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Maximizing Effectiveness Using a Flexible Inventory

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Preface & Acknowledgements

During his internship with the Graduate School of Business & Public Policy in June 2010, U.S. Air Force Academy Cadet Chase Lane surveyed the activities of the Naval Postgraduate School's Acquisition Research Program in its first seven years. The sheer volume of research products—almost 600 published papers (e.g., technical reports, journal articles, theses)—indicates the extent to which the depth and breadth of acquisition research has increased during these years. Over 300 authors contributed to these works, which means that the pool of those who have had significant intellectual engagement with acquisition issues has increased substantially. The broad range of research topics includes acquisition reform, defense industry, fielding, contracting, interoperability, organizational behavior, risk management, cost estimating, and many others. Approaches range from conceptual and exploratory studies to develop propositions about various aspects of acquisition, to applied and statistical analyses to test specific hypotheses. Methodologies include case studies, modeling, surveys, and experiments. On the whole, such findings make us both grateful for the ARP's progress to date, and hopeful that this progress in research will lead to substantive improvements in the DoD's acquisition outcomes.

As pragmatists, we of course recognize that such change can only occur to the extent that the potential knowledge wrapped up in these products is put to use and tested to determine its value. We take seriously the pernicious effects of the so-called "theory–practice" gap, which would separate the acquisition scholar from the acquisition practitioner, and relegate the scholar's work to mere academic "shelfware." Some design features of our program that we believe help avoid these effects include the following: connecting researchers with practitioners on specific projects; requiring researchers to brief sponsors on project findings as a condition of funding award; "pushing" potentially high-impact research reports (e.g., via overnight shipping) to selected practitioners and policy-makers; and most notably, sponsoring this symposium, which we craft intentionally as an opportunity for fruitful, lasting connections between scholars and practitioners.

A former Defense Acquisition Executive, responding to a comment that academic research was not generally useful in acquisition practice, opined, "That's not their [the academics'] problem—it's ours [the practitioners']. They can only perform research; it's up to us to use it." While we certainly agree with this sentiment, we also recognize that any research, however theoretical, must point to some termination in action; academics have a responsibility to make their work intelligible to practitioners. Thus we continue to seek projects that both comport with solid standards of scholarship, and address relevant acquisition issues. These years of experience have shown us the difficulty in attempting to balance these two objectives, but we are convinced that the attempt is absolutely essential if any real improvement is to be realized.

We gratefully acknowledge the ongoing support and leadership of our sponsors, whose foresight and vision have assured the continuing success of the Acquisition Research Program:

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We also thank the Naval Postgraduate School Foundation and acknowledge its generous contributions in support of this Symposium.

James B. Greene, Jr.
Rear Admiral, U.S. Navy (Ret.)

Keith F. Snider, PhD
Associate Professor

Panel 25 – Logistics Enablers for Enhanced Acquisition Outcomes

Thursday, May 12, 2011	
3:30 p.m. – 5:00 p.m.	**Chair: Lorna B. Estep,** Deputy Director of Logistics, Air Force Material Command ***Maximizing Effectiveness Using a Flexible Inventory*** Manbir Sodhi, University of Rhode Island, and James Ferguson, Marie Bussiere, and Betty Jester, USN ***Identifying and Managing Manufacturing and Sustainment Supply Chain Risks*** Nancy Moore, Elvira Loredo, and Amy Cox, RAND Corporation ***Comparing Acquisition Strategies: Maintenance-Free Operating Period vs. Traditional Logistics Support*** Nickolas H. Guertin, Open Architecture, PEO IWS, and Paul Bruhns, ManTech International Corporation

Lorna B. Estep—Deputy Director of Logistics, Directorate of Logistics and Sustainment, Headquarters Air Force Materiel Command, Wright-Patterson Air Force Base, OH. Ms. Estep is a member of the Senior Executive Service. She is responsible for the Materiel Support Division of the Supply Management Activity Group, a stock fund with annual sales of $7 billion. She directs a wide range of logistics services in support of Air Force managed spare parts, to include transformation programs, requirements determination, budgeting, acquisition, provisioning, cataloging, distribution and data management policy. She also provides supply chain management policy, guidance and direction in support of headquarters, air logistics centers, and U.S. Air Force worldwide customers.

Estep started her career as a Navy logistics management intern. She has directed the Joint Center for Flexible Computer Integrated Manufacturing, was the first program manager for Rapid Acquisition of Manufactured Parts, and has served as Technical Director of Information Technology Initiatives at the Naval Supply Systems Command. In these positions, she has developed logistics programs for the Department of Defense, implemented one of the first integrated and agile data-driven manufacturing systems, and directed the development of complex technical data systems for the Navy.

