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A New Learning Curve for Department of Defense Acquisition Programs: How to Account for the "Flattening Effect"

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A New Learning Curve for Department of Defense Acquisition Programs: How to Account for the "Flattening Effect"

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Abstract

Traditional learning curve theory assumes a constant learning rate regardless of the number of units produced; however, a collection of theoretical and empirical evidence indicates that learning rates decrease as more units are produced in some cases. These diminishing learning rates cause traditional learning curves to underestimate required resources, potentially resulting in cost overruns. A diminishing learning rate model, Boone's Learning Curve (2018), was recently developed to model this phenomenon. This research confirmed that Boone's Learning Curve is more accurate in modeling observed learning curves using production data of 169 Department of Defense (DoD) end-items. However, further empirical analysis revealed deficiencies in the theoretical justifications of why and under what conditions Boone's Learning Curve more accurately models observations. This research also discovered that diminishing learning rates are present but not pervasive in the sampled observations. Additionally, this research explored the theoretical and empirical evidence that may cause learning curves to exhibit diminishing learning rates and be more accurately modeled by Boone's Learning Curve. Only a limited number of theory-based variables were useful in explaining these phenomena. This research further justifies the necessity of a diminishing learning rate model and proposes a framework to investigate learning curves that exhibit diminishing learning rates.

Introduction

The Budget Control Act of 2011 subjected the Department of Defense (DoD) to a more fiscally constrained and financially conscious environment than ever before, contrasted with a demand for new acquisition programs of almost every type. As an increasing number of programs are terminated, managers at every level in the DoD are expected to ensure the DoD's shrinking budget is being used in the most cost-effective way. The increased scrutiny adds greater emphasis on the accuracy of program office cost estimates given that an approved program cost estimate supports every major acquisition program funded by the DoD.

In order to obtain reliable cost estimates, cost estimating models and tools within the DoD must be evaluated for their relevance and accuracy. The current learning curve methods used within the DoD's cost estimating procedures are from the 1930s (Wright, 1936). As automation and robotics increasingly replace human touch-labor in the production process, the current 80-year-old model may no longer be appropriate for accurate learning curve (and cost) estimates. Robotics and automation machines do not learn; however, they are inevitably a part of future production. New learning curve methods that incorporate automated production should be examined as a possible tool for cost estimators in the acquisition process. Additionally, we examine the flattening effect that occurs toward the end of the acquisition process and the impact this has on different learning curve formulas. Originally published by Badiru (2012), the half-life of a learning curve is the incremental production level required to reduce cumulative average cost per unit to half its initial level. Conversely, a half-life forgetting curve is the amount of time it takes for performance to decline to half of its initial level. A model that more accurately reflects true cost is critical for planning, especially over the long-term when there are hundreds of millions of dollars involved. The results of this research will inform the acquisition community as well as provide a tool that has the potential to significantly outperform the cost estimation and learning curve models currently in use.

The purpose of this research ultimately is to investigate new learning curve methods, develop learning curve theory within the DoD, and pursue a more accurate cost estimation model.

Research Approach (Methodology)

The premise of this research is to apply a new learning curve in such a way that the estimated learning rate is modeled as a decreasing function over time as opposed to the constant learning rate that is currently in use with Wright's Learning Curve. The current model in use today mathematically states that for every doubling of units there will be a constant gain in efficiency. For example, if the manufacturer observed a 10% reduction in man-hours in the time to produce unit 10 from the time to produce unit 5, then they should expect to see the same 10% reduction in man-hours in the time to produce unit 10 to the time to produce unit 20. Unfortunately, in real world situations this constant rate of decay is rarely the case. We propose that more accurate cost estimates could be made if a decay factor was taken into consideration. The proposed modification may take this form:

$$\mathrm{Cos}\, t(X) = aX^{f(x)}$$

Where:

Cost(X) = the cumulative average time (or cost) per unit

X = the cumulative number of units produced

A = time (or cost) required to produce the first unit

F(x) = the learning curve slope represented as a function of units produced

The specific function used for the slope is what this research will attempt to understand. Figure 1 shows the phenomena this research will attempt to model, where the black (flatter) line depicts the traditional curve used to model learning, the red (steeper) line represents the hypothesized learning structure, and the blue line represents actual data.

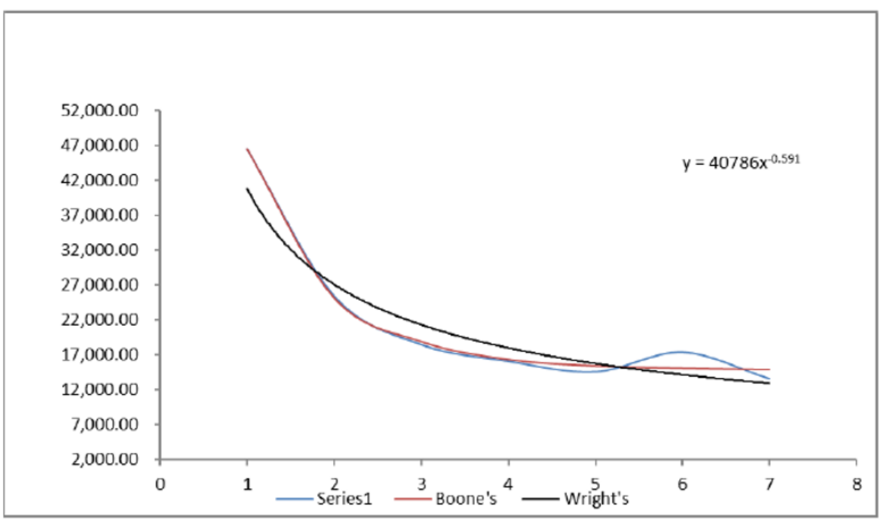

Figure 1. Learning Curve Comparison

Learning Curve Background

The concepts of learning and learning curves are intuitive: as a worker repetitively performs tasks to assemble a product, the worker will gain efficiencies. These efficiencies should decrease the time the worker spends on each unit as more units are produced. These efficiencies translate to a continuous reduction in labor hours and cost savings over time. The GAO Cost Estimating and Assessment Guide (2009) lists four reasons for these gains in efficiency that broaden the scope of learning. First, as more units are produced, workers tend to become "more physically and mentally adept" at performing tasks, and supervisors become more efficient at utilizing workers. This aspect of learning is considered to be learning-by-doing at the laborer level and is termed *autonomous learning* (Levy, 1965). Second, the work environment is improved to include the "climate, lighting, and general working conditions" (Leonard, 2009). Third, the production process is changed to "optimize the placement of tools and material and simplify tasks." Lastly, market forces in the competitive business environment will require suppliers to improve efficiency to survive. These last three aspects of learning are considered to be organizational learning by continuous improvement efforts termed *induced learning* (Levy, 1965; Dutton & Thomas, 1984).

Several terms are used to describe learning curves that include cost improvement curve, cost/quantity relationship, manufacturing process function, experience curve, and product improvement function (International Cost Estimating and Analysis Association [ICEAA], 2014, p. 7). The original and most generalized term *learning curve* will be used in this research. Although learning curves are used most popularly in aircraft manufacturing, the concept can be applied to "relatively large and complex products that require various types of fabrication and assembly skills" (Asher, 1956, p. 5). The Air Force Cost Analysis Handbook includes several specific situations in which learning curves apply that include "the manufacture of a complex end-item, limited changes to product characteristics or technology, continuous manufacturing process, constant management pressure to improve, and consistent production rates" (Department of the Air Force, 2007, pp. 8-1–8-2). The Handbook also includes other criteria, including "a high proportion of manual labor, labor efficiency/job familiarization, standardization, specialization, and methods improvements, improved materiel flow and reduced scrap, improved production procedures, tools, and equipment, improved workflow and engineering support, and product redesign improvements" (Department of the Air Force, 2007, pp. 8-1–8-2). These situations describe aspects of the manufacturing process that enable organizational learning and allow for labor efficiencies, although learning can occur without all criteria being present.

Cumulative Average Learning Curve Theory

The concept of a learning curve was first formally recorded by Theodore Paul Wright in 1936 in his work "Factors Affecting the Cost of Airplanes." Wright identified the learning curve concept in a pre-World War II production environment of a small two-seater aircraft (ICEAA, 2014, p. 16). He observed that as a worker repeatedly performs the same task, the time required to complete that task will decrease at a constant rate. More specifically, Wright formulized that as the number of aircraft produced doubles, the cumulative average labor cost would decrease at a constant rate (Wright, 1936). This relationship is described in Equation 1 and is the Cumulative Average Theory widely in use. When learning curves utilize this Cumulative Average Theory, they are frequently called "Wright Curves" (ICEAA, 2014, p. 16).

Equation 1

$$\bar{Y} = Ax^b \quad (1)$$

where Y̅ is the cumulative average cost of the first x units, A is the theoretical cost to produce the first unit, x is the cumulative number of units produced, and b is the learning curve slope (LCS) divided by the natural logarithm of 2.

The learning curve slope in the learning curve exponent "b" of Equation 1 defines how each doubling of produced units reduces cumulative average costs. For example, Wright used his empirical data to calculate a learning curve slope of 80%. Therefore, as the cumulative number of units doubles, the cumulative labor cost of the doubled units would be 80% of the original undoubled amount, resulting in a 20% cumulative average reduction in labor cost (Wright, 1936). This 20% cumulative average reduction can also be called the rate of learning. Higher rates of learning lead to greater reductions in labor costs. Although Wright's model cited 80% as a universal learning curve slope, learning curve theory evolved to realize that other slopes are possible based on an end-item's unique manufacturing characteristics (Jaber, 2006).

