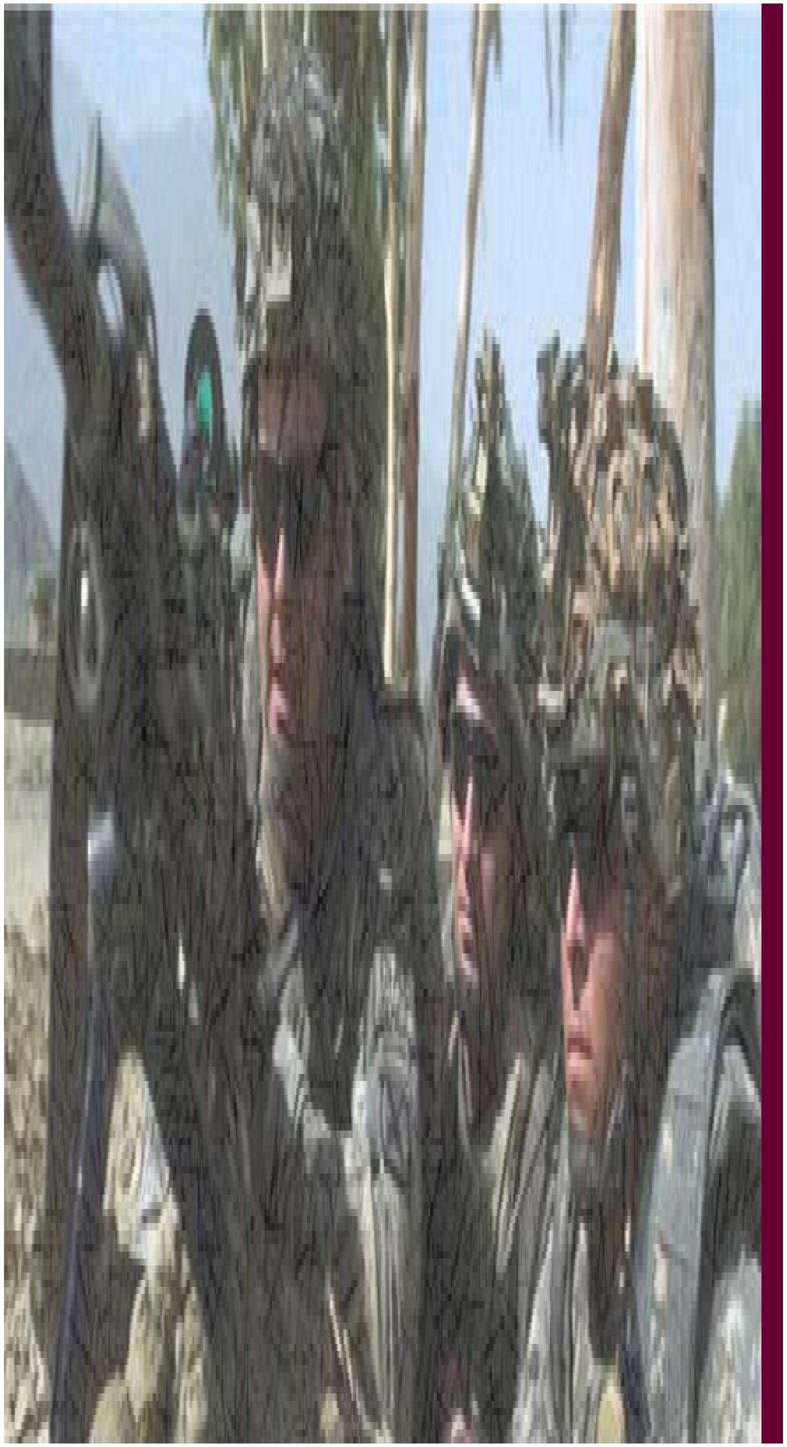

# Utilizing Statistical Inference to Guide Expectations and Test Structuring during Operational Testing and Evaluation

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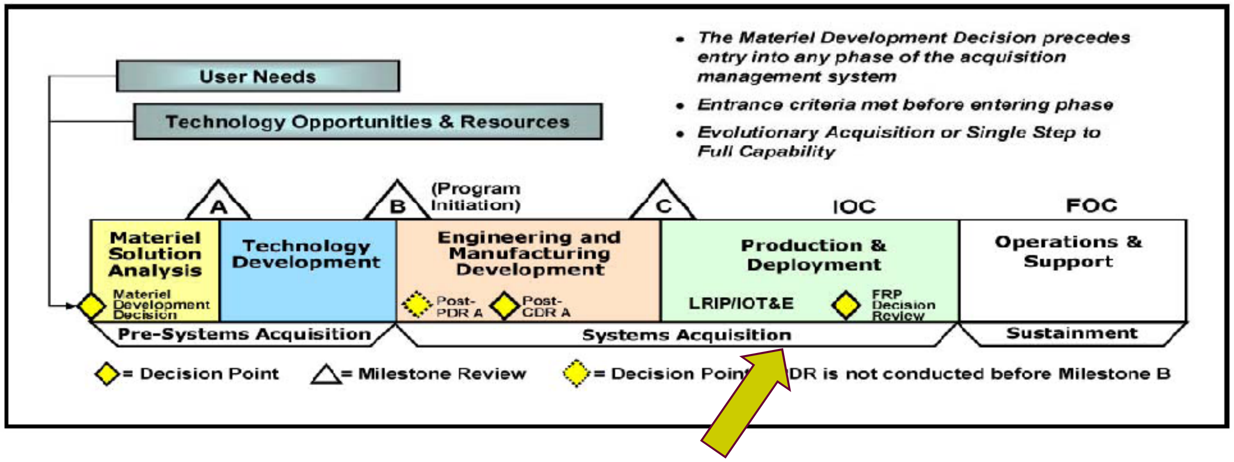
# Outline of Presentation

- Motivation
- Statistical Inference and Operational Testing
- Evaluating Potential Test Results
- Application to Situational Awareness System
- Summary

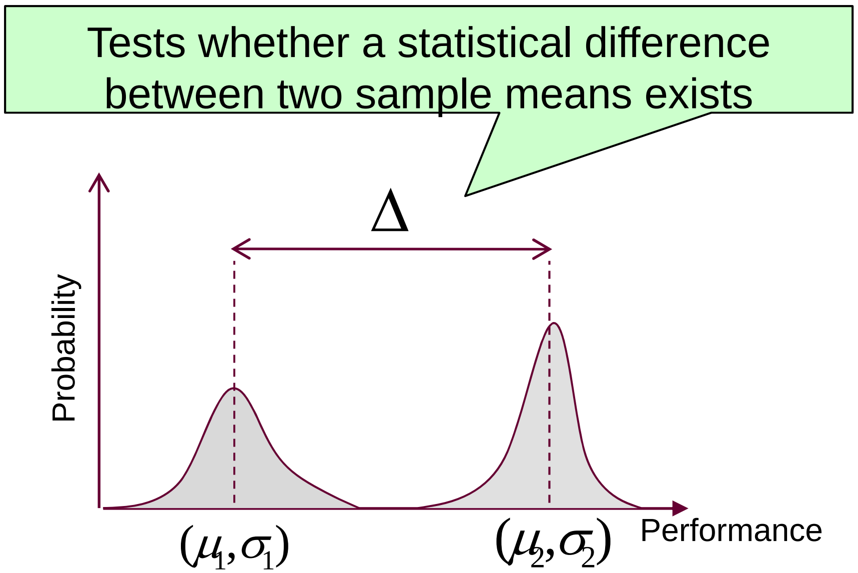
# Motivation




- Initial Operational Testing and Evaluation occurs during the Production & Deployment acquisition phase
- Congress requires testing of major weapons systems to be conducted under operationally realistic conditions to determine operational suitability
- Comparative tests are utilized during operational testing to baseline a system under test (SUT) through a series of tactical battles
  - Goal is to determine whether and by how much the unit's performance systematically improves with the SUT
  - Several approaches, both quantitative and qualitative, are used to assess a systematic improvement (e.g. statistical analysis and user evaluations)

