U.S. AIR FORCE

A New Learning Curve for Department of Defense Acquisition Programs: How to Account for the "Flattening Effect"

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Background: Diminishing Rates of Learning

- DoD must be able to estimate program costs with accuracy
- Method of forecasting production cost include learning curves
  - Workers gain efficiencies when producing end-items
  - Costs decrease by a constant percentage as units double

Crawford's Unit Theory Learning Curve

$$Y = Ax^b$$

Y = cost per unit

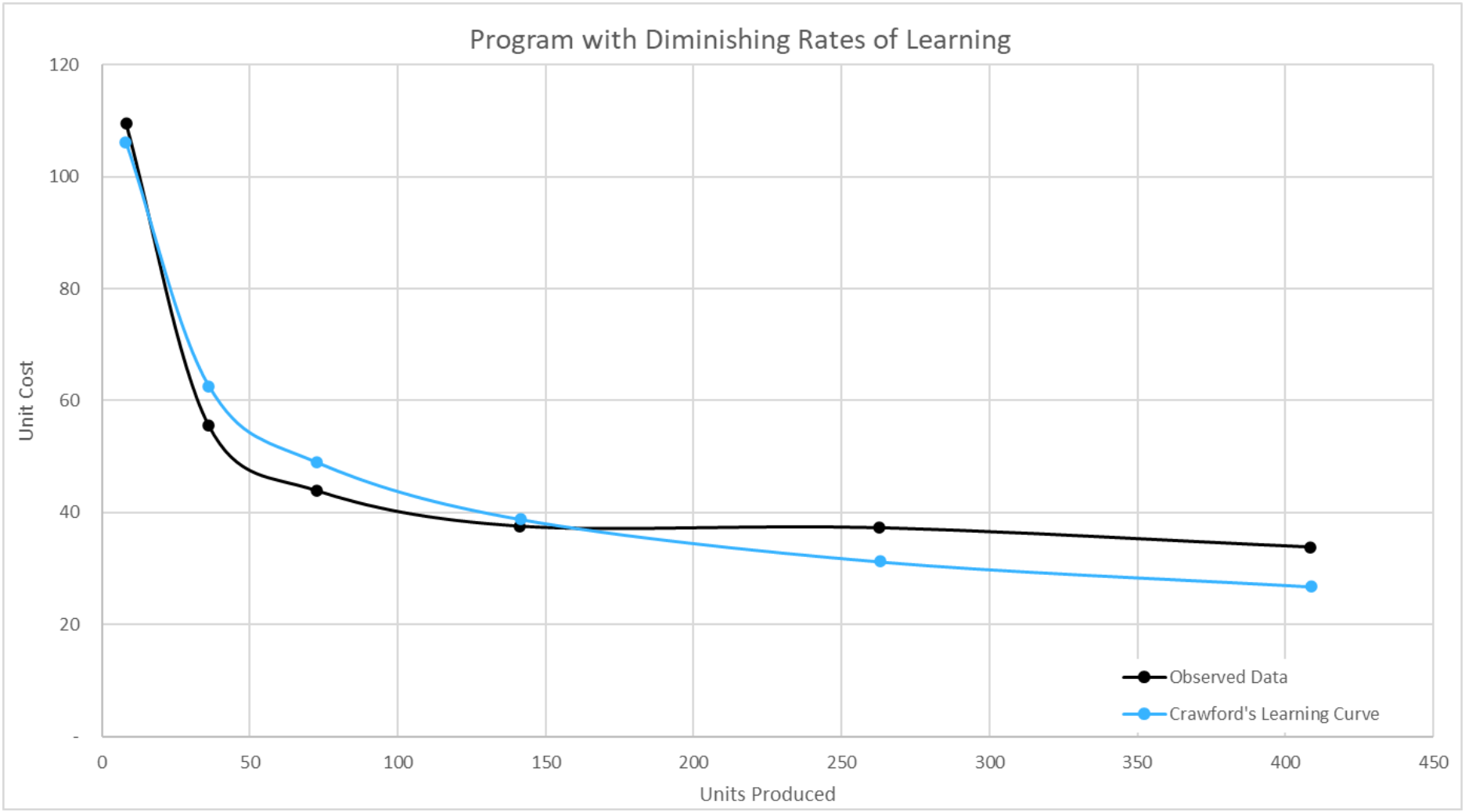
A = theoretical first unit cost (T1)

x = unit number

$$b = \frac{\ln(Learning\ Curve\ Slope)}{\ln(2)}$$

- Some programs display non-constant rates of learning
  - Cost decrease by a diminishing percentage as units double
  - Cost underestimated at beginning & end of production
  - Funding misallocated among programs' fiscal years

