it may be the case that *Top Speed* and *Range* are the most important attributes to consider among the dozens or even hundreds of other attributes. After carefully applying traditional MCDM techniques to develop measures of effectiveness for each AEOLATV alternative, use EEoA's explicit trade-off determination to ensure that the decision-maker is indeed willing to trade X amount of *Top Speed* for Y amount of *Range*, and vice versa, in the specified attribute ranges. If the explicit trade-off determination is one that the decision-maker is uncomfortable with, it is crucial to revisit the value functions and weighting schemes used to generate the measures of effectiveness for each option. While this can be a time-consuming process, it ensures that the best alternative is chosen for large procurements to satisfy the mission.

Conclusion and Avenues for Future Research

This paper offers an economic model to assist public procurement officials to rank competing vendors when benefits cannot be monetized. The problem of ranking public investment alternatives when benefits cannot be monetized has spawned an extensive literature

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[30] In this case, the standard technique of eliminating "dominated alternatives" could lead to sub-optimal decisions. For example, see Melese (2015) or the specific example of the EEoA model developed in Simon & Melese (2011).

[31] If the extra burden of normalization and swing weighting required in MCDM causes a decision-maker to "misevaluate" their trade-off preferences, then EEoA offers an alternative framework/perspective that can help to realign their weighting. Note that in theory a rational decision-maker with perfect information and infinite computational capability would never need to do this. Since in practice it is difficult to define "correct" weighting within scenarios, contrasting the development of weights in MCDM and EEoA is an empirical question worth investigating.

that underpins widely applied decision tools. The bulk of the literature, and most government-mandated decision tools, focus on the demand side of a public procurement. The EEoA extends the analysis to the supply-side.

Introducing the supply side offers multiple avenues for future research. Notably, it provides fertile ground to apply both auction and game theory literatures. An interesting extension would be to leverage auction theory and introduce strategic shading of bids by vendors. Another is to consider the risk of collusion among vendors, or allow some vendors to enjoy economies of scale (or to make engineering production function parameters a function of the budget). Whereas EEoA models vendors as proposing bundles of characteristics to win a prize (i.e., funding), alternative optimization assumptions and strategic behaviors could be assumed.

A rich opportunity also exists for both experimental and qualitative research to significantly improve public procurement. An important empirical question is whether procurement officials and managers would have an easier time using EEoA or MCDM (or some combination)? Consistency tests could be conducted in experimental settings to explore when the two techniques converge (offer identical vendor rankings), and when (and why) they diverge?

In conclusion, the EEoA captures both demand side—government procurement official decisions—and supply side—vendor optimization decisions. A unique feature of EEoA is to model vendor decisions in response to government funding projections. Given a parsimonious set of continuously differentiable evaluation criteria, EEoA provides a new tool to rank vendors. In other cases, it offers a valuable consistency check for MCDM models to guide government supplier decisions.

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