# Statistical Inference

- Statistical inference noted as a best practice in system evaluation (CBASSE 1998)
- An applied statistical approach is often used to quantify and evaluate differences between treatment and control groups (Woolbridge 2003)
- In operational testing, statistical inference evaluates the performance difference between the SUT and the current status quo




| Interval/Ratio Data | |
|---|---|
| Two independent samples | **t-test**<br>z-test<br>single factor between subjects ANOVA |
| Two dependent samples | t-test<br>z test<br>single factor between subjects ANOVA |
| **Ordinal/Rank-Order Data** | |
| Two independent samples | Mann-Whitney U test<br>van der Waerden normal-scores test |
| Two dependent samples | Wilcoxon matched pairs signed-ranks test<br>**Binomial sign test** |
| **Categorical/Nominal Data** | |
| Two independent samples | Chi-square test<br>z-test |
| Two dependent samples | McNemar test<br>Gart test |

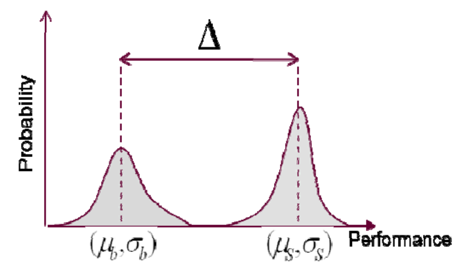
# Statistical Inference

1 **State Research Question**

Does use of the SUT improve the mean performance of a unit?

2 **Specify Null and Alternative Hypotheses**

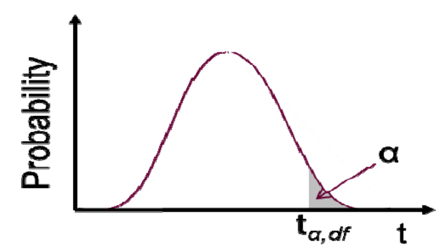
$$H_\phi : \mu_S = \mu_b$$
$$H_a : \mu_S > \mu_b$$




3 **Calculate Test Statistic**

$$t_{\alpha, df} = \frac{\overline{X}_S - \overline{X}_b}{\sqrt{\frac{s_S^2}{n_S} + \frac{s_b^2}{n_b}}}$$

4 **Compute Probability of Rejection**




5 **State Conclusions**

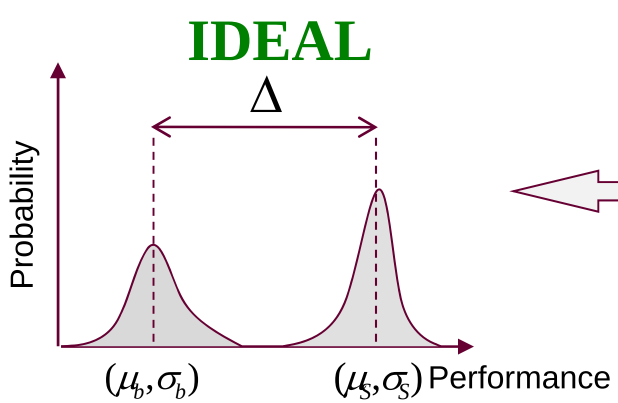
Did the SUT unit outperform the baseline unit statistically?