Wright's Cumulative Average Learning Curve is cumbersome to use because cumulative average costs are calculated in place of the unit cost. Figure 2 depicts Wright's Learning Curve with an 80% learning curve slope based on a first unit cost of 100. Figure 2 shows that as the cumulative number of units produced doubles, the cumulative average cost decreases by 20%. The corresponding unit cost is also displayed to highlight how the cumulative average cost differs from the unit cost. The unit cost is calculated by summing the cumulative costs up to but not including the unit number and subtracting it from the total cost. In Figure 2, the first unit cost for both cumulative average cost and unit cost is 100. For unit two, the cumulative average cost of the first two units is 80; this is a 20% reduction due to an 80% learning curve slope. The total cost of the first two units is 160. Because the first unit cost is known to be 100, the second unit cost can be calculated from the difference to be 60. These same calculations can be used to obtain the unit costs for the remaining units.

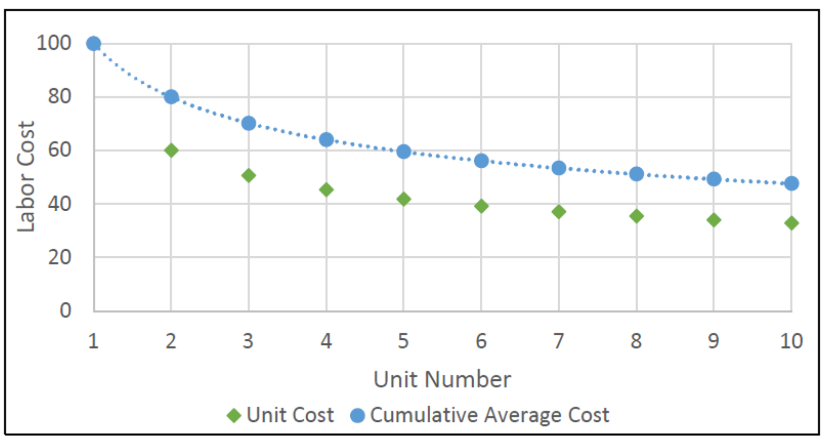

Figure 2. Wright's Cumulative Average Theory at an 80% Learning Curve Slope

Wright illustrated Equation 1 using a graph with vertical and horizontal axes displayed in logarithmic rather than linear scale. Wright illustrated his equation on this logarithmic graph in order to highlight the straight line representing a constant rate of learning (Wright, 1936). The same function can be graphed in linear scale by transforming Equation 1 into a log-linear form by taking the natural logarithm of both sides. This log-linear transformed equation is shown in Equation 2. The parameter definitions for Equations 1 and 2 are the same.

Equation 2

$$ln\,\bar{Y} = lnA + b\;ln\,x \qquad (2)$$

Equation 2 also allows analysts to apply linear regression analysis in order to estimate the parameters A and b from a set of cumulative average cost data (Mislick & Nussbaum, 2015, p. 185). Figure 3 displays Wright's Cumulative Average Theory at an 80% learning curve slope transformed into a log-linear form. The parameters are identical to the parameters of Figure 2 with a first unit cost of 100. The constant learning curve slope indicated by the linear slope in Figure 3 is a crucial concept of this traditional learning curve theory.

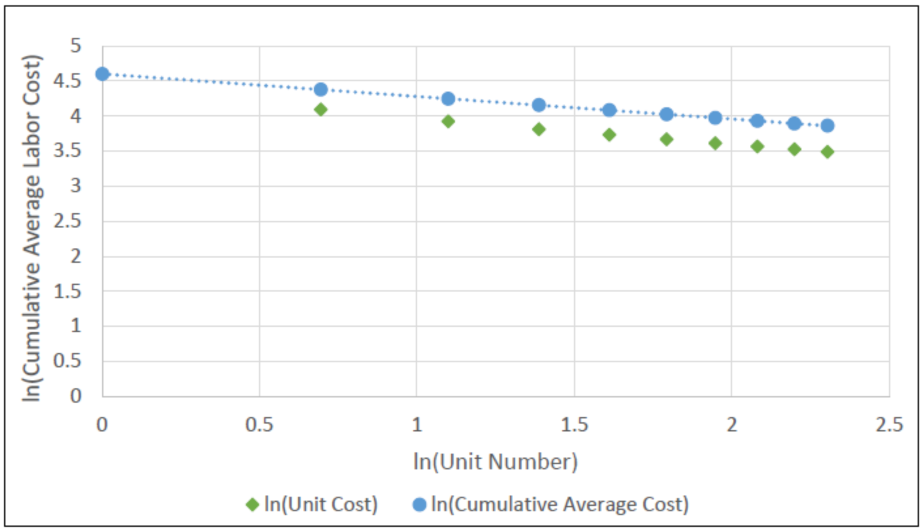

Figure 3. Wright's Cumulative Average Theory at an 80% Learning Curve Slope in Log-Linear Form

Unit Learning Curve Theory

Several years following Wright's Cumulative Average learning curve theory, J. R. Crawford formulated the Unit Learning Curve Theory, formally written in 1944. Together, these theories form the basis of the traditional learning curve theory. Crawford proposed his Unit Theory first in an undated manual prepared for Lockheed Aircraft Company personnel after realizing the difficulty of calculating unit costs from Cumulative Average Learning Curve Theory equations (Asher, 1956, pp. 21–22). As shown in Equation 3, Crawford's Learning Curve yields an estimated unit cost given the unit's sequential unit number within the production line, a learning curve slope, and a theoretical first unit cost. Crawford's Unit Theory is the same as Wright's aside from these differences in variable interpretation. Learning curves are often called "Crawford Curves" when they utilize Crawford's Unit Theory (ICEAA, 2014, p. 31).

Equation 3

$$Y = Ax^b \; (3)$$

where Y is the individual cost of unit x, A is the theoretical cost of the first unit, x is the unit number of the unit cost being forecasted, and b is the LCS divided by the natural logarithm of 2.

Using Crawford's Unit Theory with a learning curve slope of 80% and a first unit cost of 100 labor hours, the cost of the second unit is 80 labor hours. This 20% reduction in labor hours or rate of learning is due to the 80% learning curve slope. Figure 4 illustrates Crawford's Unit Theory using the same parameters from Wright's Cumulative Average Theory shown in Figures

2 and 3. The cumulative average costs are not shown in Figure 4 because these are not germane to Unit Theory.

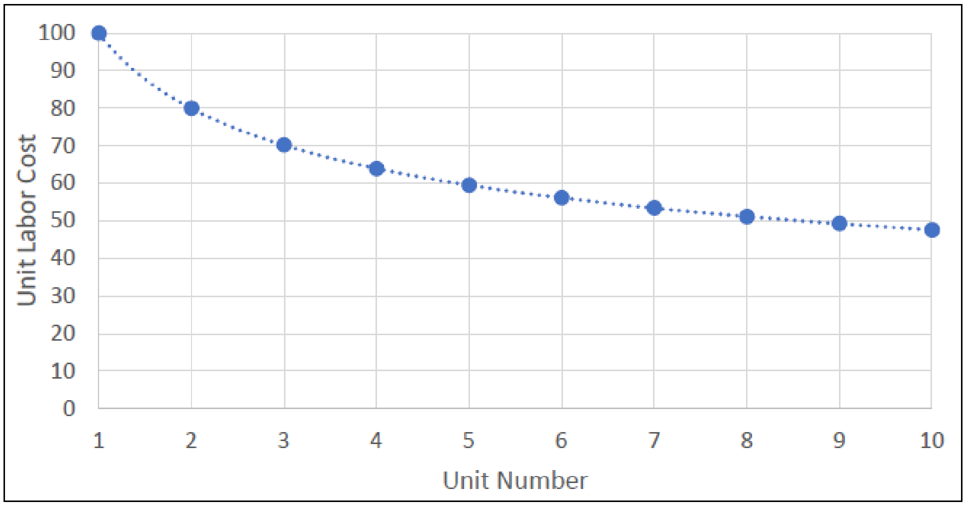

Figure 4. Crawford's Unit Theory at an 80% Learning Curve Slope

Crawford's Unit Theory Learning Curve is transformed into a log-linear form using the same methodology used to derive Equation 2. Crawford's Learning Curve in log-linear form is shown in Equation 4 using the same parameter definitions from Equation 3.

Equation 4

$$ln\, Y = lnA + b\, ln\, x \quad (4)$$

Crawford's Unit Theory Learning Curve is shown in logarithmic scale in Figure 5. Similar to Cumulative Average Theory, the constant learning indicated by the linear slope in Figure 5 is a vital concept of this theory: the rate of learning remains constant as the units double regardless of the number of units produced.

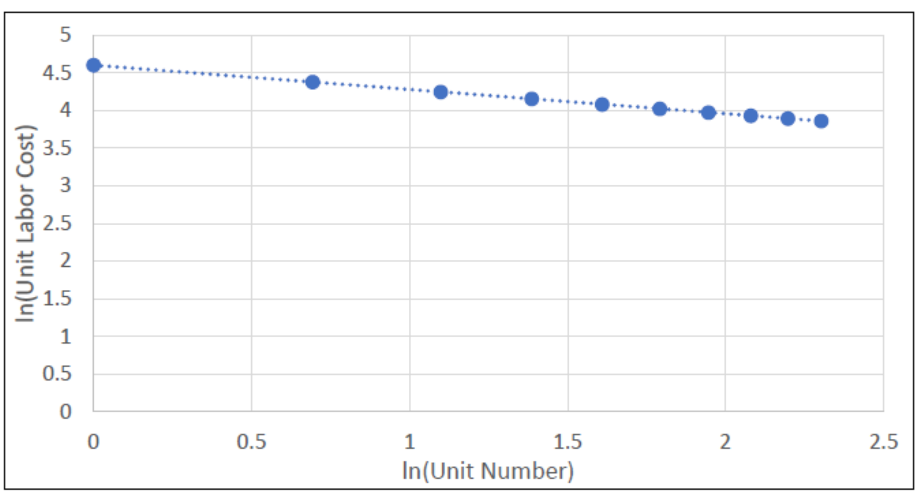

Figure 5. Crawford's Unit Theory at an 80% Learning Curve Slope in Log-Linear Form

Crawford's Unit Theory parameters are straightforward to estimate when data are available by each unit (also called unitary data); however, manufacturers generally report cost data in the form of production lots that include the total lot cost and the number of units in that lot (Mislick & Nussbaum, 2015, p. 191). Unlike Wright's Cumulative Average Theory, Crawford's Unit Theory must utilize lot midpoints to estimate parameters when unitary cost data is unavailable. The algebraic lot midpoint is defined as "the theoretical unit whose cost is equal to the average unit cost for that lot on the learning curve" (Mislick & Nussbaum, 2015, p. 192). In other words, the lot midpoint is the unit that will divide the area under the learning curve evenly within the lot (Mislick & Nussbaum, 2015, p. 192). This lot midpoint is used in the Unit Theory learning curve formula as the sequential unit number or independent variable "x" in Equations 3 and 4. The lot midpoint supplants using sequential unit numbers because sequential unit numbers are unavailable when using lot cost data. When the lot midpoint is the independent variable in Equation 3, the dependent variable will yield the average lot cost. The average lot cost results because this x-coordinate is the most representative point for the lot (ICEAA, 2014, p. 40).