Maximizing Effectiveness Using a Flexible Inventory

Manbir Sodhi—Professor of Systems and Industrial Engineering, Department of Mechanical, Industrial, and Systems Engineering, University of Rhode Island. Professor Sodhi obtained his graduate degrees from the University of Arizona and has taught courses in Systems Design, Systems Simulation, Deterministic and Stochastic Optimization, etc. In addition to consulting for several companies, he has also worked as a visiting scientist at the Naval Undersea Warfare Center (NUWC) Division, Newport, and at the NATO Undersea Research Center in La Spezia, Italy. His recent work has appeared in professional journals that include the *Journal of Scheduling*, *International Journal of Production Research*, and *IIE Transactions*. He is currently exploring decision models supporting supply chain planning in defense operations and is developing tools and concepts of operations for the use of unmanned undersea vehicles (UUVs) for a variety of search operations. [sodhi@egr.uri.edu]

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Marie Bussiere—Head, Undersea Weapons Acquisition and Life Cycle Engineering Division, Torpedo Systems Department, Naval Undersea Warfare Center (NUWC) Division, Newport. Ms. Bussiere is responsible for Torpedo Test Equipment; Propulsion and Mechanical Systems; Production, Logistics, and Quality Assurance; and Fleet Engineering Support. She is a graduate of the University of Rhode Island, where she earned a Bachelor of Science degree in Electrical Engineering. She also earned a Master of Business Administration from Salve Regina University and received a diploma from the Naval War College for coursework in the areas of Strategy and Policy, National Security Decision Making, and Joint Maritime Operations. [marie.bussiere@navy.mil]

Betty Jester—Technical Program Manager, Torpedo Production, Naval Undersea Warfare Center (NUWC) Division, Newport. For the past six years, Ms. Jester has served as the Branch Head for Quality Production and Logistics, where her primary responsibilities include resource, financial, and personnel management. Ms. Jester has extensive experience in torpedo systems, having worked on torpedo programs since graduating from the University of Rhode Island with a Bachelor of Science degree in Electronic Computer Engineering. She also earned a Master of Business Administration from Salve Regina University. She derives her technical expertise from working on all aspects of torpedo life cycle development, from requirements definition through development and test, production and support, and maintenance for both lightweight and heavyweight torpedoes. [betty.jester@navy.mil]

Abstract

Although uncertainty in production and inventory systems is not desirable, predictions for demand are inherently uncertain. When the set of products is complex, that is, composed of multiple subassemblies, and there are shared subassemblies amongst different product types, the option for storing partially completed assemblies may also help in meeting demand uncertainties. Furthermore, as new technology is developed and new models are added to the inventory, older models can sometimes be upgraded to add the new functionality and increase the overall effectiveness of the inventory in meeting demand. Thus, when faced with uncertain demand for one or more products over a geographically distributed domain, the set of recourses for a manufacturer/planner include excess production (inventory storage), rapid re-location of inventory, production surges, when to upgrade technology or procure new models, what level of assembly to store the products, and where to store these, as well as in what quantities and ratios of

product types. Factors affecting these decisions are manpower availability, budgets, ease of upgrade, cost of new procurements, and probabilities of demand realization. This paper explores related decision models in the context of the torpedo enterprise. Solutions of mathematical models are illustrated and features of some of the models leading to specific solution algorithms highlighted. Simulations to assess the utility of the solutions obtained by analytical methods are also presented.

Introduction

Managing complex products that have long lifetimes is not an easy task. However, most defense and many industrial organizations deal with such products on a daily basis. Whereas non-durable goods (i.e., goods with lifetimes of less than three years) can be sold in large volumes with very little post-sales support, durable goods such as commercial grade printing and photo-copying systems, enterprise wide computing systems, weapons, and weapon systems are designed to accommodate evolutionary updates of the design of key components, or technology refreshes and insertions that either fix existing bugs and/or introduce new features by upgrades to modules. The complicating factor here is that the upgrades/insertions have to be done to a large inventory of in-service products while meeting promised deliveries. In the context of some defense organizations such as the torpedo enterprise, there are mandates on reserve quantities for different types of weapons, scheduled rotations between training and warshot inventory, mandatory maintenance schedules, etc. Furthermore, issues such as obsolescence and part failures must also be taken into consideration, and contracts for acquiring new and replacement parts must also be matched with the budgets and promised deliveries to the fleet.

Following Keynes (2006), it is generally accepted that the main motives for holding money are transaction, precautionary and speculative. As explained in Arrow, Karlin, and Scarf (1958), precautionary motives protect against uncertainty; speculative objectives are fueled by anticipation of future gains, and transaction encapsulates the reluctance to change currencies/investments because of the fixed or variable fees incurred in flipping from one type of investment to another. Reasons for holding an inventory of goods are generally the same as those for holding currency. It can be argued that the exception is when goods are held in reserve to meet uncertain demands, with the objective of exceeding some level of customer satisfaction. The accounting of costs and benefits in defense organizations is somewhat different, and this paper seeks to develop the argument that the goal of holding inventory in this sector is to respond sufficiently to future threats. In an environment of rapidly changing threats (Hilsenrath, 2011), the utility of an inventory of weapons is not just in its ability to meet current needs, but also in its ability to meet future requirements with minimal transformation effort.

Costs Involved in Defense Logistics

The costs considered when modeling inventory decisions in commercial enterprises are typically holding, ordering, shortage, and backorder costs. Holding costs include the cost of money (opportunity loss because of the money tied up in inventory or the cost of capital borrowed to purchase inventory). Shortage costs include the cost of lost sales, which lead to lower profits. Backorder costs are the costs incurred when orders not delivered in a timely manner and must be rushed to the customer using more expensive logistics channels. Other costs considered when analyzing inventory decisions are lateral transfer cost (Lee, 1987), multiple channel supply costs, etc., .; additional issues include buyer/vendor coordination, including price discounts (Goyal & Gupta, 1989). In terms of maximizing inventory effectiveness in the commercial world, companies maximize profit, and demand serves as the primary constraint. In other words, profit is king, and demand is the main constraint to maximizing profit. As we will see (and would be expected), this is not the case when supporting weapon systems.