# Statistical Inference in OT&E

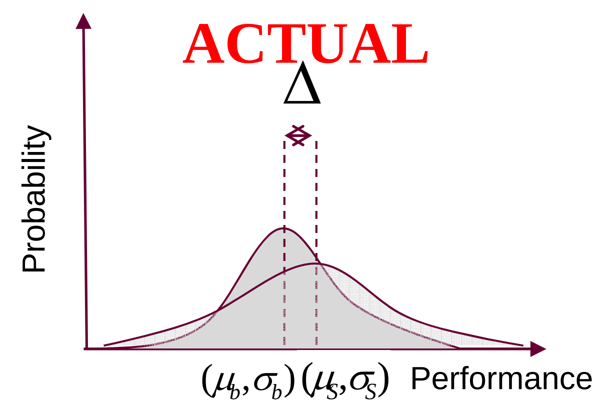
- Evaluated a Situational Awareness System as an effective tool against fratricide in 2001 (Edwards 2001)
  - **System Confidence Demonstration (SCD)**
    - No significant statistical difference between SUT and non-SUT units
    - Nearly impossible for SUT crew to statistically outperform baseline as baseline did so well
  - **Virtual Integration Exercise (VIE)**
    - Overall, no significant difference occurred in fratricide rates between baseline and SUT

Assessing the difference in performance mean between two independent samples

$$t_{\alpha,\nu} = \frac{\overline{X}_S - \overline{X}_b}{\sqrt{\frac{s_S^2}{n_S} + \frac{s_b^2}{n_b}}}$$




Small sample sizes and large variability lead to inconclusive results



# Evaluating Potential Test Results

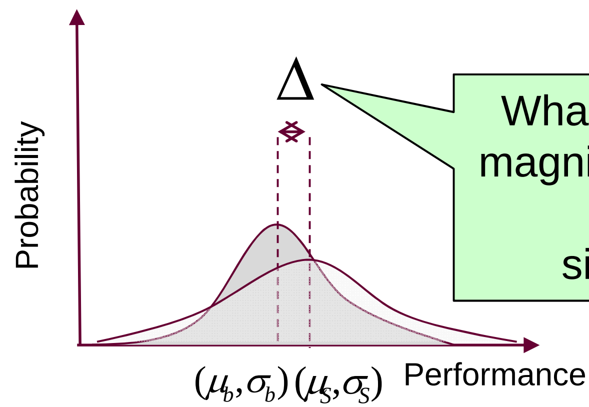
- Comparative tests are costly to administer and difficult to repeat
- Understand potential results a priori to guide expectations, test structuring and enable a more effective utilization of resources

1. What improvement in the mean performance is needed over the baseline to confidently assess whether there is a statistical difference?

2. Is the required performance of the unit needed to show a statistical difference reasonable?

$$\overline{X}_b = \overline{X}_S - t_{\alpha,\nu}\left(\sqrt{\frac{s_S^2}{n_S} + \frac{s_b^2}{n_b}}\right)$$

**ACTUAL**




What is the needed magnitude of delta for statistical significance?

# Guiding Expectations and Test Structuring

## Analysis of Systematic Difference

- Several approaches, both quantitative and qualitative, are used to assess a systematic improvement (e.g. statistical analysis and user evaluations)
- Statistical inference noted as a best practice in system evaluation (CBASSE 1998)

## Problems Experience in Previous Tests

- Evaluated a Situational Awareness System as an effective tool against fratricide in 2001 (Edwards 2001)
  - **System Confidence Demonstration (SCD)**
    - No significant statistical difference between SUT and non-SUT units
    - Nearly impossible for SUT crew to statistically outperform baseline as baseline did so well
  - **Virtual Integration Exercise (VIE)**
    - Overall, no significant difference occurred in fratricide rates between baseline and SUT

## Potential results of test a priori may:

- Provide guidance on the potential benefits of conducting test
- Provide guidance on structuring the test
- Lead to a more cost-effective test execution
- Provide maximal information given resources expended

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# Examination of the Force Effectiveness