Background: Diminishing Rates of Learning



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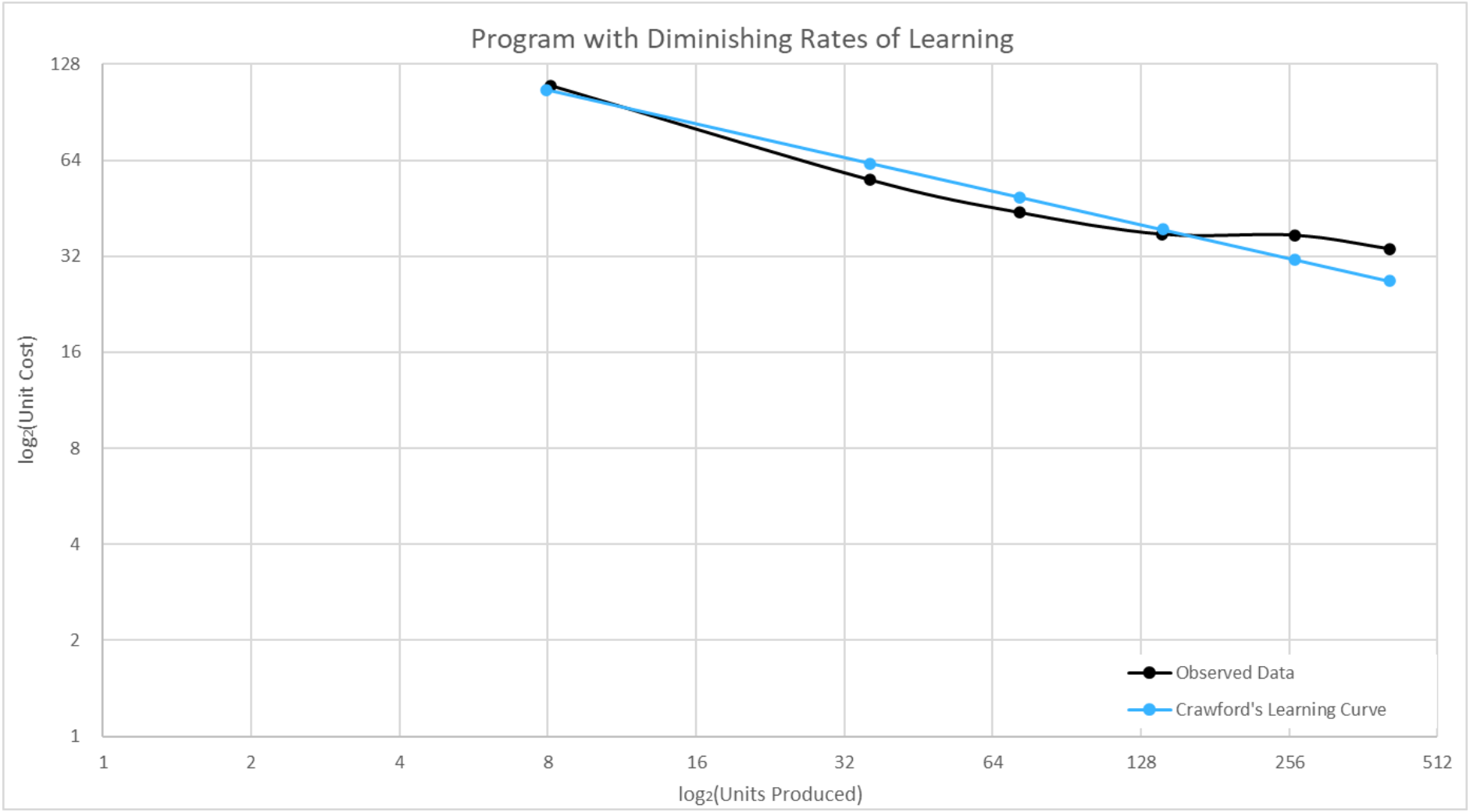