The nature of costs in the defense sector is considerably different. Defense logistics agencies are issued annual budgets for maintaining supply chains with the goal of stocking adequate levels of weapons and supplies to meet contingency demands. Stated slightly differently, the Fleet requirements drive inventory need, and the main constraint is the allowable budget; other constraints include Intermediate Maintenance Activities (IMA) capacity in terms of personnel and test equipment. To use the language from the previous paragraph, demand is king, and the budget (a type of profit) is the primary constraint when maximizing demand fulfillment. This brings out the point that in the Department of Defense (DoD), cash flow is controlled by a higher authority and cannot be increased based on "selling" more inventory. The budget is set (at some point in time), and support of the weapon system must be optimized based on that amount. This type of inventory effectiveness optimization does not lend itself to commercial enterprise, because in the retail world, profits will change based on company performance.

Logistics Costs in the Torpedo Enterprise

Another level of complexity is added to the Torpedo Enterprise's inventory system, in that its inventory is stored at three IMAs, each with differing cost models. The IMA in Pearl Harbor, HI, is contractor run and was awarded based on a competitive services contract. The IMA in Yorktown, VA, is run by the U.S. Navy; the labor at this IMA is "free," as it is supplied by sailors. The third IMA is located at NUWC, Division Keyport and is staffed with Government Civil Service labor. These differing structures (commercial, military, and federal) sometimes cause issues in regards to standardization of processes and organizational cohesiveness. Further, the torpedo enterprise, because it supports a weapon for war, is also governed by legal statutes related to safety, hazardous material, Radio Frequency Identification (RFID), and Unique Identifier (UID), to name a few; these are all cost drivers.

There are also inventory considerations below the torpedo All Up Round (AUR) level. Torpedo unique parts are inventoried by the Naval Inventory Control Point (NAVICP), and items common between torpedoes and other DoD systems are inventoried by the Defense Logistics Agency (DLA). Demands for these parts are tracked through the use of in-house databases. Problems with inventory re-order are sent to the Naval Undersea Warfare Center (NUWC) for technical recommendations (e.g., suitable replacements when obsolescence is encountered).

The torpedo enterprise inventory for purposes of this paper is the warshot and exercise inventory maintained at the AUR configuration in bunkers at or near the IMAs. These torpedo inventories are stored for both the Atlantic Fleet and the Pacific Fleet, and the torpedoes are available for the Fleet to requisition. The quantity goal for the torpedo enterprise inventory is Non-Nuclear Ordnance Requirements (NNOR), with a wartime surge capability referred to as WAR RESERVE. At one time, the planning to support the Atlantic Fleet and Pacific Fleet requirements was handled separately, but several years ago, the enterprise moved to centralized inventory planning and handling (i.e., one Planning Cell). The Planning Cell meets with the Fleet representatives quarterly, at a minimum, to discuss warshot and exercise requirements; exercise torpedoes are units capable of being fired and recovered for the Fleet to maintain proficiency. These warshot and exercise requirements are translated to IMA capacity, and torpedo build requirements are determined to workload the IMAs. So, the flexibility of the inventory at the AUR level is the IMA's capacity to build exercise and warshot torpedoes, and to turn one into the other, and vice versa. Fleet/ship requirements can also be met through a mix of torpedo configurations (i.e., MK48 Mod 6 versus MK48 Mod 7) that are tailored to the target operating theatre. Additionally, there is flexibility of inventory at the AUR torpedo level through the upgrade of operational software via download capability. Versions of operational software can be downloaded at IMAs during weapon maintenance and preparation, or even on board ships. Operational software brings flexibility to AUR torpedoes with improved and varying performance.

Since our enterprise is not in production of AUR torpedoes at this time and has not been for many years, Foreign Military Sales can both limit and enhance the Torpedo Enterprise's flexibility. To sell AUR torpedoes to other nations at this time has a negative impact on the US's inventory quantity, but provides valuable resources to reconstitute production capability or performance enhancements in both hardware and software, which are helpful in the long run of the program (i.e., financing torpedo upgrades in the future).

Use of older torpedo configuration hardware that has been "moth balled" (e.g., MK48 Mod 4) brings with it the flexibility of "quantity versus quality." Older torpedo hardware which has been slated for demilitarization can be revitalized to add quantity to the inventory with calculated performance degradation. Unrelated to the purpose of this paper, performance versus quantity models exist to evaluate overall torpedo enterprise inventory effectiveness.

Modeling Inventory Effectiveness

In the discussion that follows, details of some preliminary models investigating the impact of flexibility on inventory operations are presented. The first approach utilizes an established two-level service model with conversion options between different part types to estimate the benefit that may be garnered by pooling inventory. The second approach presents a mathematical programming approach for determining optimal inventory decisions, with transfers and conversions between different part types and common subassemblies. A brief literature review is first presented.

A two class inventory system for modeling consumable items in a defense setting has been presented in Deshpande, Cohen, and Donohue (2003). The authors construct a model approximating the management of consumables by the DLA and propose a threshold for determining backorders for different classes of items. This model is useful when considering the allocation of pooled inventory items, but requires the setting of priorities for different classes externally. Clearly, this is difficult to do. However, this paper explains many of the issues particular to inventory management in defense settings.