- Operational needs statements from theater called for ground and aerial robotic capability to enable better situational awareness
- Evaluation of a SUT to improve the unit situational awareness on the battlefield
- Data on SUT performance gathered from its LUT 09
- Operational performance evaluation of a battalion with and without the SUT systems

| Mission | Mission Type | Success | BLUFOR Starting Strength | BLUFOR Casualties | OPFOR Starting Strength | OPFOR Casualties |
|---|---|---|---|---|---|---|
| 1 | Raid | yes | 130 | 10 | 50 | 26 |
| 2 | Raid | yes | 130 | 7 | 50 | 25 |
| 3 | Defend | yes | 130 | 25 | 50 | 0 |
| 4 | Attack | yes | 130 | 15 | 50 | 10 |
| 5 | Attack | yes | 130 | 25 | 50 | 8 |
| 6 | Cordon and Search | yes | 130 | 8 | 50 | 7 |
| 7 | Defend | yes | 130 | 16 | 50 | 15 |
| 8 | Cordon and Search | yes | 130 | 12 | 50 | 6 |
| 9 | Raid | partially | 130 | 7 | 50 | 3 |
| 10 | Cordon and Search | yes | 130 | 20 | 50 | 8 |
| 11 | Attack | no | 130 | 14 | 50 | 10 |
| 12 | Stability Operations | yes | 130 | 2 | 50 | 5 |
| 13 | Raid | yes | 130 | 10 | 50 | 22 |

# Performance Metrics of Interest

1. **Missions Not Accomplished** — Considered missions which had a conclusive result