Lot midpoints are calculated in a two-step approach due to the lack of a closed-form solution. A closed-form solution does not exist because the lot cost is a function of the learning curve exponent *b* from Equations 3 and 4 used to estimate the lot midpoint. However, the lot midpoint is also used to estimate the learning curve exponent "b." The first step in calculating a lot midpoint utilizes a parameter-free approximation formula to estimate the lot midpoint. These lot midpoint estimates are then used to estimate the learning curve exponent "b." The second step is to use a lot midpoint formula that includes an estimate of the learning curve exponent "b" and iterate until successive values of the estimated lot midpoints and "b" are sufficiently small (Mislick & Nussbaum, 2015, pp. 200–201). The parameter-free lot midpoint approximation is shown in Equations 5 (Mislick & Nussbaum, 2015, p. 193).

Equation 5

$$\text{Lot Midpoint (LMP)} = \frac{F+L+2\sqrt{FL}}{4} \qquad (5)$$

where F is the first unit number in a lot and L is the last unit number in a lot. These lot midpoint estimates are then used to estimate the learning curve parameters for Crawford's model (Equation 3) using the GRG non-linear optimization algorithm.

Several parameter lot midpoint approximations exist, but a simple and popular lot midpoint approximation is Asher's Approximation shown in Equation 6. The same parameter definitions presented in Equation 5 also apply to Equation 6, and the learning curve exponent "b" is the same as shown previously in Equations 1–4 (Mislick & Nussbaum, 2015, p. 201).

Equation 6

$$\text{Lot Midpoint} \approx \left[\frac{\left(L+\frac{1}{2}\right)^{b+1}-\left(F-\frac{1}{2}\right)^{b+1}}{(L-F+1)(b+1)}\right]^{\left(\frac{1}{b}\right)} \qquad (6)$$

where F is the first unit number in a lot, L is the last unit number in a lot, and b is the estimated value from Equation 1.

Comparison of Cumulative Average and Unit Theory

Cumulative Average Theory and Unit Theory will produce different predicted costs provided the same set of data despite all predicted costs being normalized to unit costs. Figure 6 demonstrates this point where Unit Theory was used to generate data using a first unit cost of 100 and a learning curve slope of 90%. The original Unit Theory data was converted to cumulative averages in order to estimate Cumulative Average Theory Learning Curve parameters. Cumulative Average Theory estimated a learning curve slope of 93% and a first unit cost of 101.24. These Cumulative Average Theory parameters were then used to predict cumulative average costs. These predicted costs were then converted to unit costs. This conversion allows for the Cumulative Average predictions to be directly compared to the original Unit Theory generated data.

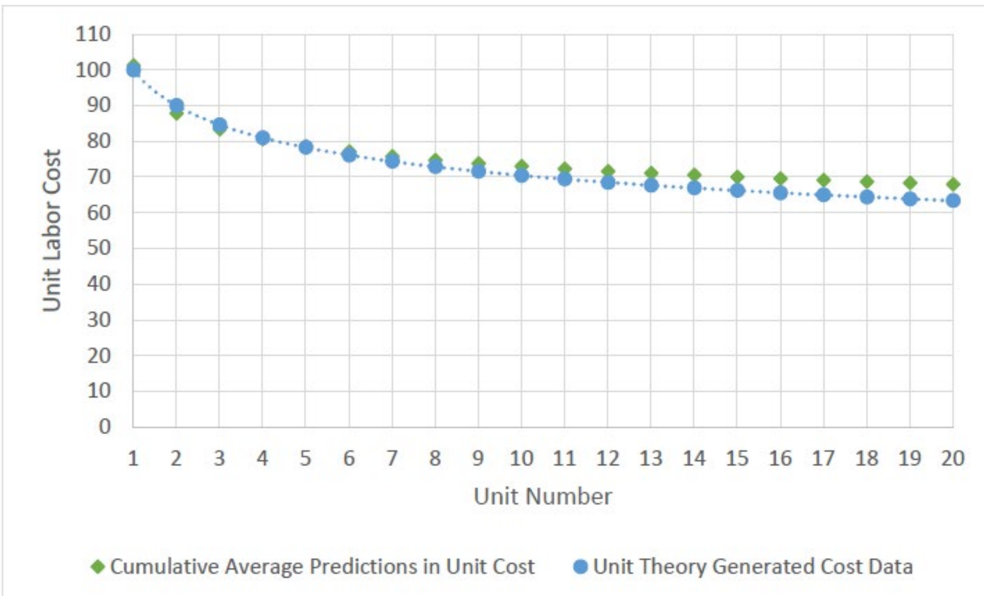

Figure 6. Cumulative Average and Unit Learning Curve Theory Comparison

As shown in Figure 6, the Cumulative Average Learning Curve predictions first overestimate, then underestimate, and ultimately overestimate the generated Unit Theory data for all remaining units. A similar case would occur if Cumulative Average Theory were used to generate data and Unit Theory learning curve parameters were estimated from this data. Figure 6 highlights that these two theories are inherently different due to differences that occur when estimating learning curves parameters using unit costs or cumulative average costs. Several factors can assist an analyst in deciding which theory to apply; however, solely relying on goodness-of-fit statistics will likely bias the decision toward Cumulative Average Theory (Mislick & Nussbaum, 2015, p. 215; Cullis et al., 2008). Frequently goodness-of-fit statistics to include the coefficient of determination (R2) and standard error are used to determine which model best explains variation in a dataset. The coefficient of determination is the total variation of the dependent variable explained by the independent variable (Hilmer, 2014, p. 90). The standard error is the measure of how far on average the data tend to fall from the predicted learning curve (Hilmer, 2014, p. 91). These goodness-of-fit statistics can be used when comparing models of the same units, which is not the case when comparing Cumulative Average and Unit Theory learning curves.

Researchers investigated this Cumulative Average Theory goodness-of-fit statistic bias and presented at a Society of Cost Estimating and Analysis (SCEA) conference (Cullis et al., 2008). The researchers used a methodology similar to that used to produce the example illustrated in Figure 6. Cumulative Average data were first generated, and a Unit Theory learning curve was fit to this data. Unit Theory data were then generated, and a Cumulative Average learning curve was fit to this data. The goodness-of-fit statistics reliably indicated the correct learning curve theory to model these perfect data. Next, artificial variation was injected into the generated data, and the researchers repeated the process. When the researchers injected variation or error in the data, the goodness-of-fit statistics overwhelmingly favored selecting Cumulative Average Theory over Unit Theory even with small amounts of variation in the data (Cullis et al., 2008). In other words, Cumulative Average Theory Learning Curves will tend to have a higher coefficient of determination and lower standard error when compared to Unit Theory learning curves. This bias in the goodness-of-fit statistics is because Cumulative Average Theory is a cumulative running average, so the curves are generally smoother and closer to the data points (Mislick & Nussbaum, 2015, p. 215). Therefore, bias exists in favor of Cumulative Average Theory, so more subjective judgments are warranted to determine which learning curve theory to utilize.

The GAO Cost Estimating and Assessment Guide (2009) provides factors to consider when choosing which learning curve theory to model data. Analysts should review which theories were applied to analogous systems that are similar in form, fit, or function to the current system being considered (GAO, 2009, p. 369). Next, some industries have standards that prefer one theory over the other (GAO, 2009, p. 369). Experience should also be considered by reviewing which theory has been applied to the contractor in the past (GAO, 2009, p. 369). Lastly, some aspects of the production environment can indicate which theory is best to apply (GAO, 2009, p. 369). For example, Cumulative Average theory is best when “the contractor is starting production with prototype tooling, has an inadequate supplier base, expects early design changes, is subject to short lead times,” or where there is a risk of concurrency between development and production phases (GAO, 2009, p. 369). In contrast, Unit Theory is more suited for contractors that are well-prepared to begin production (GAO, 2009, p. 369).

Other factors must be considered when deciding which learning curve theory to use. Cumulative Average learning curve theory will provide more conservative estimates and is less responsive to trends than the Unit Learning Curve theory (Mislick & Nussbaum, 2015, p. 215). For these reasons, Unit Learning Curve theory is frequently favored by government negotiators when negotiating contracts (Mislick & Nussbaum, 2015, p. 215). Cumulative Average Learning Curve theory also relies on continuous data and is unable to be calculated with missing prior data using traditional estimation techniques.

Finally, when ordinary least squares (OLS) regression is used to estimate Cumulative Average learning curve parameters, the cumulative averaging technique violates the OLS regression assumption of independence. For OLS regression to provide an unbiased estimator, the data must be obtained through independent random sampling (Hilmer, 2014, pp. 111–112). In other words, the unit labor cost and its associated unit number cannot be statistically related to other observations of unit labor cost and their associated unit numbers. This assumption is violated due to the costs of earlier observations being a function of the costs of later observations from cumulative averaging calculations. This violation biases the learning curve parameters to produce expected values that are not equal to the population parameter being estimated (Hilmer, 2014, p. 109). Despite this violation, Cumulative Average Learning Curves estimated using OLS regression are widely used and remain a valid method for estimating learning curves.