Multi-echelon models for inventory management of spares in the defense industry have been considered by Simon (1971) and Yanmei, Jiangsheng, Sujian,and Weimin (2008), among others. However, most multi-echelon models consider single item types and the location of inventory pools at different levels to meet demand changes at different end points by cross-shipping when necessary. A fundamental analysis of the two-level case for repairable items is in Simon (1971), Muckstadt (1973), and Graves (1985). Although substitution of items, examined in Karaesmen and Van Ryzin (2004), can result in significant savings, it has not generally been considered in these multi-echelon models. Begnaud, Benjaafar, and Miller (2009) do consider multi-echelon inventory planning with flexible substitution opportunities, but the decision for interchanging items with an associated transaction cost is not developed.

There is a vast body of literature related to mathematical programming models for lot sizing. Starting with Wagner and Whitin (1958), Crowston and Wagner (1973), etc., the solution approaches for such problems have involved either dynamic programming approaches, specialized algorithms, or integer programming formulations and solutions (Belvaux & Wolsey, 2000; Wolsey, 2002). As noted in Wolsey (2002), many real-world lot sizing problems can now be adequately solved using commercial-off-the-shelf mathematical programming software. Wolsey (2002) further classifies lot sizing problems using three fields: [x, y, z]. The first field, x, indicates the problem version, and its choices are LS (lot sizing), WW (Wagner Whitin), DLSI (Discrete Lot Sizing with Initial Stock), and DLS (Discrete Lot Sizing without initial stock). The second field describes the production capabilities: C for capacitated, CC for constant production, and U for uncapacitated. When multiple items share production capacities, the additional qualifier BB is prepended to DLSI. The third field describes extensions/variants and includes B (Backlogging), SC (startup costs), ST (startup times), LB (minimum production levels), SL (sales constraints), and SS (safety stock considerations). The first two fields of problem considered here could then be described as DLSI-CC. Since the nomenclature proposed does not capture transformations, we suggest an extension to the nomenclature—T for transformation whereby items can be transformed from one product type to another. Although there are a large number of additional combinations that can be proposed, for now, the nomenclature used to describe the multi-item lot sizing problem with transformations can be BB/DLSI-CC-T.

Based on the discussion above, we propose the thesis that for a defense logistics operation, a fundamental measure of inventory effectiveness is the flexibility to meet a variety of potential needs for future operations. Based on this assumption, two preliminary models are developed to show how the increase in flexibility can indeed result in improvements to service levels. The first approach is based on an established two-level service operation, first explored in Sherbrooke (1968), further developed in Simon (1971), Muckstadt (1973), and others. The second model presented is a multi-product lot sizing model with transformations between different product types.

A Preliminary Investigation of the Impact of Flexibility in 2-Level (Base–Depot) Operations

Following Sherbrooke (1968), a two-level operation for recoverable parts is described as follows: Several distributed maintenance facilities (j = 1,…, N) restore incoming recoverable parts. While most parts can be repaired locally, some fraction of incoming parts has to be sent to the central depot for repair. The base and depot each maintain their own levels of inventory independently, and this inventory of parts is used for immediate replacement of incoming parts that undergo repair. When this inventory is depleted, the turnaround of outgoing parts is delayed until some refurbished units are available. The organization of this system is shown in Figure 1. As indicated in the figure, the parts are assumed to arrive at base j with exponential inter-arrival times, at rates λ_j respectively. The service time at each base is μ_j. The depot is designated by the index 0. The total transfer time between the base and the depot is denoted as τ_j, and the stock levels maintained at the depot and bases are (S_0, S_1,…, S_n).

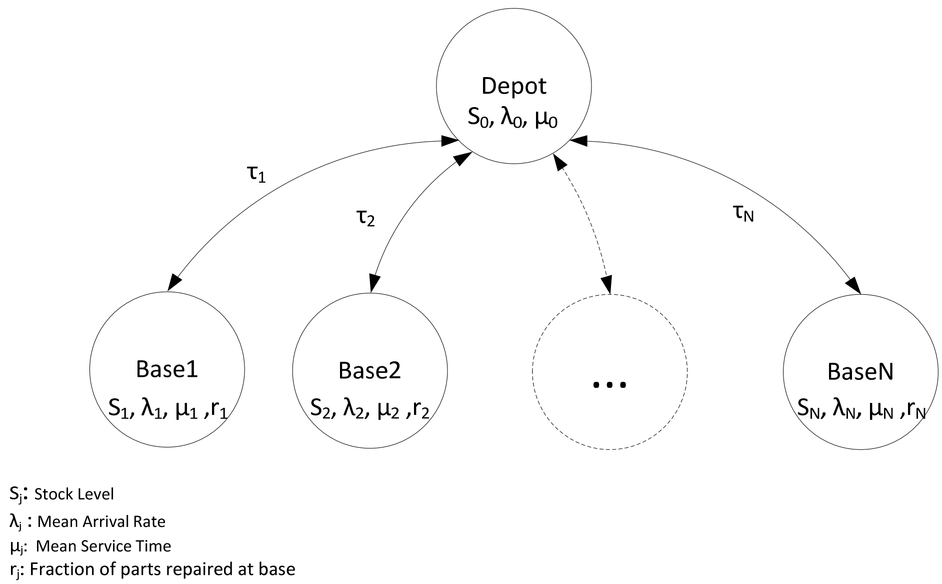

Figure 1. 2-Level Structure for Repairable Items

For such a scenario, given an allocation of spares ($S_0,\ldots,S_N$) among the bases and depot, the average number of parts waiting in the system at the base and the depot (L_0 , L_1, …, L_N) is computed in the following way:

$$L_0 = \sum_{k=s_0}^{\infty}(k - S_0)e^{-\lambda_0\mu_0}\frac{(\lambda_0\mu_0)^k}{k!}$$

$$L_j \approx \sum_{k=s_j}^{\infty}(k - S_j)e^{-\lambda_j\beta_j}\frac{(\lambda_j\beta_0)^k}{k!}, j = 1,\ldots,N$$

where,

$$\lambda_0 = \sum_{j=1}^{N}(1 - r_j)\lambda_j \text{ , and}$$

$$\beta_j = r_j\mu_j + (\tau_j + \frac{L_0}{\lambda_0})(1 - r_j) .$$

A detailed discussion can be found in Tijms (2003).

Now, let us assume that the system handles two part types, k=1, 2. The repair protocol is the same—that is, base *j* repairs incoming parts with probabilities r_{j1} and r_{j2} respectively. The stock levels at the depot and the bases are (S_{01}, S_{02}, S_{11}, S_{02},…, S_{N1}, S_{N2}) respectively. A simulation experiment was conducted in which arrival and service rates were randomly selected (with a service ratio of ½ for the bases and the depot). The transportation time between the base and the depot was set to $2^*\mu_j$. The total inventory level was varied, as shown in Figure 2. This was done for each product type, and an optimal distribution of inventory was determined. The expected number of items in the system for each product type was recorded as $\mathcal{L}_1$ and $\mathcal{L}_2$. Finally, an optimal allocation of inventory for the combined system was determined using an evolutionary algorithm, and the total number of items in the system was noted as $\mathcal{L}_3$. A graph comparing $\mathcal{L}_1 + \mathcal{L}_2$ and $\mathcal{L}_3$ is shown in Figure 2. As expected, the performance of the pooled system is significantly superior to that of the separate systems. For the parameters used here, the number of parts in the system required to maintain an equivalent service level is smaller by a factor of 4 on the average.

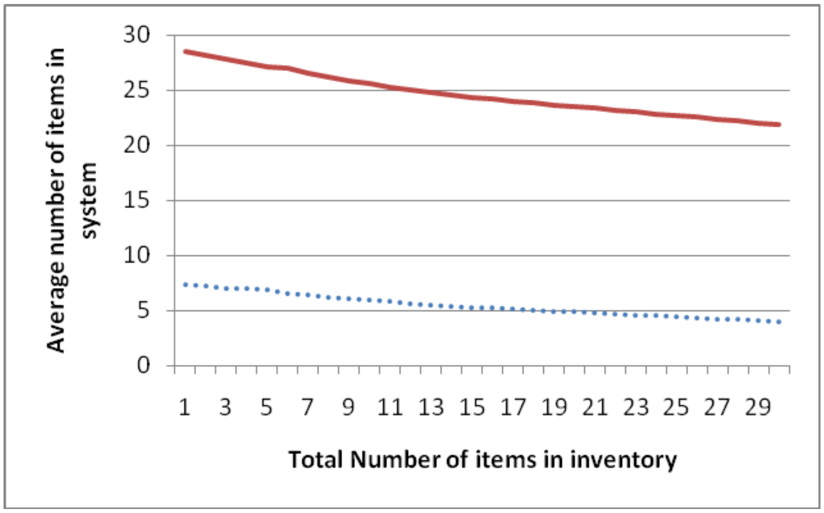

Figure 2. Comparison of Pooled vs. Segregated Inventory Performance

The example presented here emphasizes the advantages of a pooled inventory and transformations between two product types. This analysis is a part of ongoing work focused at developing metrics for effective inventory with transformations in the context of defense organizations.

Basic Lot Sizing Model

The model being expanded in this section that seeks to mimic the Torpedo Enterprise's inventory is a lot sizing problem. The assumptions of this model are unlimited and instantaneous production, unlimited inventory storage, no incoming or outgoing inventory, and deterministic demand. However, these assumptions can easily be altered by adding the proper constraints. The constraining costs in the model are inventory carry-over ($/period/unit), set-up costs ($/set-up), and production costs ($/production unit). The objective of this model is to meet demand for each period, while minimizing cost over the periods being studied, and allowing transformations between products/subassemblies during the planning horizon.