2. **Mission Success Rate**

$$MSR = \frac{Number\ of\ Missions\ Accomplished}{Total\ Missions\ Conducted}$$

3. **BLUFOR Casualty Rate**

$$B\ Casualty\ Rate = \frac{BLUFOR\ Losses}{BLUFOR\ Starting\ Strength}$$

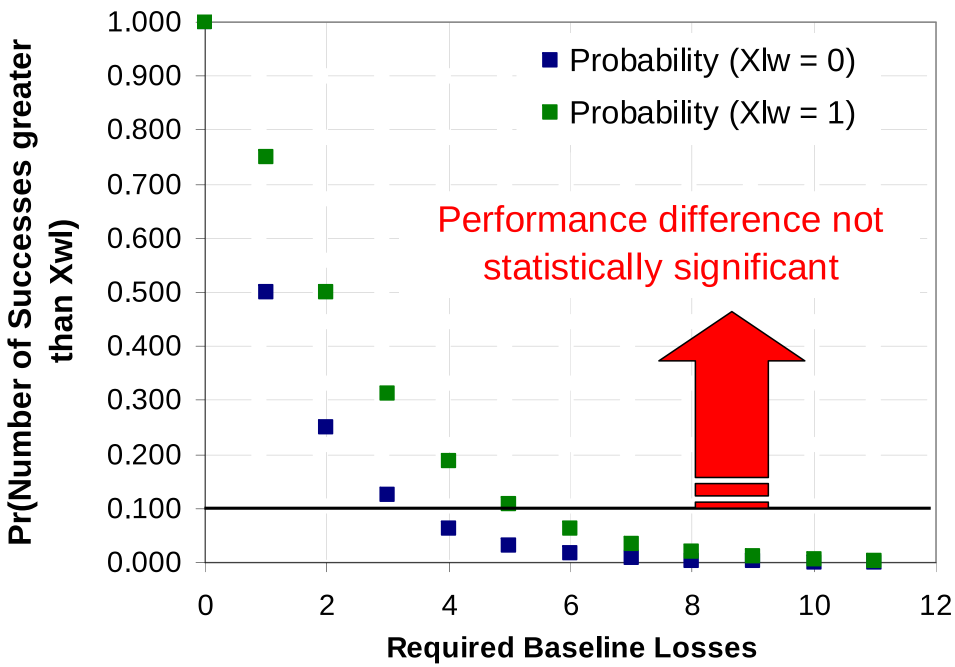
4. **OPFOR Casualty Rate**

$$O\ Casualty\ Rate = \frac{OPFOR\ Losses}{OPFOR\ Starting\ Strength}$$

5. **BLUFOR Fratricide Rate**

$$Fratricide\ Rate = \frac{BLUFOR\ Fratricides}{BLUFOR\ Losses}$$

# Missions Not Accomplished




Xwl – Number of missions accomplished by SUT unit but not baseline unit

Xlw – Number of missions accomplished by baseline unit but not SUT unit

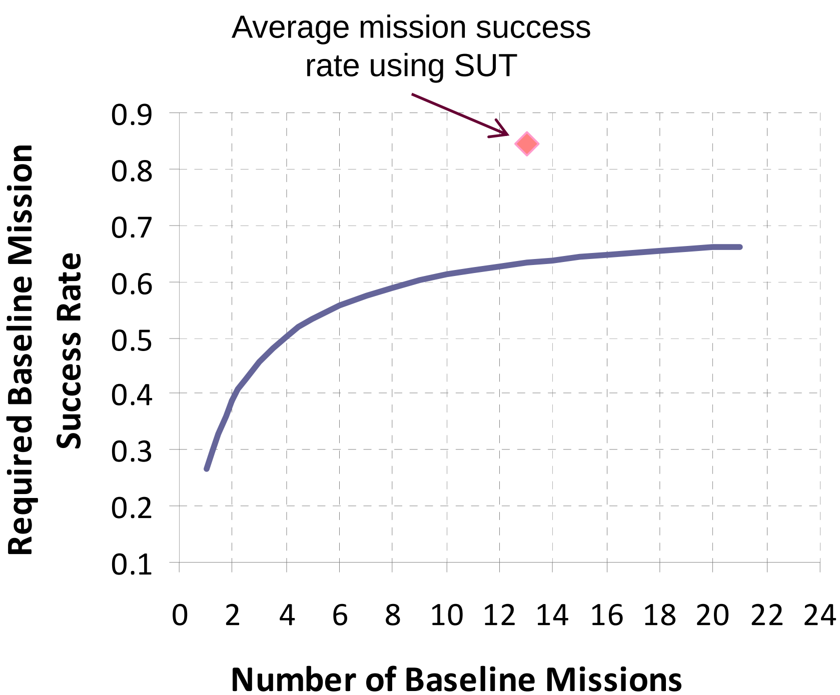
- Comparative evaluation using binomial sign test at 90% confidence level (Sheskin 2004)
- Given the results of the LUT 09, the baseline unit would have to lose 4 or more missions to statistically underperform the SUT unit

- Given the starting strength ratio of 2:1, it is unlikely the baseline unit will lose 4 missions
- Modify test structure to use a lower starting strength ratio

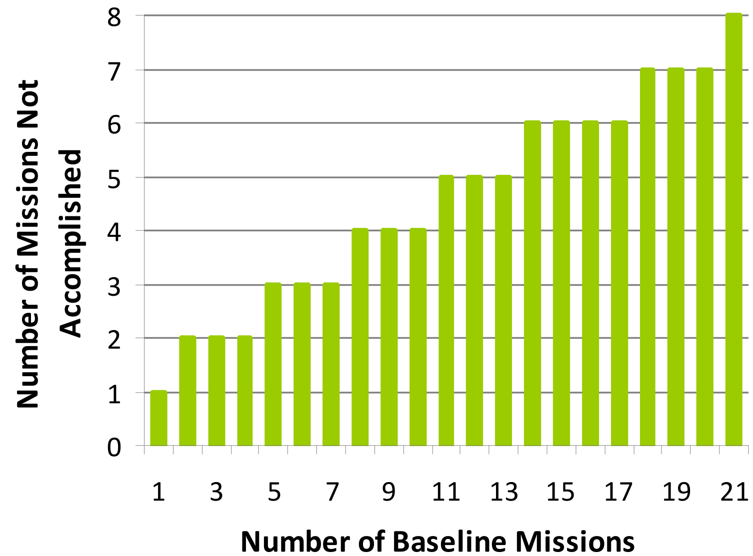
# Mission Success Rate

- Comparative evaluation using two proportion z-test at 90% confidence level

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}}$$




- Given an expected 13 baseline missions to be conducted, the required performance of the baseline unit is a maximum mission success rate of 63%