Cost Accounting for Learning Curves

The fundamental aspects of traditional learning curve theory apply only to a subset of total program costs. Hence appropriate costs must be considered when applying the theory to yield viable parameter estimates and predictions. In a complex program, costs can be presented in units of hours or dollars and organized as recurring and non-recurring for various cost elements of the end-item or the program as a whole. For each cost element, labor costs are also categorized into further groups. The analyst must select the applicable subset of costs and consider their units when utilizing learning curve theory.

Costs are generally categorized as recurring and non-recurring costs. Non-recurring costs are one-time costs that are not directly attributable to the number of end-items being produced (Mislick & Nussbaum, 2015, p. 26). Recurring costs are costs that are incurred repeatedly for each unit produced (Mislick & Nussbaum, 2015, p. 26). At the basis of learning curve theory is the idea that costs vary as more units of an end-item are produced. Therefore, non-recurring costs are excluded from learning curve analysis due to the inability to relate these costs with the number of units produced (Mislick & Nussbaum, 2015, p. 180). Traditional research also has limited insight into how learning applies to nonrecurring costs. For these reasons, learning curve analysis focuses solely on recurring costs in estimating learning curve parameters and predicting recurring costs (Mislick & Nussbaum, 2015, p. 180).

Manufacturing program costs are also organized broadly as labor, material, and overhead. T. P. Wright initially claimed that all these categories vary with the number of units produced, although he specifically focused on labor hour costs when forming his seminal theory (Wright, 1936). Due to this focus and the intuitive idea of learning at the laborer level, researchers have since focused solely on labor costs, including J. R. Crawford. Crawford exclusively studied labor learning and elaborated at length on how learning occurs from the laborer's perspective (Asher, 1956, p. 24). Both fundamental theorists also focused on the laborers who manufactured the aircraft by considering touch labor (Asher, 1956, pp. 16, 21). Additionally, cost estimating standard practice and guidance concerning learning curves also provides the basis for considering touch labor costs only (ICEAA, 2014, p. 7; Department of the Air Force, 2007, p. 8-1).

Defense contractors are often contractually required to submit costs incurred when producing large, complex end-items for the U.S. government. These costs have historically been submitted on the Defense Department (DD) Form 1921 report series to include the Functional Cost-Hours Report DD Form 1921-1 and Progress Curve Report DD Form 1921-2. The DoD transitioned to using the Cost and Hour Report "FlexFile" and Quantity Data reports on May 15, 2019 (Burke, 2019). However, historical program data will likely remain in the legacy 1921 series forms, and "FlexFile" and Quantity Data reports can easily be manipulated to legacy 1921 forms. The 1921-1 form is organized by work breakdown structure (WBS) elements that include various functional cost categories both in units of hours and dollars. Three broad functional cost categories: labor, material, and other costs are included in both forms of recurring and non-recurring costs. This form also has four functional labor categories that include manufacturing, tooling, engineering, and quality control labor. These four labor category costs, when summed with the material costs and other costs, comprise the total cost for each WBS element for recurring and non-recurring costs. This total cost is provided in units of dollars due to the underlying units of material and other costs. A document accompanies the 1921-1 to describe the elements of the form called a 1921-1 Data Item Description (DID). The 1921-1 DID defines these various functional cost categories to include the four labor categories whose definitions are useful in determining which categories pertain to learning curve analysis.

The definition for the manufacturing labor cost category most clearly aligns with the extant literature to be the focus as the pertinent labor cost category for learning curve research.

According to the 1921-1 DID, the manufacturing labor category "includes the effort and costs expended in the fabrication, assembly, integration, and functional testing of a product or end-item. It involves all the processes necessary to convert raw materials into finished items" (1921-1 Data Item Description, p. 12). This manufacturing labor category aligns with the categories examined by Wright (1936), which he called "assembly operations" (p. 124), along with those cost categories Crawford studied, which he called "airframe-manufacturing processes" (Asher, 1956, p. 21). A RAND learning curve study also defined the direct labor used in the study as "those expended to manufacture the airframe and install the equipment required to transform the airframe into a complete, flyable airplane" (Asher, 1956, p. 49). Therefore, the manufacturing labor cost category as defined by the 1921-1 DID is associated with the types of labor costs studied by traditional learning curve theorists and succeeding research. However, data availability can prompt analysts and researchers to use total costs instead. Although these curves remain valid according to Wright (1936), they are composite learning curves with caveats to be discussed later. The 1921-1 is organized into WBS elements defined for each program. A WBS element is a method to display, define, and organize the overall end-item into sub-products while maintaining their relationship with the end-item and other sub-products (DoD, 2018, p. 4). For example, WBS elements for an aircraft program could include the airframe, wings, and engines, among other elements. These WBS elements are comprised of lower-level elements as well. For example, the airframe element may include elements such as the forward, middle, and aft airframe. WBS elements can also comprise activities instead of physical components such as testing the aircraft. Although some of these activities may experience efficiencies over time, traditional learning curve theory focuses exclusively on the production of physical components. WBS elements are frequently organized into various cost categories that can comprise elements suitable and unsuitable for learning curve analysis.

WBS elements are organized into various cost categories to include procurement costs, weapons system costs, and flyaway/rollaway/sail-away costs among others. Not all WBS elements and their respective cost categories are pertinent for learning curve analysis. The group of WBS elements in which learning is relevant is prime mission equipment and its sub-elements. Prime mission equipment is all hardware and software WBS elements installed on the weapon system such as "propulsion equipment, electronics, armament, etc." (Flyaway Costs, n.d.). The prime mission equipment WBS aligns with those elements that experience learning according to the traditional learning curve theorists. The prime mission equipment WBS group excludes elements such as systems engineering and program management (SE/PM) and system test and evaluation (STE); these costs are instead included in flyaway costs (Flyaway Costs, n.d.). These latter elements are activities tangentially related costs that may not experience learning as theorized by the traditional learning curve literature.

Recent learning curves research has considered equivalent WBS elements. Moore et al. (2015) scoped their research to consider airframe costs, which is a sub-element of prime mission equipment due to the homogeneity of the programs they analyzed. Honious et al. (2016) also used airframe costs. Boone used prime mission equipment WBS elements to perform analysis due to the wide variety of programs researched (Boone, 2018, pp. 22–23).

Another cost accounting item to consider is whether to use hours or dollars as the units for labor cost. The total cost for each WBS element is provided in dollars due to material and other costs not having associated labor hours. Therefore, if the total WBS cost is used for analysis, units of dollars will be used. In contrast, the four labor categories to include manufacturing labor has both dollars and hours associated with each. Ideally, labor hours would be analyzed for a variety of reasons as discussed by the Air Force Cost Analysis Handbook (2007). First, labor dollars must be normalized to remove the effects of escalation (Department of the Air Force, 2007, p. 8-65). Escalation effects comprise economy-wide price changes as well as industry-specific price changes. Normalization removes these price variations that allow for labor costs to be compared across different fiscal years. The escalation indices used to normalize data are estimates of the escalation that the industry experienced that year. Therefore, the use of escalation indices can inject error into the data. In contrast, if labor hour data were used, labor costs between years could easily be compared without normalization. Furthermore, changes in labor rates can also bias the labor cost learning curve. Senior personnel are brought on to a program initially due to the initial complexity (Department of the Air Force, 2007, p. 8-65). Once the program stabilizes and production increases, the program usually transitions to more junior labor (Department of the Air Force, 2007, p. 8-65). This labor rate effect, when combined with the effect of normal learning, artificially steepens the learning curve (Department of the Air Force, 2007, p. 8-65). Therefore, the slopes of learning curves utilizing labor dollars will likely be steeper due to the influence of declining average labor rates as the workforce builds towards full-rate production (Department of the Air Force, 2007, p. 8-65). Although using labor cost data in dollars does not invalidate analysis and is frequently utilized due to data availability, labor hour data would ideally be used for these reasons.

In summary, the literature indicates using direct, recurring, manufacturing labor costs in the form of hours. These costs should be considered only for the WBS elements that include prime mission equipment and its lower-level elements. Using these specific WBS costs in the form of hours ensures alignment with the original costs and elements considered to be affected by learning in the traditional models. Although this review exclusively examined the DD 1921-1 form, the 1921-2 form reports the same cost data, albeit in a different format for specific use in learning curve analysis. The methodology on which WBS elements and costs to consider is translatable between the two legacy forms along with the current Cost and Hour Report "FlexFile" and Quantity Data reports.

Variations to Traditional Learning Curve Theory

The traditional learning curve models assume a constant learning environment comprised of a stable production line and invariable end-item design. Due to the realities of changing production environments and modifications to the end-item configuration, many researchers have investigated non-stable learning environments. These areas include production rate changes, changes to the end-item design during production, and breaks in production, among other topics.

Production rates of end-items can vary as the program proceeds through the production life cycle. Researchers in a 1974 RAND report first formally proposed that production rate effects can alter unit costs (Large et al., 1974). The researchers hypothesized that as more units are produced, fewer costs would be allocated to each unit due to fixed costs within the manufacturing process (Large et al., 1974). When fixed costs are allocated over more units, each unit will be less expensive. The researchers also hypothesized that as more units are produced, the contractor may be able to take advantage of volume discounts, resulting in lower material costs per unit (Large et al., 1974). This modified learning curve equation, termed Unit Theory with Rate Adjustment, is shown in Equation 7.