Mathematically this model can be written as follows:

$$P_{it} = \text{production of product } i \text{ in time } t$$

$$I_{it} = \text{inventory carry} - \text{over of product } i \text{ in time } t$$

$$S_{it} = \text{setup of production for product } i \text{ in period } t$$

$$\chi_{ij} = \text{cost of producing } (i = j) \text{ products}$$

$$\sigma_i = \text{setup cost of product } i$$

$$\phi_i = \text{cost of holding product } i$$

$$\delta_{it} = demand\ of\ product\ i\ in\ time\ t$$

$$Min\ Z = \{\sum_i \sum_j \sum_t (\chi_{ij} P_{ijt} | i = j) + \sum_i \sum_t (\varphi_i I_{it}) + \sum_i \sum_t (\sigma_i S_{it}) | \forall\ i,t\} \tag{1}$$

$$S.t.$$

$$P_{it} + I_{i(t-1)} = \delta_{it} + I_{it} \tag{2}$$

$$P_{it} \leq S_{it} * M \tag{3}$$

$$P_{it}, I_{it} \geq 0 \tag{4}$$

$$P_{it}, I_{it} = Integer \tag{5}$$

$$S_{it} = Binary \tag{6}$$

Equation 1 is the objective function which minimizes the production inventory and setup costs of the system. Equation 2 ensures the conservation of material within the model flow. Equation 3 uses Big M logic to set the setup decision for product *i* to 1 if production for product *i* is needed. Equations 4–6 incorporate the necessary non-negativity, integer, and binary constraints, respectively. A flowchart of the base model can be seen in Figure 3.

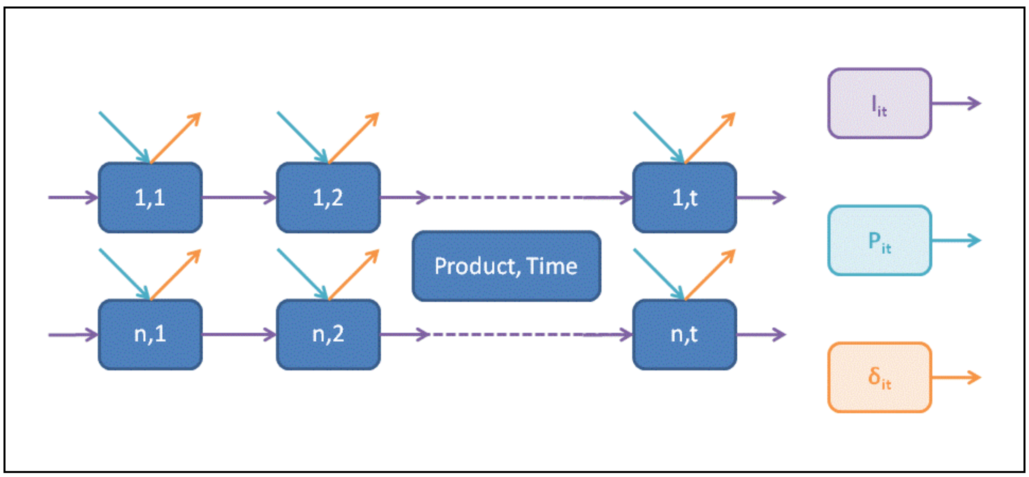

Figure 3. Simple Model for Transformations Among Different Part Types

Transformation Expansion

The first expansion to be integrated into the lot sizing model is that of product transformation. Consider the problem where two distinct products can, at a price, be converted from one to the other. An example is the production of modern automobiles, where the base model can be upgraded to more “deluxe” or “luxury” models. Another similar example that this model was developed for, is the transformation of torpedoes from one model to another. The ability to transform products in an inventory creates a more flexible inventory and provides the opportunity for cost savings depending on the transformation and setup costs of a particular system.

In order to expand the model to include transformations, the following variable is added to the model’s environment.

$$T_{ijt} = transformation\ of\ product\ i\ into\ j\ in\ time\ t$$

And the following constant is changed to include transformation costs from one product to another.

$$\chi_{ij} = cost\ of\ producing\ (i = j) or\ transforming\ (i \neq j)\ products$$

Furthermore, Equations 1 and 2 are expanded to include the new variable and constant.

$$Min\ Z = \{\Sigma_i \Sigma_j \Sigma_t (\chi_{ij} P_{ijt} | i = j) + \Sigma_i \Sigma_t (\phi_i I_{it}) + \Sigma_i \Sigma_t (\sigma_i S_{it}) + \Sigma_i \Sigma_j \Sigma_t (\chi_{ij} T_{ijt} | i \neq j) \,|\, \forall\ i, t\} \quad (7)$$

$$P_{it} + I_{i(t-1)} + \Sigma_j T_{jit} = \delta_{it} + I_{it} + \Sigma_j T_{ijt} \quad (8)$$

Note that in Equation 7, the same cost matrix is used for both production and transformation. For Production *i* = *j*, while for transformation, *i* ≠ *j*. For the conservation of material constraint, the left-hand side (incoming) of the constraint adds the summation of the transformations from all products *j* into product *i* for the given period, while the right-hand side (outgoing) adds the summation of the transformations from product *i* into all products *j* for the given period. A flowchart of the transformation expanded model can be seen in Figure 4.

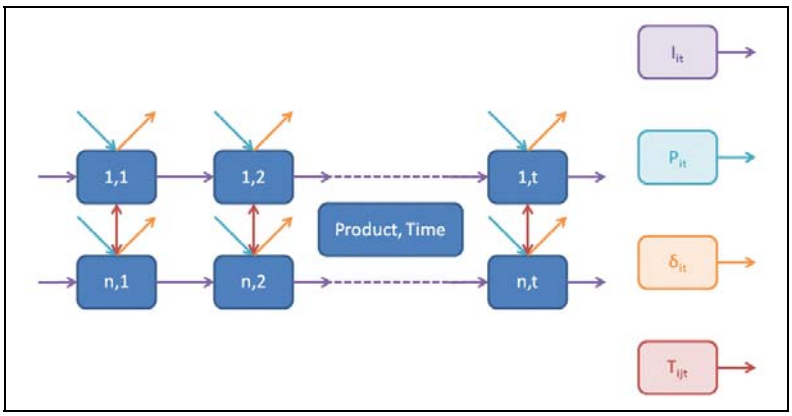

Figure 4. Transformation Expansion

Move Expansion

The next model expansion considers the system where there is more than one location for producing and storing products. Each distinct location can have its own associated production, storage, inventory, and setup costs. It is assumed that movement of products between locations is instantaneous. This assumption can, however, be dropped by manipulating the time (*t*) values associated with the move variables in the conservation of material constraint.