Background: Diminishing Rates of Learning

- To address these diminishing rates of learning a 2018 AFIT graduate created Boone's Learning Curve
  - Explored making the learning curve exponent a function of the numbers of units produced
- Functional form of learning curve exponent created from trial and error
  - Decreases "b" as the number of units produced "x" increases
  - As "b" decreases, the rate of learning decreases
  - Effects of "x" on "b" are tempered using the Boone's Decay Value "c"
  - All parameters are empirically estimated

$$Y = Ax^{f(x)} \longrightarrow Y = Ax^{b/(1+\frac{x}{c})}$$

Boone's Unit Theory Learning Curve

Y = cost per unit

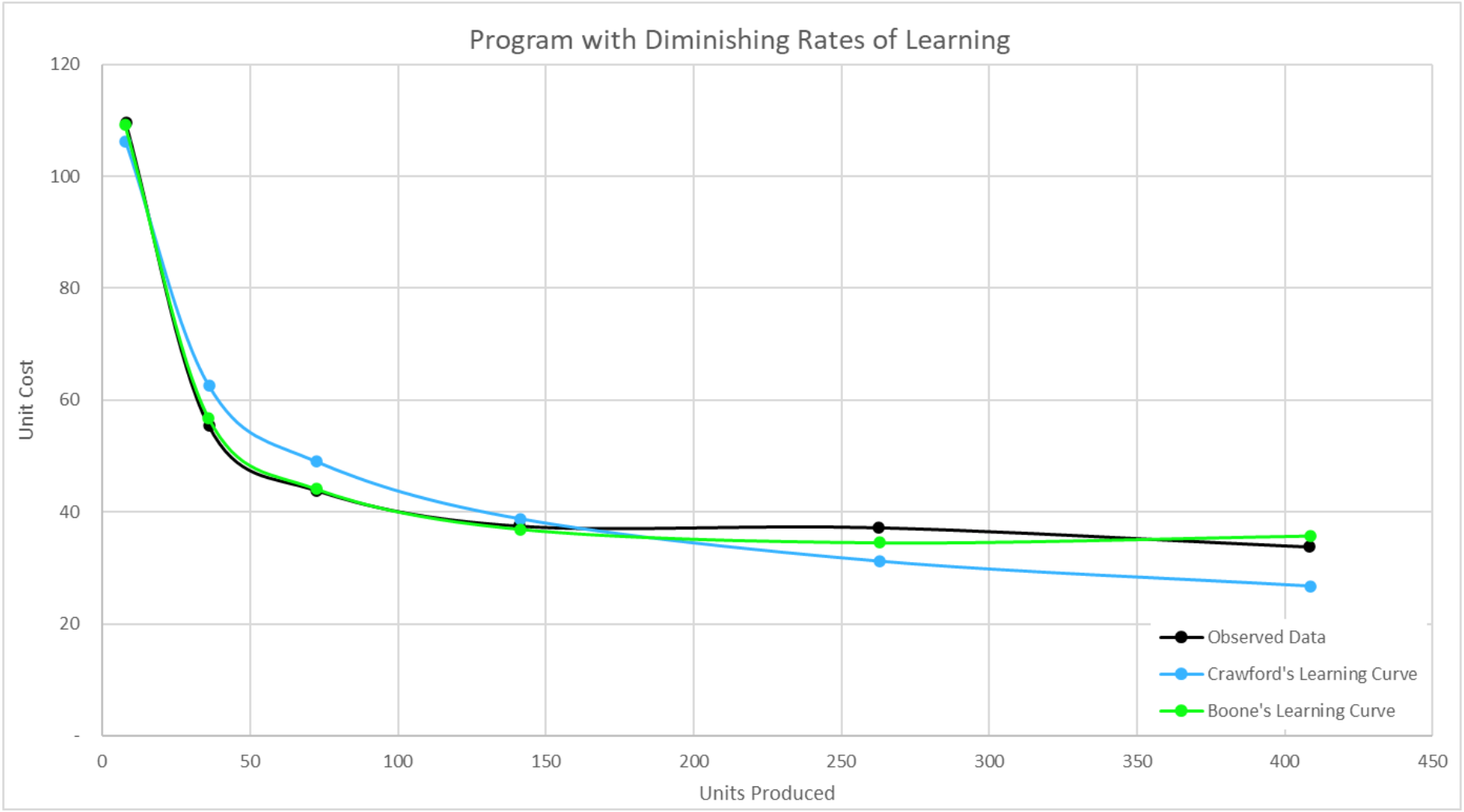
A = theoretical first unit cost (T1)