Equation 7

$$Y = Ax^b R^c \quad (7)$$

Where:
Y = cost of the xth unit
A = theoretical cost to produce the first unit (T1)
x = sequential unit number of the unit being calculated

b = $\frac{\ln Learning\ Curve\ Slope}{ln}$
R = Annual production rate
c = $\frac{\ln Rate\ Slope}{\ln 2}$

Despite these logical hypotheses, the researchers rarely found the rate term to be statistically significant. Several factors can cause the rate term to be not statistically significant, such as how the contractor responds to the production rate changes. There are also statistical challenges when investigating rate effects due to the likely presence of multicollinearity. The independent variables "x" and "R" in Equation 7 do not make independent contributions to describe the dependent variable because there is no means to hold "x" constant while changing "R." For these reasons, statistical analysis is unable to discern the effects that either variable has on the dependent variable. The presence of multicollinearity tends to cause one or both independent variables to be not statistically significant when using linear regression analysis (Department of the Air Force, 2007, pp. 8-31–8-32). Also, researchers have studied how production breaks alter the learning curve. Production breaks occur when the manufacturer of the end-item stops production for a period. During the time lapse between the completion of a unit and the start of another unit, a loss of learning can occur. This learning loss results in an increased cost for the first unit following a production break and all subsequent units (Honious et al., 2016).

One popular method to assess the loss of learning and subsequent unit costs is the Anderlohr Method (1969). This method identifies five factors that when weighted appropriately account for the amount of learning lost during the production break. The amount of lost learning is used to regress up the original learning curve before production resumes. Once production resumes, the manufacturing process resets to the cost of a previously produced unit and progresses back down the revised learning curve at the original learning rate for all units produced after the production break (Anderlohr, 1969).

The final area in which research has focused is changes to the end-item during production. These changes include additions, deletions, and substitutions of components of the end-item. These modifications can occur due to conventional engineering change orders to the configuration of the end-item or due to concurrency between development testing and production that reveals necessary design changes. The addition, deletion, and substitutions of components of an end-item during production can cause the following units' costs to differ significantly from what is predicted using traditional learning curve theory (Department of the Air Force, 2007, pp. 8-50–8-56). Additionally, configuration changes can also affect the rate at which the manufacturing process learns, which alters and often steepens the original learning curve slope (Honious et al., 2016). The learning curve slope is especially affected during concurrency between development testing and production due to the continual flow of design changes, which tend to flatten the learning curve slope (Department of the Air Force, 2007, p. 8-50–8-56).

This research into production rates, production breaks, and changes to end-items demonstrates the importance of a stable production environment with a constant end-item configuration. Without these tenets, traditional learning curve analysis becomes challenging due to confounding variables. The influence of confounding variables obscures how unit costs are related to the number of units produced.

Forgetting & Plateauing Phenomena

An implicit assumption in the traditional learning curve theories is that knowledge obtained through learning does not depreciate (Epple et al., 1991). However, empirical evidence demonstrates that knowledge depreciates in organizations (Argote, 1993; Argote et al., 1990).

Argote et al. (1990) have shown that knowledge depreciation occurs at both the laborer level and the organizational level. Many variations of the traditional models make use of a concept of performance decay and forgetting to model non-constant rates of learning. Forgetting and its effects on lost learning can take many forms and is essential to consider in contemporary learning curve analysis. Forgetting is the concept that laborers and the organization as a whole will experience a decline in performance over time resulting in non-constant rates of learning. Badiru (2012) theorizes that forgetting and resulting performance decay is a result of factors "including lack of training, reduced retention of skills, lapse in performance, extended breaks in practice, and natural forgetting" (p. 287). According to Badiru (2012), these factors may be caused by internal processes, including training policy or external factors such as breaks in production.

Badiru (2012) lists three cases in which forgetting arises. First, forgetting may occur continuously as a worker or organization progresses down the learning curve due in part to natural forgetting (Badiru, 2012). In other words, the impact of forgetting may not wholly eclipse the impact of learning but will hamper the rate of learning while performance continues to increase at a slower rate. Second, forgetting may occur at distinct and bounded intervals such as during a scheduled production break (Badiru, 2012). Third, forgetting may intermittently occur at random times and for stochastic intervals such as during times of employee turnover (Badiru, 2012). Figure 7 illustrates this third case where intermittent periods of forgetting degrade the regular learning curve path to result in a degraded performance curve. In this illustration, the learning curve is shown as an increase in performance rather than a decrease in time or cost (Badiru, 2012). Others have expanded on the causes of forgetting and have drawn similar conclusions to Badiru (2012), such as Glock et al., 2019; Jaber, 2006; and Jaber & Bonney, 1997.

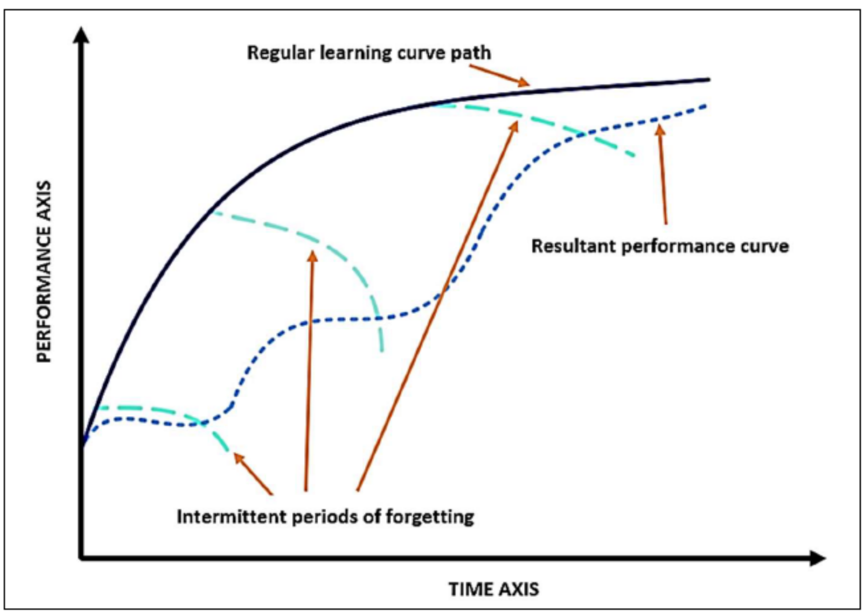

Figure 7. Effects of Forgetting on the Traditional Learning Curve

This decline in performance decays the rate of learning that causes longer manufacturing times and higher costs than would be forecasted using traditional learning curve theory. Although forgetting is common when production breaks occur as previously discussed, forgetting can occur without interruptions to the production line as discussed by Argote et al. (1990) and Badiru (2012). Many contemporary learning curve models attempt to incorporate the concept of forgetting. Models that incorporate variations to traditional learning curves—including rates of production, breaks in production, and configuration changes to end-items—attempt to model Badiru's (2012) second case of forgetting. Badiru's (2012) third case is challenging to model due to the stochastic nature of when and for how long forgetting will occur.

The concept of forgetting and its impact on decaying and non-constant rates of learning has proven relevant in contemporary learning curve research. Several forgetting models have been developed to include the learn-forget curve model (LFCM; Jaber & Bonney, 1996), the recency model (RCM; Nembhard & Uzumeri, 2000), the power integration and diffusion (PID) model (Sikström & Jaber, 2002), and the Depletion-Power-Integration-Latency (DPIL) model (Sikström & Jaber, 2012) among others (Glock et al., 2019). However, these forgetting models focus solely on the phenomenon of forgetting due to interruptions of the production process and most directly model Badiru's (2012) second case of forgetting (Glock et al., 2019; Anzanello & Fogliatto, 2011; Jaber, 2006). Jaber (2006) states that "there has been no model developed for industrial settings that considers forgetting as a result of factors other than production breaks" (p. 30-13) and mentions this as a potential area of future research. Although forgetting models have emerged after Jaber (2006), a review of the popular forgetting models cited confirms Jaber's statement. Therefore, Badiru's (2012) first case of forgetting along the learning curve and its effect on the curve should be investigated.

A related concept to the forgetting phenomenon is the plateauing phenomenon. According to Jaber (2006), plateauing occurs when the learning process ceases. This ceasing of learning results in a flattening or partial flattening of the learning curve corresponding to rates of learning at or near zero. These near-zero rates of learning are in contrast to forgetting curves where rates of learning may become negative resulting in inverted learning curves. There remains debate as to when plateauing occurs in the production process or if learning ever ceases completely (Asher, 1956; Crossman, 1959; Honious et al., 2016; Moore et al., 2015). Jaber (2006) provides several explanations to explain the plateauing phenomenon that include concepts related to forgetting. Baloff (1966, 1970) recognized that plateauing is more likely to occur when capital is used in the production process as opposed to labor. According to some researchers, plateauing can be explained by either having to process the efficiencies learned before making additional improvements along the learning curve or to forgetting altogether (Corlett & Morcombe, 1970). According to other researchers, plateauing can be caused by labor ceasing to learn or management's unwillingness to invest in capital to foster induced learning (Yelle, 1980). Related to this underinvestment to foster induced learning, management's doubt as to whether learning efficiencies related to learning can occur is cited as another hindrance to constant rates of learning (Hirschmann, 1964).

Li and Rajagopalan (1998) investigated these explanations and concluded that no empirical evidence supports or contradicts them while ascribing plateauing to depreciation in knowledge or forgetting. Jaber (2006) concludes that "there is no tangible consensus among researchers as to what causes learning curves to plateau" and alludes that this is a topic for future research (p. 30-9).

Boone's Learning Curve: Accounting for the Flattening Effect

In an attempt to address the previous issues highlighted in constant learning rates, Boone (2018) developed a learning curve model with a rate of learning that diminishes as more units are produced. The traditional learning curve theories diminish the rate of cost reductions as more units are produced because costs will decrease at a constant rate only when the number of units produced doubles. Because the rate at which units double decreases as more units are produced, the rate of cost reductions will also decrease as more units are produced. However, the literature review cited various theoretical and empirical evidence indicating that the cost reductions that occur with each doubling of units may not be constant as the number of units produced increases. Therefore, Boone (2018) sought to attenuate the cost reductions that occur with each doubling of produced units by reducing the amount that each doubled unit's cost decreases as the number of units increases. This attenuation of cost reductions was accomplished by decreasing the rate of learning as the number of units increases.