In order to expand the model to include transformations, the following variable is added to the model's environment.

$$M_{itkl} = move\ of\ product\ i\ in\ time\ t\ from\ location\ k\ to\ location\ l$$

And the following constant is changed to include movement costs from one location to another.

$$\rho_{ikl} = cost\ of\ moving\ product\ i\ from\ location\ k\ to\ location\ l$$

Furthermore, all of the other constraints and variables must have a location subscript added to their definitions.

Equations 7 and 8 are expanded to include the new variable, constant, and location subscript:

$$Min\ Z = \left\{ \begin{matrix} \sum_i \sum_j \sum_t \sum_k (\chi_{ijk} T_{ijtk} | i \neq j) + \sum_i \sum_j \sum_t \sum_k (\chi_{ijk} P_{itk} | i = j) + \\ \sum_i \sum_t \sum_k (\phi_{ik} I_{itk}) + \sum_i \sum_t \sum_k \sum_l (\rho_{ikl} M_{itkl}) \end{matrix} \middle| \forall i, t, k \right\} \quad (9)$$

$$P_{itk} + I_{i(t-1)k} + \sum_j T_{jitk} + \sum_l M_{itlk} = \delta_{itk} + I_{itk} + \sum_j T_{ijtk} + \sum_l M_{itkl} \quad (10)$$

The expansion of Equation 7 adds the term for the movement cost and movement variable. Also, the subscript for location is added to all of the costs and variable definitions. In Equation 10 (conservation of material constraint), the left-hand side (incoming) of the constraint adds the summation of the movements from all locations *l* to location *k* for the given period, while the right-hand side (outgoing) adds the summation of the movements from location *k* to all location *l* for the given period. A flowchart incorporating the movement expanded model can be seen in Figure 5.

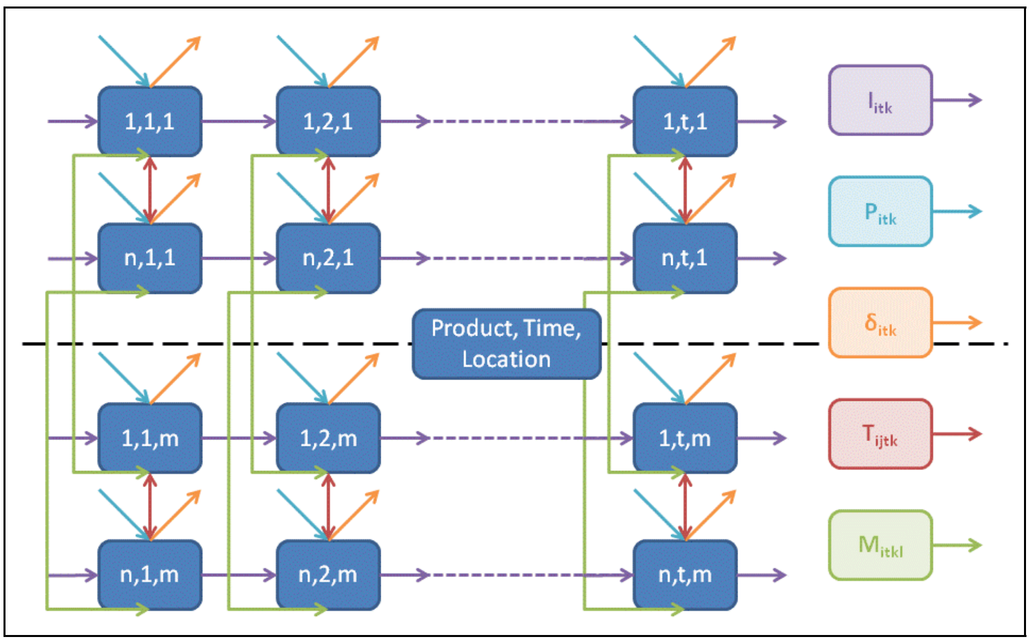

Figure 5. Movement Expansion

Multi-Level Product Expansion

Another possible expansion of this model would be to consider not only the finished products, but also the subassemblies that are used to build them. In order to evaluate such a model, the subassemblies would need their own cost constants for production/purchase, storage, movement, transformation (if applicable), and setup (if applicable). Demand for the subassemblies would be a function of the demand on the finished products. A simple flow chart showing finished products as compositions of subassemblies can be seen in Figure 6.