x = unit number

$b = \frac{\ln(Learning\ Curve\ Slope)}{\ln(2)}$

c = Boone's Decay Value (0-5000)

Background: Diminishing Rates of Learning



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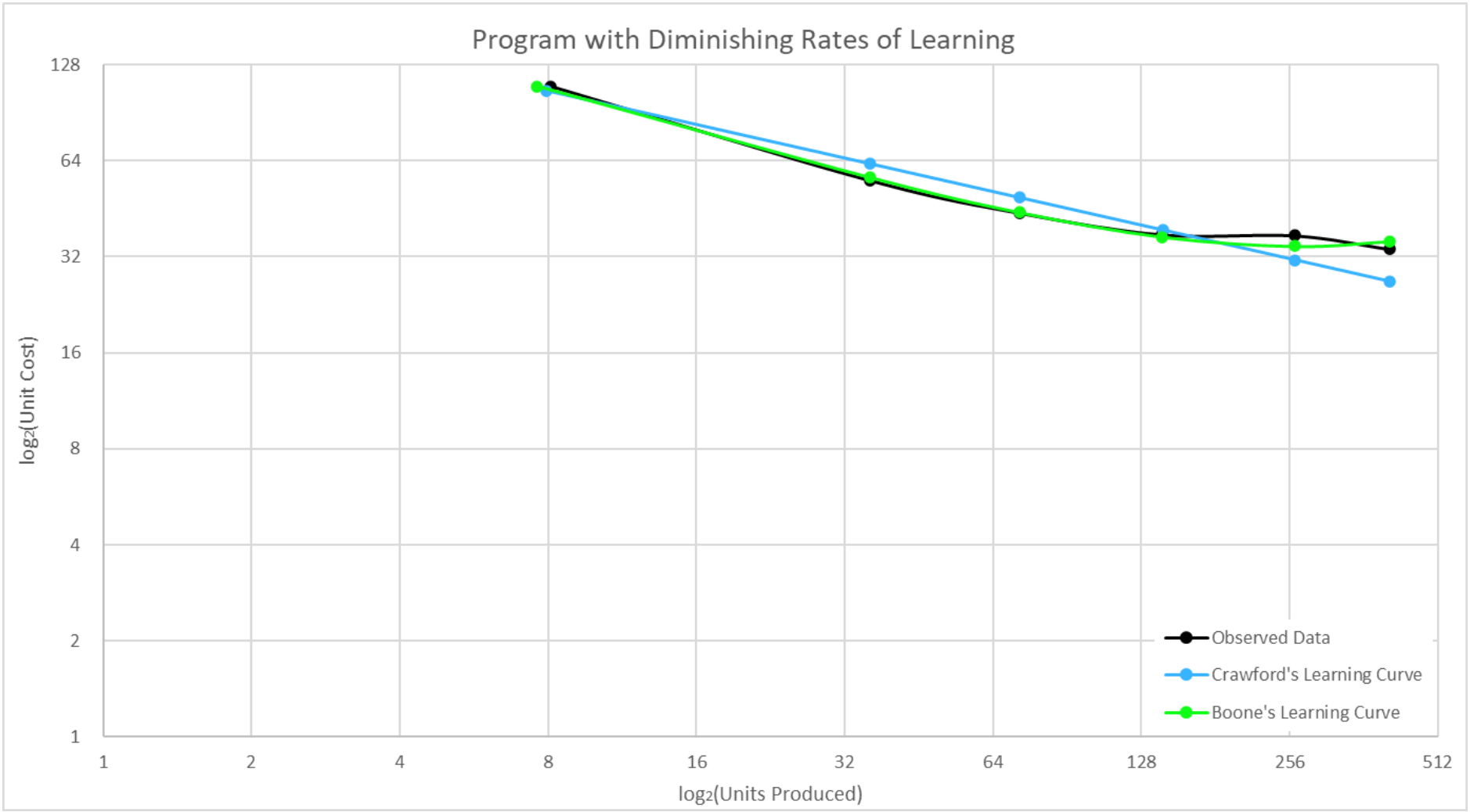