Boone (2018) began by formulating a model where the exponent of the traditional learning curve equation is a function of the number of units produced. This amendment was intended to vary the learning curve exponent with the independent variable "x" in order to alter the degree of cost efficiencies experienced as the number of units produced changes. He then devised a series of specific models that decreased the learning curve exponent "b" as the number of units produced "x" increased. Boone (2018) first created a model without an additional parameter that aimed to reduce the learning curve exponent "b" directly by the unit number. However, he claimed that it resulted in too drastic of changes to the exponent value and did not model data appropriately (Boone, 2018, p. 20). To temper the effect each additional unit has on the parameter "b," a qualifier was added. This qualifier "c" was named Boone's Decay Value with an initially studied value ranging from zero to 5,000 (Boone, 2018, p. 21). The resulting Boone's Learning Curve is shown in Equation 8.

Equation 8

$$\bar{Y} = Ax^{b/(1+\frac{x}{c})} \tag{8}$$

Where:
$\bar{Y}$ = the cumulative average time (or cost) per unit
X = the cumulative number of units produced
a = time (or cost) required to produce the first unit
b = slope of the function when plotted on log-log paper = log of the learning rate/log of 2
c = Boone's Decay Value (range can be anything except 0)

Boone found this curve was not only flatter near the end of production but was also steeper in the early stages in comparison to the traditional theory learning curve (Boone, 2018, p. 21). Holding the cumulative number of units produced "x" constant, as Boone's Decay Value approaches zero, the parameter "b" approaches zero representing a learning curve slope approaching 100%. As Boone's Decay Values approaches infinity, the parameter "b" remains unchanged, and Boone's Learning Curve simplifies to the traditional learning curve (Boone, 2018, p. 23).

Boone (2018) proceeded to test his model using Cumulative Average Theory against Wright's model. Boone (2018) utilized prime mission equipment cost data in units of total dollars for fighter, bomber, and cargo aircraft programs along with missile and munition programs (p. 21). He constrained his data to require at least five lots per program to prevent overfitting the data (Boone, 2018, p. 22). In total, 46 weapon system platforms were tested (Boone, 2018, p. 23). The OLS regression method is unable to estimate the parameters for Boone's Learning Curve because of the non-constant rate of learning; Boone's Learning Curve is convex in logarithmic scale. Instead, Boone utilized Microsoft Excel's Solver package to minimize the sum of squares errors (SSE) by iteratively adjusting the theoretical first unit cost "T1," the learning curve slope "b," and Boone's Decay Value "c" (Boone, 2018, p. 24). The conventional OLS methodology was maintained to estimate the parameters for Wright's Cumulative Average learning curve because this model remains linear in logarithmic scale (Boone, 2018, p. 24). Microsoft Excel's Solver requires bounds for each parameter when solving for the combination of parameter values that minimize the SSE. Wright's Learning Curve parameters informed these bounds to assist in estimating Boone's Learning Curve parameters (Boone, 2018, pp. 24–25).

Boone's theoretical first unit cost parameter minimum bound was equal to half of Wright's theoretical first unit cost and twice the value of Wright's first unit cost for the maximum bound (Boone, 2018, pp. 24–25). Boone's "b" parameter bounds were set between -3 and 3 times Wright's "b" specific to each estimated learning curve (Boone, 2018, pp. 24–25). These two bounds' values varied for each learning curve estimated due to their dependence on Wright's Learning Curve parameters. Lastly, Boone's Decay Value was bound from zero to 5,000 for all estimated learning curves (Boone, 2018, pp. 24–25). The only limits that were found to be binding when solving for optimal values were the upper limit of Boone's Decay Value (Boone, 2018, p. 25).

Boone then estimated the parameters for his curve and Wright's curve for each of the 46 programs using Cumulative Average Learning Curve theory. He obtained goodness-of-fit statistics in the form of the SSE and MAPE for each estimate in order to compare the accuracy of both curves (Boone, 2018, pp. 25–26). Boone then performed a paired difference t-test for both SSE and MAPE statistics (Boone, 2018, pp. 25–26). Both the SSE and MAPE paired difference t-tests rejected the null hypothesis that the means were equal to zero (Boone, 2018, pp. 29–30). These tests indicate that Boone's Learning Curve more accurately explains the cost data in comparison to Wright's Learning Curve at a significance level (α) of 0.05. Therefore, Boone demonstrated his learning curve to be more accurate to a statistically significant degree in comparison to Wright's Learning Curve (Boone, 2018, pp. 30–31).

Boone did not assess the predictive capacity of his model in comparison to Wright's by estimating parameters for a subset of early units and then extrapolating to future lots using the same estimated parameters. This approach departs from previous research including Moore et al. (2015) and Honious et al. (2016); however, this may be due to data availability as several lots are required for predictive analysis. Additionally, Boone did not hold constant the traditional learning curve parameters and estimate his new parameter, Boone's Decay Value, with these values fixed. Instead, Boone allowed all three of his parameters to change from Wright's estimated parameter values. Boone's methodology is also a departure from the previous methodology where Moore et al. (2015) and Honious et al. (2016) estimated the traditional learning curve, held the estimated parameters constant, and then added additional parameters. Boone's methodology may also be justified because Boone's Decay Value is a result of estimating learning curves and requires a different learning curve slope and first unit cost be considered. This Decay Value is unlike the parameters Stanford-B parameter and incompressibility parameter "M," which are measurable values that describe the manufacturing process itself.

Methodology Population and Sample

In order to test Boone's Learning Curve against the traditional learning curve theories, quantitative data from a diverse set of DoD programs was gathered. The population studied is DoD programs that have produced several complex end-items over time. These complex units can include aircraft, land vehicles, and missiles along with their complex sub-systems and sub-components. The data sample consists of programs with the necessary information required for learning curve analysis. This required program data included direct recurring labor costs in units of labor hours or total dollars per production lot along with the number of units per lot.

The direct recurring manufacturing labor cost category for each applicable WBS element was used to obtain labor hour data for each program. If this labor hour data were unavailable, the total recurring cost for each applicable WBS element was utilized instead. The total recurring dollar cost comprises the costs of all functional categories of labor along with materials costs and other costs for each WBS element. Unlike labor hours, costs in units of dollars must be normalized to be compared over time; therefore, all costs in units of dollars were normalized using escalation rates based on Producer Price Index (PPI) 3364 Aerospace Products and Parts. Removing the effects of escalation using PPI 3364 is common practice when normalizing costs in the aerospace industry. Additionally, total costs in units of dollars were provided and maintained in units of thousands of dollars. These labor cost data included costs at the prime mission equipment. WBS level prime mission equipment costs are directly related to touch labor and experience learning. Depending on data availability, additional elements below the prime mission equipment WBS elements were also analyzed to include engines and wings among other sub-systems and sub-components. Therefore, one program may contribute several unique components for learning curve analysis.

This final sample included direct recurring cost data from bomber, cargo, and fighter aircraft along with missiles and munitions. The programs in this dataset are both historical and contemporary spanning 1957–2018 and include a variety of defense contractors. This diverse dataset tested the generalizability of Boone's Learning Curve due to the varying levels of analysis along with the multitude of platforms, contractors, and production periods that foster various learning environments. In total, data from 123 weapon system programs were gathered with 258 unique components. Learning curve analysis will be performed on these unique components.

Data Collection

This dataset was created using DD Form 1921-1 "Functional Cost-Hour Report" and DD Form 1921-2 "Progress Curve Report" data obtained from the CADE Defense Automated Cost Information Management System (DACIMS). CADE DACIMS is a repository of DoD weapons system program cost data available to DoD analysts. Some historical data was also extracted from the AFLCMC Cost Research Library.

Business rules were created to avoid overfitting the data and to ensure learning curve analysis was appropriate to model each component's cost. The first business rule omitted programs with production lots of four or fewer. This business rule is consistent with Boone (2018, p. 22) and limited the sample from 258 to 169 unique components. A second business rule was also necessary when performing Cumulative Average Theory analysis. Cumulative Average Theory relies on continuous data because each lot's cumulative average cost and cumulative quantity is a function of all previous lots' costs and quantities. Therefore, if a program's production lot was missing cost or quantity data, all lots after that missing lot were removed for that program; however, all lots before the missing lot were retained. These lot removals decreased the number of lots in the total program. This reduced number of lots warranted the complete removal of some programs by applying the first business rule. This second business rule limited the dataset to 140 unique components for Cumulative Average Theory analysis. Despite these business rules, there was not a systematic elimination of any characteristic of the program labor cost data; a diverse dataset remained with the previously stated attributes.

Data Analysis

This analysis will examine whether Boone's Learning Curve more accurately explains variability in program labor cost data than the traditional theories. Both Cumulative Average Theory and Unit Theory will be used to make these comparisons. In order to test these hypotheses, learning curve parameters were estimated using each program's labor cost data for Boone's Learning Curve and the traditional learning curve models. Next, parameters from Boone's Learning Curve and the respective traditional theory were used to predict a learning curve. These predicted learning curves were then compared to the observed data. In order to utilize Unit Learning Curve Theory, Boone's Learning Curve was adapted from its original Cumulative Average Theory form to Unit Learning Curve Theory form. When Boone's Learning

Curve utilized Cumulative Average Theory, Boone's Learning Curve was compared to Wright's Learning Curve. When Boone's Learning Curve utilized Unit Theory, Boone's Learning Curve was compared to Crawford's Learning Curve.

Parameters for each learning curve were estimated using non-linear optimization techniques in Microsoft Excel. The traditional learning curve theories could be estimated using OLS regression. However, non-linear optimization was utilized to estimate the traditional curves for equitable comparison with Boone's Learning Curve. In contrast, Boone's Learning Curve required the use of nonlinear optimization techniques. This requirement spawns from the fact that Boone's Learning Curve is not linear when logarithmically transformed due to the decaying learning curve slope. This non-linearity of Boone's Learning Curve precludes the parameters from being estimated using OLS regression. The learning curve parameters for Boone's Learning Curve (i.e., "A," "b," and "c") and the traditional theories (i.e., "A" and "b") were estimated by minimizing the SSE using Excel's Generalized Reduced Gradient (GRG) Nonlinear Solver and Excel's Evolutionary Solver engines. The SSE is calculated by squaring the vertical difference of the observed data and predicted data for each lot and summing these squared differences across all lots. The SSE is calculated separately for Boone's Learning Curve and the traditional learning curves. For each model, Excel Solver is set to minimize the objective cell that is set as the SSE. The changing variable cells are the learning curve parameters specific to each learning curve model. These parameters are iteratively solved for using optimization techniques specific to each engine for each learning curve model. When using Evolutionary Solver, it is also necessary to bound the changing variable cells and choose a set of values from which to begin the optimization process. Due to the inherent differences in both Cumulative Average Theory and Unit Learning Curve Theory, different specific processes were used to estimate parameters for each.