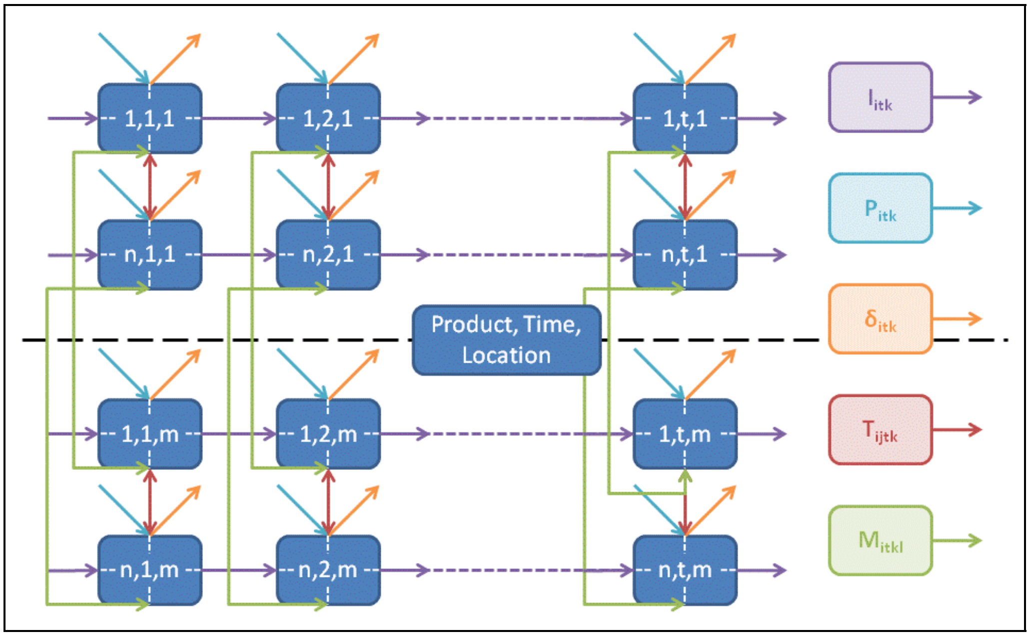

Figure 6. Subassembly Expansion

Expanded Model

The fully expanded model (not including the subassembly expansion) is as follows:

$$P_{itk} = \text{production of product } i \text{ in time } t \text{ at location } k$$

$$I_{itk} = \text{inventory carry} - \text{over of product } i \text{ in time } t \text{ at location } k$$

$$S_{itk} = \text{setup of production for product } i \text{ in period } t \text{ at location } k$$

$$T_{ijtk} = \text{transformation of product } i \text{ into } j \text{ in time } t \text{ at location } k$$

$$M_{itkl} = \text{move of product } i \text{ in time } t \text{ from location } k \text{ to location } l$$

$$\chi_{ijk} = \text{cost of producing } (i = j) \text{ or transforming } (i \neq j) \text{ products at location } k$$

$$\phi_{ik} = \text{cost of holding product } i \text{ at location } k$$

$$\sigma_{ik} = \text{setup cost of product } i \text{ at location } k$$

$$\rho_{ikl} = \text{cost of moving product } i \text{ from location } k \text{ to location } l$$

$$\delta_{itk} = \text{demand of product } i \text{ in time } t \text{ at location } k$$

$$Min\ Z = \left\{ \begin{matrix} \sum_i \sum_j \sum_t \sum_k (\chi_{ijk} T_{ijtk} | i \neq j) + \sum_i \sum_j \sum_t \sum_k (\chi_{ijk} P_{itk} | i = j) + \\ \sum_i \sum_t \sum_k (\phi_{ik} I_{itk}) + \sum_i \sum_t \sum_k \sum_l (\rho_{ikl} M_{itkl}) \end{matrix} \middle| \forall\, i, t, k \right\} \quad (11)$$

$$P_{itk} + I_{i(t-1)k} + \sum_j T_{jitk} + \sum_l M_{itlk} = \delta_{itk} + I_{itk} + \sum_j T_{ijtk} + \sum_l M_{itkl} \quad (12)$$

$$P_{itk} \leq S_{itk} * M \quad (13)$$

$$P_{itk}, T_{ijtk}, I_{itk}, M_{itkl} \geq 0 \quad (14)$$

$$P_{itk}, T_{ijtk}, I_{itk}, M_{itkt} = Integer \quad (15)$$

$$S_{itk} = Binary \quad (16)$$

As mentioned, it is possible to use commercial integer programming solvers, with appropriate reformulations, to attempt solution of this problem; research on this topic is ongoing.

Conclusion

This paper examines inventory costs in the context of defense operations. Based on the argument that inventory costs in defense operations are not the same as those in commercial enterprises, it is proposed that inventory effectiveness, in this context, should be measured in terms of the ability to meet a range of anticipated and sometimes unanticipated threats. This does not necessarily mean that planning can only be for “known knowns” and “known unknowns,” but not for “unknown unknowns.” Initial models have been developed to examine inventory decisions for complex products, that is, those composed of multiple subassemblies in which there are shared subassemblies among different product types. It is possible that the option for storing partially completed assemblies may also help in meeting demand uncertainties. Thus, when faced with uncertain demand for one or more products over a geographically distributed domain, the set of recourses for a manufacturer/planner include excess production (inventory storage), rapid re-location of inventory, production surges, when to upgrade technology or procure new models, what level of assembly to store the products, and where to store these, as well as in what quantities and ratios of product types. Solutions of mathematical models are illustrated, and simulations to assess the utility of the solutions obtained by analytical methods are also presented.

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