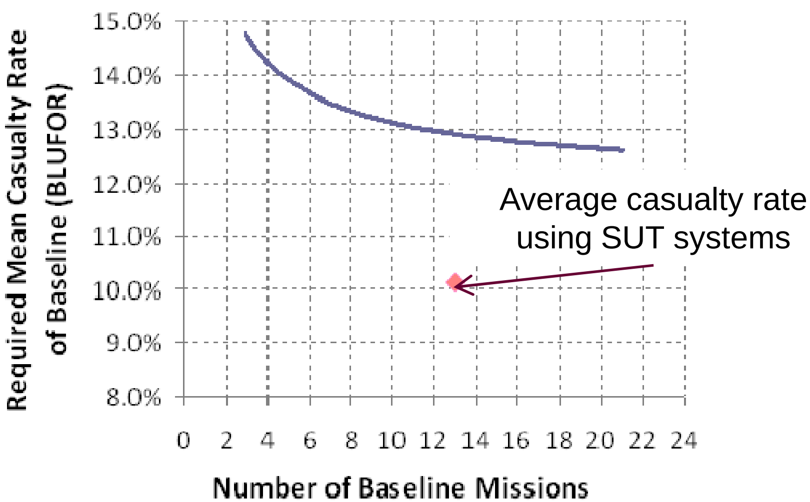

- Given the starting strength ratio of 2:1, it is unlikely that a 63% mission success rate will be observed
- Modify test structure to use a lower starting strength ratio

# Casualty Rates

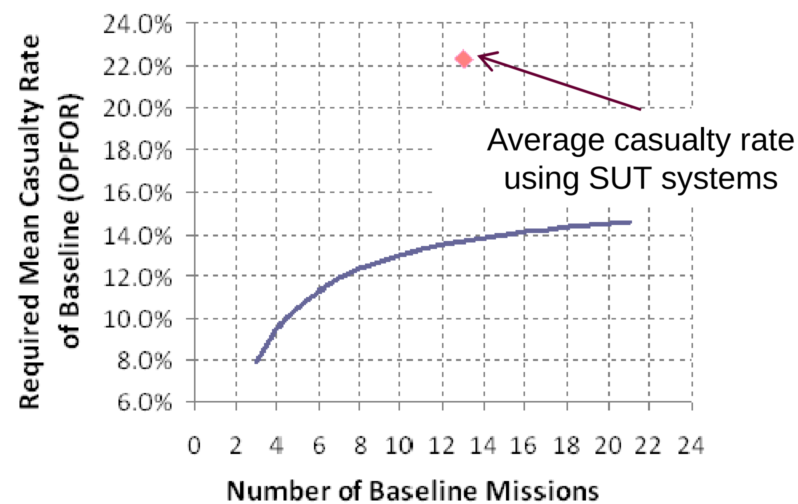
- Comparative evaluation using t-test at 90% confidence level (Sheskin 2004)

$$t_{obt} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

- Assume variability is the same for both baseline and SUT unit




- Given an expected 13 baseline missions to be conducted:
  - Minimum required BLUFOR rate is 12.9%
  - Maximum required OPFOR rate is 13.7%




- Typical observed BLUFOR and OPFOR rates are around 10% and 25% respectively
- Possible to observe positive impact of SUT on BLUFOR rate, but highly unlikely for OPFOR rate