Cumulative Average Learning Curve Theory

The following process was implemented to estimate parameters for Wright's Learning Curve and Boone's Learning Curve using Cumulative Average Theory for each program.

1. Wright's Learning Curve parameters "A" and "b" were initially estimated using OLS regression. a. Cumulative Average Cost was the dependent variable, while Cumulative Number of Units Produced was the independent variable.
2. These initial learning curve parameter estimates were used as starting values to more precisely estimate Wright's Learning Curve parameters using GRG Non-Linear Solver. This process generated final estimates for Wright's Learning Curve parameters.
3. Boone's Learning Curve parameters "A," "b," and "c" were estimated using Excel's Evolutionary Solver. This process generated initial estimates for Boone's Learning Curve parameters.
    a. Final estimates for Wright's Learning Curve parameters were used to calculate the upper and lower bounds of Evolutionary Solver.
    b. The starting values were calculated from the upper and lower bounds.
4. The Evolutionary Solver learning curve parameter estimates for Boone's Learning Curve were used as starting values to more precisely estimate parameters using GRG Non-Linear Solver.

This process produced final estimates for Boone's Learning Curve parameters. When estimating Wright's Learning Curve parameters, the GRG Nonlinear Solver technique should produce SSE at least equal to the SSE using OLS regression. The GRG Nonlinear Solver technique was appropriate to estimate Wright's Learning Curve parameters because this technique is used to find locally optimal solutions of smooth and non-linear functions (Solver Technology–Smooth Nonlinear Optimization, 2012). Because OLS regression was used to provide starting values for GRG Nonlinear Solver, it was reasonable to assume that the global minimum is within this local region approximated using OLS regression. However, GRG Nonlinear Solver cannot guarantee that a global minimum is found. The GRG Nonlinear Solver Multistart method was also utilized to ensure this technique yielded a global minimum. However, the Multistart method failed to provide consistent, reliable parameter estimates, unlike GRG Nonlinear Solver.

When estimating Boone's Learning Curve, the GRG Nonlinear Solver optimized values for Wright's Learning Curve parameters were used to calculate bounds for Boone's Learning Curve. For the "A" parameter, the lower bound was half of Wright's "A" parameter, and the upper bound was twice that of Wright's "A" parameter. For bounds on Boone's Learning Curve "b" parameter, the absolute value of Wright's Learning Curve "b" parameter was multiplied by 3 to yield upper (positive) and lower (negative) bounds. Lastly, for Boone's Decay Value "c," bounds were set between 0 and 500,000 in contrast to Boone's original bounds of 0 and 5,000. This bound was increased in comparison to Boone (2018) due to the upper bound being binding in Boone's analyses. Except for Boone's Decay Value, these bounds are consistent with Boone (2018) and provide the optimization model with a restricted range to decrease the search time for an optimal solution but a broad enough range to not constrain the model. Similar to Boone (2018), none of these constraints were binding except for the Decay Value "c" upper bound despite relaxing this constraint in comparison to Boone (2018). Further relaxing this constraint would not have led to substantive changes to Boone's Learning Curve; as Boone's Decay Value "c" approaches infinity, Boone's Learning Curve transforms into Wright's Learning Curve.

In order to estimate Boone's Learning Curve, the starting values were set as the midpoint of the negative values of the slope parameter "b" because negative values are those that represent learning curve slopes below 100%. This starting point is reasonable because most programs in this analysis will experience cost efficiencies from learning. Thus, these programs will have learning curve slopes at or below 100% that translate to a negative learning curve exponent "b" parameter. The starting values for Boone's Learning Curve first unit cost "A" parameter were the midpoint of the upper and lower bounds. These two starting values and bounds depend on the parameters estimated by Wright's Learning Curve. The last starting value was set as the midpoint between the upper and lower bounds of the Boone's Decay Value "c," which was static at 250,000 due to the static bounds.

Once these starting values and bounds were set, Evolutionary Solver was used to estimate a globally optimal solution. The Evolutionary Solver technique was appropriate to estimate Boone's Learning Curve parameters because this technique is used to find a globally optimal solution of smooth and non-smooth functions (Solver Technology–Global Optimization, 2016). In contrast to estimating Wright's Learning Curve, a local region with the global minimum cannot be reliably approximated before using this optimization technique. This local region cannot be reliably approximated because OLS regression cannot be used to provide starting values for Boone's Learning Curve. Similar to GRG Nonlinear Solver, Evolutionary Solver cannot guarantee the parameter estimates produce a global minimum. However, using the Evolutionary Solver solution as starting values, the GRG Nonlinear Solver was then used to ensure the solution is locally optimal.

Unit Learning Curve Theory

Unit Learning Curve Theory parameter estimation maintained the Cumulative Average Learning Curve Theory parameter estimation methodology for calculating various bounds and starting values. Additionally, the justification for utilizing both Excel Solver engines also remains the same. However, the inclusion of lot midpoint calculations required different analysis techniques. The following process was implemented to estimate parameters for Crawford's Learning Curve and Boone's Learning Curve using Unit Theory for each program.

1. Parameter-free lot midpoint approximations (Equation 6) were calculated for each production lot.
2. Crawford's Learning Curve parameters "A" and "b" were initially estimated using OLS regression. a. Average Unit Cost was the dependent variable while Lot Midpoint, calculated in Step 1, was the independent variable.
3. These initial learning curve parameter estimates were used as starting values to more precisely estimate Crawford's Learning Curve parameters using GRG Non-Linear Solver. This process generated intermediate estimates of Crawford's Learning Curve parameters.
4. The intermediate estimate of Crawford's Learning Curve "b" parameter was used to calculate a more precise set of lot midpoints using Asher's Approximation (Equation 7).
5. Applying these more precise lot midpoint approximations, Crawford's Learning Curve parameters "A" and "b" were more accurately estimated using GRG Nonlinear Solver.
    a. Steps 4 and 5 were repeated until the iterative process converged on a solution to produce final estimates of Crawford's Learning Curve parameters and lot midpoint approximations.
6. Parameter-free lot midpoint approximations (Equation 6) were used to estimate Boone's Learning Curve parameters "A," "b," and "c" using Excel's Evolutionary Solver. This process generated intermediate estimates of Boone's Learning Curve parameters.
    a. The final estimates for Crawford's Learning Curve parameters were used to calculate upper and lower bounds.
    b. The starting values were calculated from the upper and lower bounds.
7. The intermediate estimates of Boone's Learning Curve parameters "b" and "c" were used to approximate a more precise set of lot midpoints using Asher's Approximation adapted for Boone's Learning Curve (Equation 14).
8. Applying these more precise lot midpoint approximations, Boone's Learning Curve parameters were more accurately estimated using Evolutionary Solver. This process generated Evolutionary Solver parameter estimates of Boone's Learning Curve.
9. These Evolutionary Solver parameter estimates were used as starting values to improve further the accuracy of Boone's Learning Curve parameters estimates "A," "b," and "c" using GRG Non- Linear Solver.
    a. Steps 7, 8, & 9 were repeated until the iterative process converged on a solution to produce a final estimate of Boone's Learning Curve parameters and lot midpoint approximations.

For both Crawford's and Boone's Learning Curves, an iterative process is used to calculate precise parameter estimates and lot midpoint approximations. This iterative process was repeated until a solution converged. A solution converged when small changes in the learning curve exponent "b" parameter were calculated between iterations. This process of iterative solving was adapted from Hu and Smith's "Accuracy Matters" (2013). For Boone's Learning Curve, a limit of 10 iterations was placed on the iterative process. This limit of 10 iterations was reached a limited number of times and still produced relatively small differences of Boone's Learning Curve exponent "b" between iterations. In order to estimate lot midpoints and Boone's Learning Curve parameters, Asher's Approximation from Equation 6 was adapted to incorporate Boone's decaying learning curve slope.

Statistical Significance Testing

The estimated parameters for Boone's Learning Curve and the traditional learning curves were used to create predicted learning curves. These predicted curves were then compared to observed data. Total model error was calculated by comparing the difference between observations and predicted values to determine which learning curve theory more accurately explained variability in the data. Two measures were used to determine the overall model error. The first error measure was RMSE. RMSE is calculated by dividing the total SSE by the number of observations; in our case, the number of lots in that program (McClave et al., 2014, p. 14-25). RMSE is a scale-dependent measure. Additionally, RMSE is not robust to outliers; therefore, the greater the magnitude of an outlier from the average error values, the more influence this outlier will have on RMSE. RMSE was used instead of total SSE because RMSE transforms units from squared units to original units. This transformation eases interpretation. RMSE can be interpreted as the average amount of error of the model in the model's original units.

The second measure used to determine the overall model error was MAPE. MAPE is calculated by subtracting the predicted value from the observed value, dividing this difference by the observed value, taking the absolute value, and multiplying by 100%; these absolute percent errors are then summed over all observations and divided by the total number of observations (McClave et al., 2014, p. 14-25). MAPE provides a unitless measure of accuracy and can be interpreted as the average percentage the model is inaccurate. MAPE is robust to outliers, so the effects of outliers do not unduly influence this measure.