Justification for a Diminishing Learning Model

- Diminishing learning rate models justified in research
- Psychology research indicates knowledge depreciates:
  - Forgetting, production breaks, & employee turnover
- Other research shows learning slows over time:
  - Highly automated production & inability to process learning
- Asher's 1956 RAND study concluded higher aggregations of learning curve relate to diminishing learning rates
  - Manufacturing learning curves contain several constituent learning curves
  - Most learning curve analysis performed at highly aggregated levels

Theory & empirical analyses justify diminishing rates of learning

Analysis: Data & Methodology

- Gathered diverse dataset of 123 weapons systems with 258 unique components
  - Unit of analysis: weapon system components
  - System types: aircraft, engine, helicopter, missile, & UAV
  - Analysis levels: PME, airframe, & sub-components
- Estimated four learning curves and generated predictions
- Compared error from Boone's Curve and traditional curves
  - Boone's Curve vs. Wright's Cumulative Average Curve
  - Boone's Curve vs. Crawford's Unit Curve
- Performed statistical tests to determined if Boone's Learning Curve systematically reduces error

Boone's Curve tested using large, diverse dataset

Analysis: Results

- Based on six paired difference t-tests, Boone's Curve reduced error when compared to both traditional curves
  - Right-tailed rejection region
  - Distributions centered on zero & right skewed

						Hypothesis Test: $H_0: \mu \leq 0$ $H_A: \mu > 0$		
Learning Curve Theory	**Error Measure**	**Units of Measure**	**Sample Mean ($\bar{x}$)**	**Sample Standard Deviation (s)**	**Number of Observations**	**Test Statistic**	**P-Value**	**Result**
Cumulative Average Theory	Root Mean Squared Error Percentage Difference	Total Dollars (K)	19.3%	28.9%	118	7.23	<0.001	Reject H_0
		Labor Hours	15.2%	31.2%	22	18.50	0.280	Fail to Reject H_0
	Mean Absolute Percent Error Percentage Difference	Total Dollars (K) & Labor Hours Combined	18.6%	29.5%	140	7.45	<0.001	Reject H_0
Unit Theory	Root Mean Squared Error Percentage Difference	Total Dollars (K)	13.8%	22.7%	141	7.23	<0.001	Reject H_0
		Labor Hours	6.0%	14.8%	28	74.00	0.046	Reject H_0
	Mean Absolute Percent Error Percentage Difference	Total Dollars (K) & Labor Hours Combined	11.3%	23.1%	169	6.36	<0.001	Reject H_0

Analysis: Results

- 93% of observations had comparable percentage error or less
  - Defined insignificant percentage error reductions between -0.25% & 0.25%
  - 42% of observed learning curves significantly better explained
  - 51% of observed learning curves error approximately equal in error
- Cannot estimate an expected error reduction: highly variable reductions
  - Mean percentage difference error reductions: 6% to 19%
  - Coefficients of variation: 150% to 247%
- Applicability of Boone's Curve is an area of future research
  - Attempted to predict the error reductions from the use of Boone's Curve
  - Utilized program attributes and theory in OLS regression analysis
  - Could not explain more than 5% of the variation in the data

Promising results but further analysis necessary to fully leverage

Questions/References

Questions

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Journal Article:
MDPI's *Forecasting*: "Cost Estimating Using a New Learning Curve Theory for Non-Constant Production Rates."
https://doi.org/10.3390/forecast2040023

Original work:
Thesis: "An Analysis of Learning Curve Theory & Diminishing Rates of Learning."
https://scholar.afit.edu/etd/3607