# BLUFOR Fratricide Rate

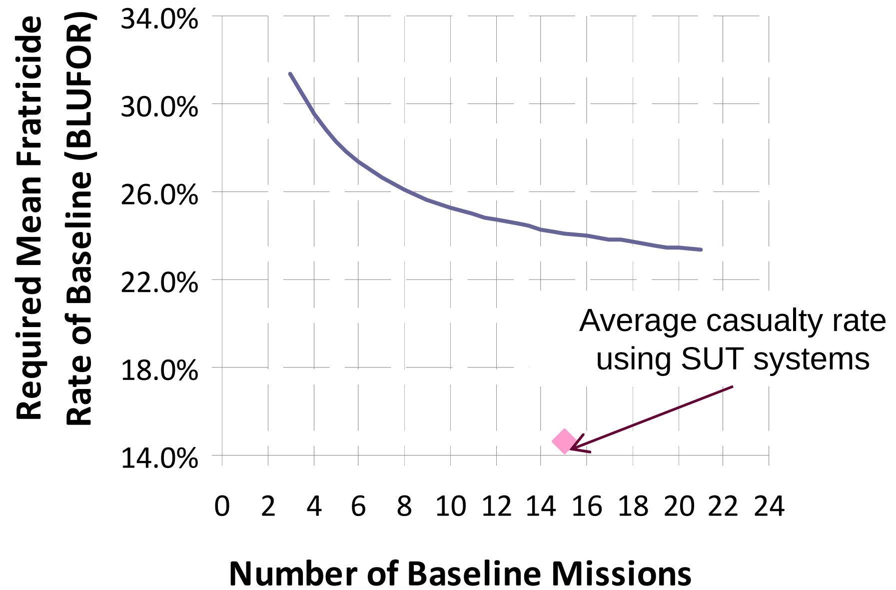
- Comparative evaluation using t-test at 90% confidence level (Sheskin 2004)

$$t_{df} = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

- Assume variability is the same for both baseline and SUT unit

- Observed BLUFOR fratricides rates are around 13% (Gadsden & Outteridge 2006)
- Highly unlikely to observe significant performance difference between the two units

- Given an expected 13 baseline missions to be conducted, minimum required BLUFOR fratricide rate is 25%



# Sensitivity Analysis

- Analysis predicated on a number of assumptions
  - Variability in performance measures is identical for the SUT and baseline unit
  - 90% confidence interval is the more appropriate confidence interval for the analysis
  - Performance of SUT unit in LUT 09 is representative of future performance in subsequent OT&E

Required casualty rates and mission success metrics are consistent with observed values

Required improved performance of SUT raised concerns about being able to provide conclusive results in a comparative test

| | | Required Values for Statistical Significance in IOT&E | | | |
|---|---|---|---|---|---|
| **Metrics** | **Observed LUT 09** | **Initial Results** | **50% Variability Reduction** | **Confidence Level = 80%** | **SUT** |
| **Missions Not Accomplished** | 1 | 4-6 | N/A | 4 | -- |
| **Mission Success Rate** | 0.85 | 63.2% | N/A | 71.1% | 98.2% |
| **BLUFOR Casualty Rate** | 10.1% | 12.9% | 12.1% | 11.9% | 4.7% |
| **OPFOR Casualty Rate** | 22.3% | 13.7% | 16.2% | 16.7% | 31.0% |
| **BLUFOR Fratricide Rate** | 14.6% | 24.5% | 21.6% | 21.2% | 7.3% |

Required fratricide rate remains high

Required OPFOR casualty rate remains low

# Summary

- Using statistical inference insight may be gained about possible outcomes of comparative tests
  - Guide expectations
  - Point to areas where test may need restructuring
  - Enable a more effective utilization of resources

- For case study, it is likely that a comparative evaluation of these quantitative metrics will lead to statistically inconclusive results as performance requirements are high
  - Possible restructuring of test needed
  - Given current performance of SUT, a comparative test may not be an effective utilization of limited resources

- Extend analysis to qualitative measures of operational effectiveness which are gathered from surveys and interviews

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