After calculating these measures of overall model error, a series of paired difference t-tests were conducted to determine if reductions in error from Boone's Learning Curve are significantly different than zero or due to random chance. In order to conduct the first paired difference t-test, Boone's Learning Curve RMSE using Cumulative Average Theory will be subtracted from Wright's Learning Curve RMSE, and the difference will be divided by Wright's Learning Curve RMSE by observation. This calculation will yield a percentage difference rather than raw difference to compare programs of varying differences in magnitude equitably. Only programs with errors reported in total dollars will be examined first to examine the results of the different unit measures. The null hypothesis for this test is that the percentage difference in RMSE is equal to or less than zero. This null hypothesis represents that Boone's Learning Curve results in an equal amount of or more error in predicting observed values in comparison to Wright's Learning Curve. The alternative hypothesis is that the percentage difference is greater than zero. The alternative hypothesis represents that Boone's Learning Curve results in less error in predicting observed values than Wright's Learning Curve. This same methodology was repeated five more times to examine both learning curve theories, each with two measures of model error for programs examined using labor hours and total dollars.

Results

The diverse dataset utilized in Phase 1 was augmented with observed learning curves estimated at the flyaway cost level to form the basis of the dataset used in Phase 2. Phase 2 utilized Unit Theory learning curve predictions generated from analyses in Phase 1. Observed learning curve data, Crawford's Learning Curve data, and Boone's Learning Curve data were used to answer a variety of research questions. These analyses sought to determine 1. if the plateauing and forgetting phenomena occur using traditional learning curve theory, 2. how Boone's Learning Curve models observed learning curve data in comparison to traditional learning curve theory, and 3. which program attributes, if any, can explain the programs that are best modeled by Boone's Learning Curve as well as explain the prevalence of plateauing and forgetting phenomena.

In order to understand if the plateauing and forgetting phenomena occur using the traditional learning curve theory, the mean percentage error of Crawford's Learning Curve in the fourth quarter was isolated. These fourth quarter mean percentage errors indicated that Crawford's Learning Curve did not systematically underestimate observed learning curves. In fact, the statistical tests across all quarters indicated Crawford's Learning Curve systematically overestimated observed learning curves with a high amount of variability. These findings suggest that the plateauing and forgetting phenomena are not systematically present in observed data when modeled with Crawford's Learning Curve. This conclusion limits instances where Boone's Learning Curve can more accurately model observed learning curve data in comparison to Crawford's Learning Curve.

The mean percentage error of Crawford's Learning Curve in the fourth quarter were then analyzed using proportions. Crawford's Learning Curve underestimated 47.6% of the observed learning curves tested. Therefore, these hypothesis tests of proportions also failed to indicate that Crawford's Learning Curve systematically underestimates observed learning curve data in the fourth quarter. Lastly, for the proportion analysis, the proportion of learning curves that Crawford's Learning Curve underestimated in both the first and last quarter was investigated. These instances represent learning curves with high rates of learning decaying to low rates of learning; Boone's Learning Curve models these observed learning curves exceptionally well. The proportion of learning curves that experienced high rates of learning decaying to low rates of learning was 35% when modeled using Crawford's Learning Curve. This proportion highlights that if an observed learning curve is experiencing high rates of learning in the first quarter, it is also more likely to experience low rates of learning in the last quarter when modeled with the Crawford's Learning Curve. Although these results failed to show a systematic presence of plateauing, they emphasize opportunities for Boone's Learning Curve to improve upon Crawford's Learning Curve.

Next, Boone's Learning Curve was analyzed to determine how it models observed learning curve data in comparison to the traditional learning curve theory. Because Boone's Learning Curve contains an additional, empirically-estimated parameter, it is expected to improve upon Crawford's Learning Curve at an aggregate level. However, if Boone's Learning Curve more accurately models observed learning curves that plateau and remains approximately equal to Crawford's Learning Curve in terms of error for observed learning curves that do not plateau, then Boone's Learning Curve provides inherent value in modeling the plateauing phenomenon. To investigate this, a confusion matrix was used to determine how learning curves that plateau interact with learning curves that are more accurately explained by Boone's Learning Curve. The confusion matrix indicated that Boone's Learning Curve more accurately modeled observed learning curves based on if those observed learning curves plateaued for 77% of observations. These results provide mixed conclusions as to if Boone's Learning Curve improves upon Crawford's Learning Curve by more accurately modeling the plateauing phenomenon of observed learning curves or because of its an additional, empirically-estimated parameter.

To further investigate how Boone's Learning Curve models observed learning curves, the MAPE percentage differences between Boone's Learning Curve and Crawford's Learning Curve in the first, middle, and last quarters were investigated. Hypothesis tests were conducted to determine in which quarter Boone's Learning Curve improved upon Crawford's Learning Curve to a statistically significant degree. Outliers were excluded, and the dataset was limited to observations in which Boone's Learning Curve was a significant improvement in error. The hypothesis tests indicated that Boone's Learning Curve improved upon Crawford's Learning Curve to a statistically significant degree in the first quarter only. Furthermore, the tests indicated that the error improvements in the last quarter were not statistically different from zero. The middle quarters' hypothesis test indicated that Boone's Learning Curve was systematically worse at explaining observed data in comparison to Crawford's Learning Curve. Each of these tests had a high amount of variability. These results suggest that Boone's Learning Curve improves upon Crawford's Learning Curve more than it merely having an additional empirically estimated parameter. However, Boone's Learning Curve was not shown to model the plateau effect in the last quarter to a statistically significant degree. There is a possibility observed learning curves improved in different sections of the learning curves while performing worse in others in comparison to Crawford's Learning Curve. These error improvements in different sections of the learning curve would have caused substantial variations of error in each section. These substantial variations of error improvement within each section could significantly obscure the statistical results due to the variability included in each hypothesis test.

Regression analysis was also used to investigate which program attributes, if any, can explain the programs that are best modeled by Boone's Learning Curve as well as explain the plateauing and forgetting phenomena. For the first part of this regression analysis, the percentage difference between Boone's Learning Curve MAPE and Crawford's Learning Curve MAPE was used as a dependent variable. A variety of independent variables were created using the Literature Review and operationalized using available data. The independent variables representing the level of aggregation of the learning curve was significant in the model. The regression analysis indicated that Boone's Learning Curve MAPE would decrease 26% on average when modeling learning curves at the highest (flyaway cost) and second highest (air vehicle cost) levels of aggregation, respectively, in comparison to Crawford's Learning Curve. This finding is consistent with the research hypotheses. The independent variable that measured the amount of time that spanned between the first and last years of production was also statistically significant although the coefficient estimate's sign was opposite of the hypothesized sign. In a separate regression, the independent variable representing the learning curve units of measure in total dollars was also statistically significant. This independent variable indicated that Boone's Learning Curve is associated with a 14.9% decrease in error on average when modeling observed learning curves with units of measure in total dollars rather than hours in comparison to Crawford's Learning Curve. Despite these pool of significant variables, several hypothesized were not significant in the model. Additionally, the Adjusted R2 for these models was relatively low at 6.7% and 5.3%, respectively. These Adjusted R2 values highlight that a large amount of variability remains to be explained in the dependent variable. These theory-based independent variables were also tested using data-mining independent variables using a mixed stepwise regression. This methodology was employed to potentially expose statistically significant theory-based independent. Some groups of data-mining independent variables were statistically significant and able to explain more variability in the dependent variable; however, no additional statistically significant theory-based independent variables were revealed. Furthermore, the stepwise regression model results and statistical inferences may have been invalid due to not passing overall model tests.

For the second part of this regression analysis, the percentage error between Crawford's Learning Curve and observed data in the fourth quarter of the observed learning curve served as a dependent variable. This variable provided a measure of plateauing indifferent to how Boone's Learning Curve models observed data. After testing various independent variables and validating the model with statistical tests, no hypothesized independent variables were significant in the model. This finding suggests that either the operationalization of plateauing was inappropriate, there exists too much variability in the data to explain instances of plateauing, or the hypothesized independent variables are unsuitable or were not appropriately operationalized. These theory-based independent variables were also tested using a mixed stepwise regression in order to potentially expose other statistically significant theory-based independent. Some groups of data-mining independent variables were statistically significant and able to explain more variability in the dependent variable; however, no statistically significant theory-based independent variables were revealed. Furthermore, the second set of stepwise regression model results and statistical inferences may have also been invalid due to not passing overall model tests.

Summary

This research sought to determine 1. if the plateauing and forgetting phenomena occur using traditional learning curve theory, 2. how Boone's Learning Curve models observed learning curve data in comparison to traditional learning curve theory, and 3. which program attributes, if any, can explain the programs that are best modeled by Boone's Learning Curve as well as explain the plateauing and forgetting phenomena. The various statistical results indicate that the plateauing and forgetting phenomena do not systematically occur using the traditional Unit Learning Curve theory. Despite these results, there remain instances where plateauing and forgetting occur that provide opportunities for Boone's Learning Curve to improve upon Crawford's Learning Curve.

Additionally, results provide mixed conclusions as to if Boone's Learning Curve improves upon Crawford's Learning Curve by more accurately modeling the plateauing phenomenon of observed learning curves or because of its additional, empirically-estimated parameter. Tests also did not show that Boone's Learning Curve improved upon Crawford's Learning Curve to a statistically significant degree in the last quarter but instead in the first quarter. This finding brings into question if Boone's Learning Curve does in fact more accurately model the plateauing phenomenon.

Finally, few hypothesized independent variables were significant in explaining when Boone's Learning Curve more accurately models observed learning curves, and a significant amount of variability in the data remained. Despite these results, OLS regression analysis confirmed that the level of aggregation, the units of measure, and the time span of a program explain a small amount of variability of the degree to which Boone's Learning Curve more accurately models observed learning curves. The OLS regression results for explaining plateauing independent of Boone's Learning Curve indicated that no hypothesized independent variables were significant in the model even when non-theoretically based independent variables were included in the OLS regression model.

These results indicate that Boone's Learning Curve may provide value by more accurately explaining observed learning curves. However, the theoretical explanations as to where Boone's Learning Curve improves upon the traditional learning curve theory and under what circumstances Boone's Learning Curve is more appropriate to use than the traditional learning curve theory remains to be discovered.

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